Holographic Meissner effect

Makoto Natsuume (KEK)

arXiv: 2207.07182 [hep-th]
work in progress

w/ Takashi Okamura (Kwansei Gakuin Univ.)

2023/3 Univ. of Alabama

Introduction

AdS/CFT (holographic duality): useful tool to study "real world"

It has been applied to various areas:

- Quark-gluon plasma/QCD
- Non-equilibrium physics (hydrodynamics)
- Non-linear physics (turbulence, chaos)
- Condensed-matter physics
- quantum information

I would like to talk about a CMP application: holographic superconductors.

AdS/CFT

Finite temperature gauge theory = Gravitational theory
at strong coupling (large-Nc limit)   in AdS black hole

⇩ thermal

⇩ thermal due to the Hawking radiation

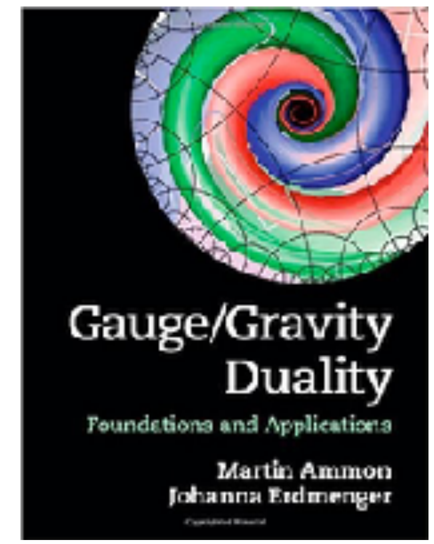
4d spacetime "Boundary"

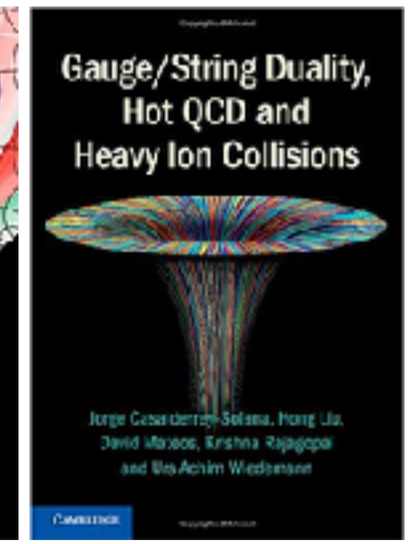
5d spacetime "Bulk"

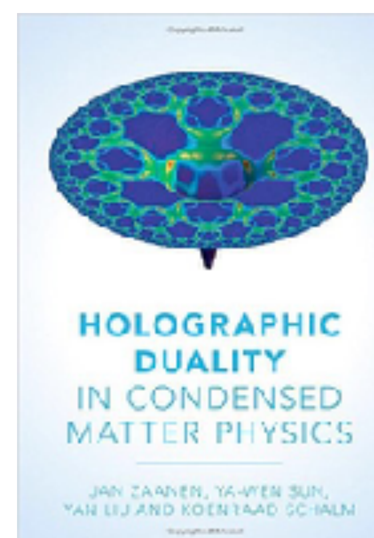
"Holographic duality"

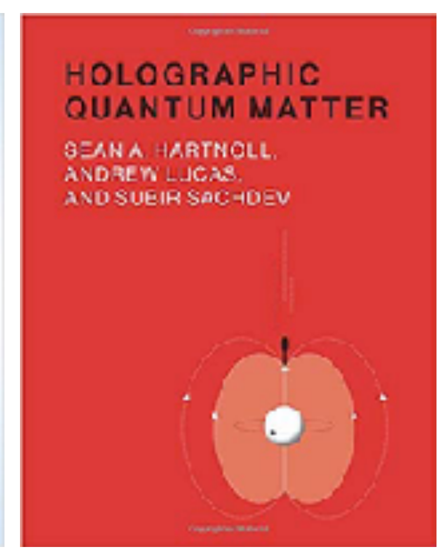
Many textbooks available by now

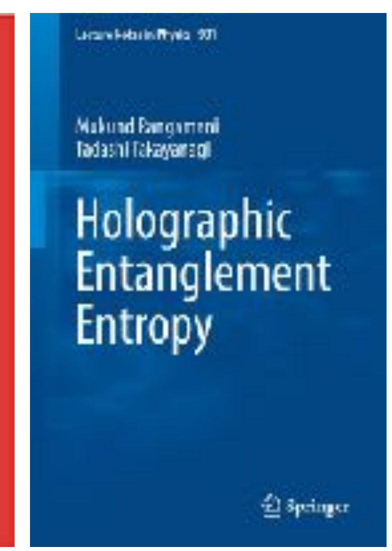
- *"AdS/CFT duality user guide"* (2015)
- *"Gauge/Gravity duality: foundations and applications"* (2015)
- *"Gauge/String duality, hot QCD and heavy ion collisions"* (2014)
- *"Holographic duality in condensed matter physics"* (2015)
- *"Holographic quantum matter"* (2018)
- *"Holographic entanglement entropy"* (2017)









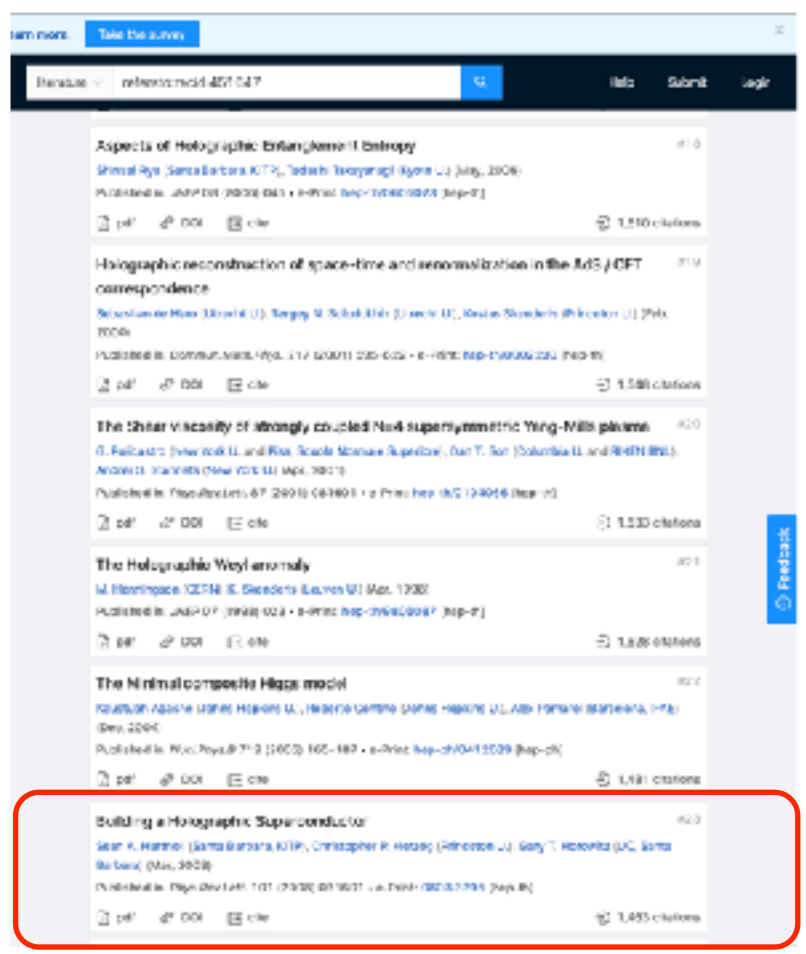


Introduction

In AdS/CFT, there is a system called holographic superconductor (HSC) which is dual to a superconductor.

This system has been studied extensively. For example, here is the list of citations of Maldacena's original paper (presumably, it covers most of AdS/CFT papers).

HSC is #23



Introduction

Although HSC was extensively studied, 2 important issues remain:

- First one is Meissner effect, which is the characteristic feature of superconductivity.
  - In conventional HSCs, Meissner effect was rarely discussed.
  - The reason is simple: in conventional HSCs, there is no Meissner effect!
  - I explain how to implement the Meissner effect in HSCs and show the effect analytically.
- Second one is the dual Ginzburg-Landau (GL) theory. We identify the GL theory analytically (for the bulk 5-dim case).

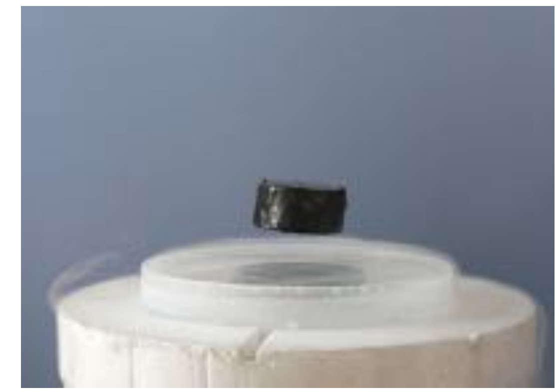
Plan

- Review: Superconductivity & Ginzburg-Landau theory
- Review: Holographic superconductors
- Holographic Meissner effect (bulk 4-dim)
- The bulk 5-dim. case and the dual GL theory

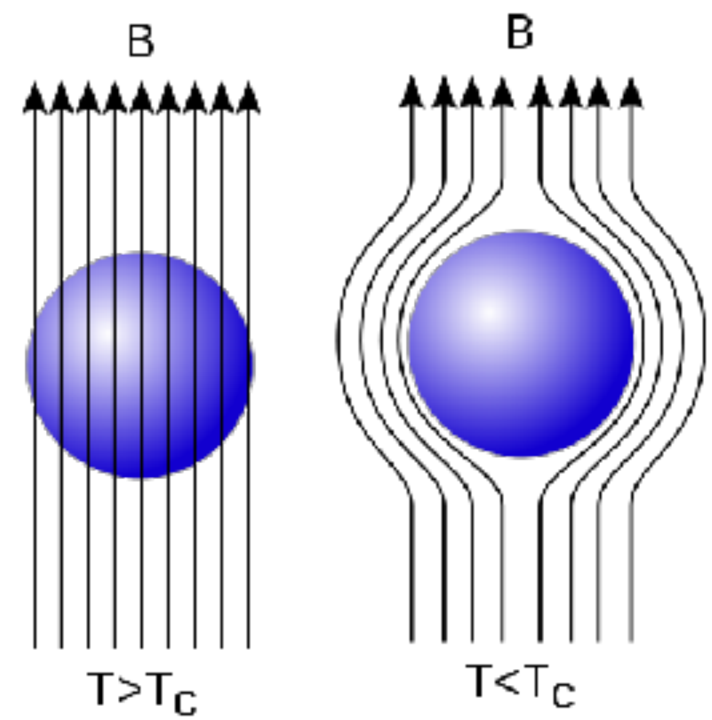
Superconductivity

2 characteristic features of SC:

- Zero resistivity/diverging conductivity
- Meissner effect:
  Magnetic field cannot enter the material



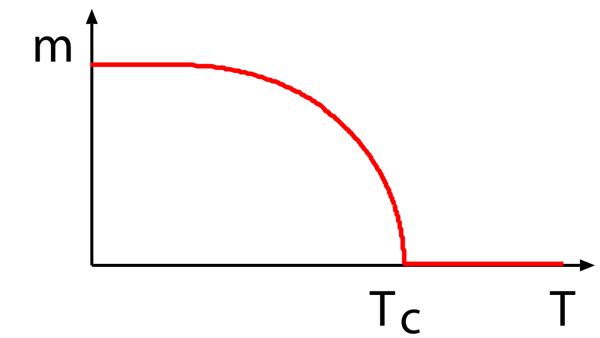
Phenomenologically,
Ginzburg-Landau theory describes SC well.



Wikipedia

Ferromagnet

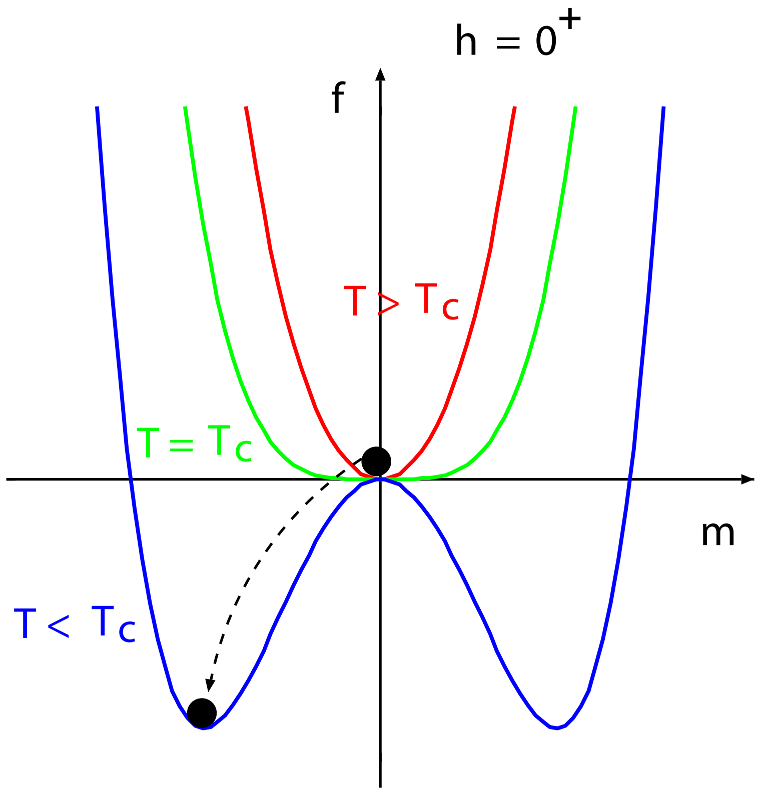
Spontaneous magnetization m ≠ 0 for $T < T_c$



The GL theory:

$$f = \frac{1}{2} a m^2 + \frac{1}{4} b m^4 + \cdots - mh$$

$$a = a_0 (T - T_c) + \cdots$$



effective theory near critical pt (small m)
Familiar Higgs-like potential
Mass term is proportional to temperature

- m = 0 for $T > T_c$
- m ≠ 0 for $T < T_c$ and SSB $m^2 = -\frac{a}{b}$

GL theory of superconductivity

For SCs,

$$f = |(\partial_i - iA_i)\psi|^2 + a|\psi|^2 + \frac{b}{2}|\psi|^4 + \frac{1}{4}F_{ij}^2$$

- Difference from the previous one
  - ψ is complex "macroscopic wave function"
  - The system is coupled w/ Maxwell field

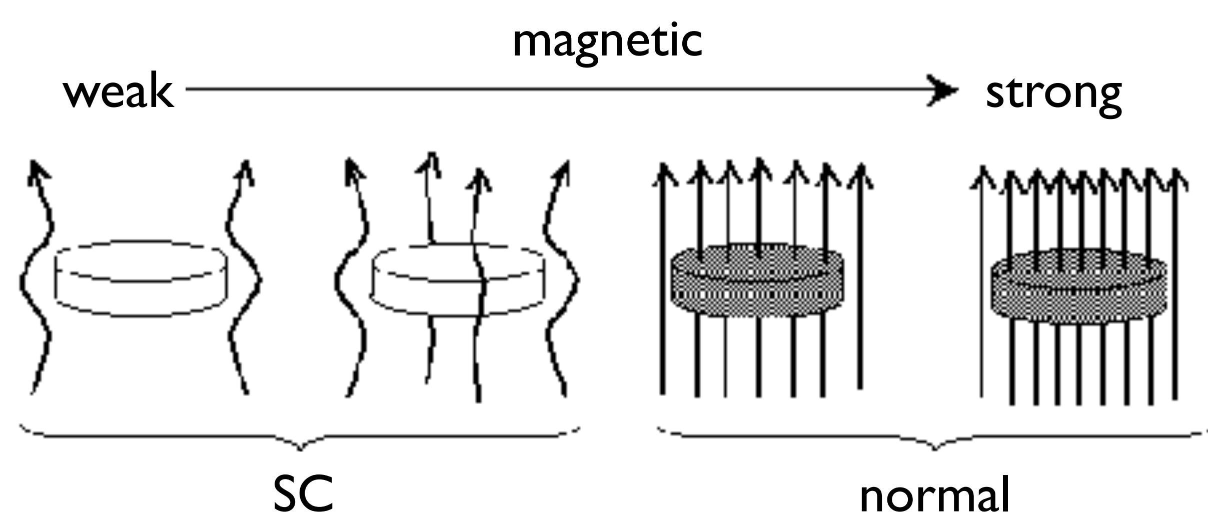
At low temperature, $\psi \neq 0$, and Maxwell field becomes massive (just like Higgs mechanism)

→ Meissner effect
  Magnetic field cannot enter the SC.

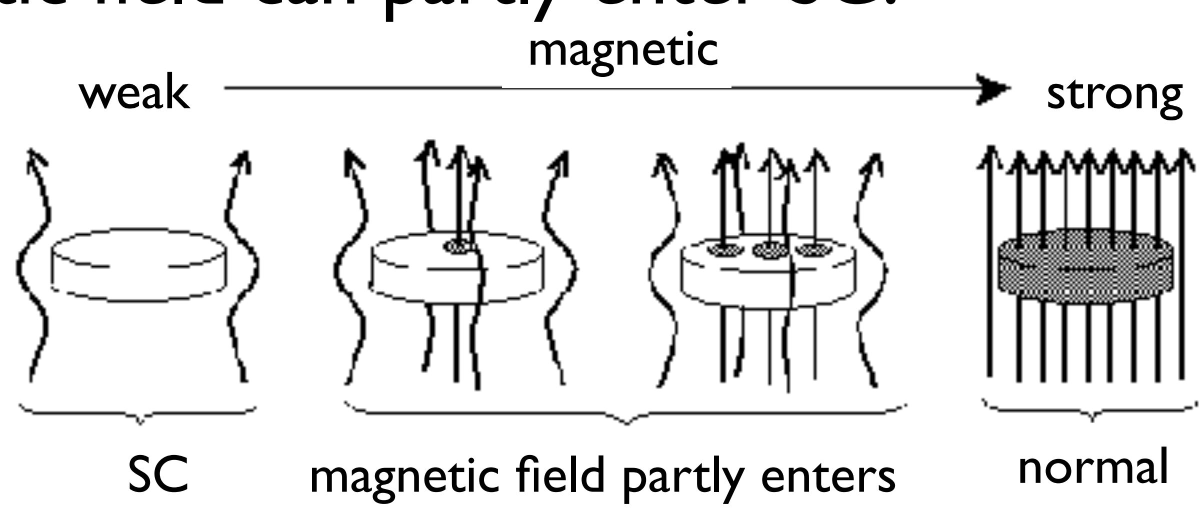
Type I & II

There are 2 kinds of SCs:

- Type I: magnetic field is completely expelled. The SC state is broken at high enough magnetic field.



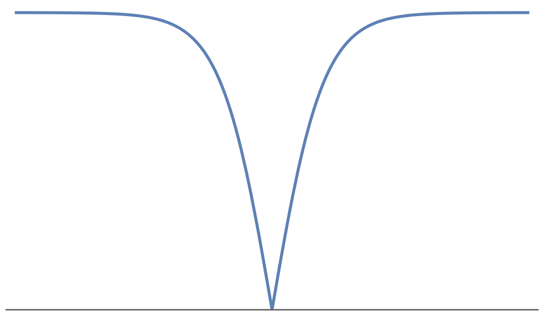
- Type II: magnetic field can partly enter SC.



Wikipedia

In Type II SCs, magnetic field can enter SC keeping SC state.

The magnetic field enters by forming vortices.
ψ =0 at vortex core and magnetic field enters there.



According to GL theory,

$$\nabla_j F^{ij} = -2e^2 |\psi|^2 A^i$$

→ supercurrent (diamagnetic)

$$A_\phi \propto \sqrt{r} e^{-r/\lambda}$$

$$\lambda^2 = \frac{1}{2e^2 |\psi|^2}$$

This is one way to see the Meissner effect.

We see analogous expression in HSC, but this is impossible in the standard HSC.

- The vortices create supercurrent from London eq: $J = -2|\psi|^2 A$
- The Maxwell eq. or the Ampere law $\nabla \times B = J$ then tells the magnetic field is induced, and compensates the external magnetic field.

As we see below, in standard HSCs, the London eq holds, but there is no Maxwell eq on bdy: Maxwell field on bdy is not dynamical.
The magnetic field can always enter and no Meissner effect. In a sense, the standard HSC is an "extreme" type II.

Plan

- Review: Superconductivity & Ginzburg-Landau theory
- Review: Holographic superconductors
- Holographic Meissner effect (bulk 4-dim)
- The bulk 5-dim. case and the dual GL theory

Holographic superconductors

Typically, Einstein-Maxwell-complex scalar system:

$$\mathcal{L} = \sqrt{-g}\left[R - 2\Lambda - \frac{1}{g^2}\left(F_{MN}^2 + \left|\nabla_M \psi - iA_M \psi\right|^2 + m^2 |\psi|^2 \right)\right]$$

Hartnoll - Herzog - Horowitz, 0803.3295; 0810.1563
Gubser, 0801.2977

M,N...: bulk indices
μ,ν...: bdy indices

- Phase structure
  - T>Tc: AdS BH w/ ψ = 0
  - T<Tc: AdS BH w/ ψ ≠ 0 → ψ: order parameter
    ~ dual to "macroscopic wave fn"
- Dual to some kind of superconductors → diverging conductivity

Setup

$$\mathcal{L} = \sqrt{-g}\left[R - 2\Lambda - \frac{1}{g^2}\left(F_{MN}^2 + \left|\nabla_M \psi - iA_M \psi\right|^2 + m^2 |\psi|^2 \right) \right]$$

Matter fields are coupled w/ gravity, and the system is hard to solve. So, we employ the "probe approx" $g \gg 1$, where matter fields are decoupled from gravity.

Then, one can simply use pure gravity solution (Schwarzschild-AdS BH) and solve matter fields in the background.

SAdS4 BH

We first consider the 4-dim bulk (for simplicity):

$$ds_4^2 = r^2(-fdt^2 + dx^2 + dy^2) + \frac{dr^2}{r^2 f}$$

$$= \frac{1}{u^2}(-fdt^2 + dx^2 + dy^2) + \frac{du^2}{u^2 f}$$

AdS radius: $L = 1$

$f = 1 - r^{-3} = 1 - u^3$ horizon radius: $r_0 = 1$

$u = 1/r$

$r = u = 1$: horizon

$u = 0, r = \infty$: asymptotic infinity, "boundary"

Hawking temp: $4\pi T = 3r_0$

BH: only T as scale → no characteristic T/no phase transition
→ chemical potential μ

In high temp. phase, $$A_t = \mu(1-u)$$

System parametrized by μ/T. We fix T and vary μ.

- μ/T < (const) → Normal phase
- μ/T > (const) → SC phase

Plan

- Review: Superconductivity & Ginzburg-Landau theory
- Review: Holographic superconductors
- Holographic Meissner effect (bulk 4-dim)
- The bulk 5-dim. case and the dual GL theory

Bdy. Maxwell

We have the bulk Maxwell field A_M, but normally the bdy Maxwell field $\mathcal{A}_\mu$ plays the role of an external source or a background and is not dynamical.

e.g. chemical potential μ

In other words, the bdy Maxwell has the action but no Maxwell action.

$$\delta S = \int d^3x\, \mathcal{A}_\mu \left\langle J^\mu \right\rangle$$

Here $\langle J^\mu \rangle$ is bdy U(1) current computed by standard AdS/CFT recipes.

So, normally one cannot discuss dynamical U(1) in holography. One cannot discuss the Meissner effect…

If there is none, let's add it!

$$S_{bdy} = -\frac{1}{4e^2}\int d^3x \mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu}$$

Then, w/ the source term, $\delta S = \int d^3x\, \mathcal{A}_\mu \left\langle J^\mu \right\rangle$ one gets the bdy Maxwell eq:

$$\frac{1}{e^2}\partial_\nu \mathcal{F}^{\mu\nu} = \left\langle J^\mu \right\rangle$$

All quantities are bdy ones including U(1) coupling *e*.

We call it "holographic semiclassical eq."

AdS/CFT dictionary

In the language of holography, the issue is the choice of the BC.

Compere - Marolf, 0805.1902

No dynamical U(1) because we usually impose the Dirichlet BC on the bdy.

Here is the standard dictionary:

- Solve the bulk EOM (e.g. A_M)
- Extract u→0 behavior, one gets

$$A_\mu \sim \mathcal{A}_\mu + \left\langle J^\mu \right\rangle u + \cdots \quad (u \to 0)$$

$\mathcal{A}_\mu$: bdy Maxwell

J^μ : bdy U(1) current

fixed: Dirichlet ↓

or

$$A_\mu\Big|_{bdy} = \mathcal{A}_\mu$$

Under the Dirichlet BC, we fix the bdy U(1), and U(1) is just a background.

So, change the BC.
Here is the procedure:

- Solve the bulk EOM and obtain e.g. $\left\langle J^{\mu} \right\rangle$ by standard AdS/CFT recipes.
- But impose the semiclassical eq as BC instead of the Dirichlet BC.

$$\frac{1}{e^2}\partial_\nu \mathcal{F}^{\mu\nu} = \left\langle J^{\mu} \right\rangle$$

We show Meissner effect analytically by imposing the holographic semiclassical eq. on holographic superconductors.

The argument is very simple, so we look at the details.

Example

Probably you are not familiar to the procedure, so let me explain more using the well-known system. Consider the bulk 5-dim.

- Pure gravity: dual to $\mathcal{N} = 4$ SYM ~ QGP

$$\mathcal{L} = \sqrt{-g}\,\left[R - 2\Lambda\right]$$

- Einstein-Maxwell theory: $\mathcal{N} = 4$ SYM + background U(1) (at finite chemical potential if A_t)

$$\mathcal{L} = \sqrt{-g}\,\left[R - 2\Lambda - F_{MN}^2\right]$$

- w/ semiclassical eq BC:

$\mathcal{N} = 4$ SYM+ dynamical U(1) ~ QGP + photon

But we do not really have QGP in mind. Instead, we consider HSC.

Dirichlet BC case

First, let us check that standard HSC has no Meissner effect.

For T<Tc, a uniform condensate ψ is a solution, so add a small magnetic field there.

$$A_y = Y(u,x) \propto e^{iqx}$$

Bulk EOM: $$0 = \left\{ -\partial_u (f\partial_u) + q^2 + 2|\varphi_0|^2 \right\} Y, \quad \psi = u\varphi$$

One can integrate the eq formally:

$$Y = \mathcal{Y} - \int_0^u \frac{du'}{f(u')} \int_{u'}^1 du''(q^2 + 2|\varphi_0|^2) Y$$

$$= \mathcal{Y} \left\{ 1 - \int_0^u \frac{du'}{f(u')} \int_{u'}^1 du''(q^2 + 2|\varphi_0|^2) + \cdots \right\}$$

$\mathcal{Y}$: bdy value

$$Y = \mathcal{Y}\left\{1 - \int_0^u \frac{du'}{f(u')} \int_{u'}^1 du''(q^2 + 2|\varphi_0|^2) + \cdots\right\}$$

- The 1st term reps. the magnetic field.
  Imposing the Dirichlet BC means to fix $\mathcal{Y}$. Then, by construction, $B \neq 0$.
  The magnetic field can enter SC (no matter how small) and no Meissner effect.

$$B = \partial_x Y\big|_{u=0} = iq\mathcal{Y}$$

- The 2nd term reps. current.
  According to the standard AdS/CFT recipe,

$$\begin{aligned}\langle J_y \rangle &= \partial_u Y\big|_{u=0} \\ &= \mathcal{Y}\left(-q^2 - \int_0^1 du\, 2|\varphi_0|^2 + \cdots\right)\end{aligned}$$

$$\langle J_y \rangle = \mathcal{Y}\left(-q^2 - \int_0^1 du\, 2\,|\,\varphi_0\,|^2 + \cdots \right)$$

normal current↓ (diamagnetic)

↓supercurrent (diamagnetic)

This is London eq. w/ added normal component. $J = -2\,|\,\psi\,|^2 A$

- Supercurrent itself exists, but there is no Ampere law on the bdy $\nabla \times B = J$, so the magnetic field does not decrease and no Meissner effect.

- The 1st term exists even for pure bulk Maxwell theory. This is interpreted as the magnetization current due to magnetization.

Holographic semiclassical eq case

We now impose holographic semiclassical eq as the BC:

$$\partial_j \mathcal{F}^{ij} = e^2 \left\langle J^i \right\rangle$$

What is the role of the normal current?
Consider only normal current & external current. The semiclassical eq gives

$$\left\langle J_y \right\rangle = \mathcal{Y}\left(-q^2 - \int_0^1 du\, 2 |\varphi_0|^2 \right)$$

$A_y = Y e^{iqx}$

$$q^2 \mathcal{Y} = -e^2 q^2 \mathcal{Y} + e^2 J_{\text{ext}}$$

$$q^2 \mathcal{Y} = \frac{e^2}{1+e^2} J_{\text{ext}}$$

cf. $\nabla \times B = \mu_m J$

It shifts magnetic const. from $\mu_0 = e^2$ to $\mu_m = e^2/(1+e^2)$

Magnetization current

Recall elementary EM.
In a material or a medium, magnetic moment produces magnetization M and steady magnetiz. current Jm.

$$\nabla \times B = \mu_0 (J_{\rm ext} + J_m) \\ = \mu_0 (J_{\rm ext} + \nabla \times M)$$

At linear order

$$M = \frac{1}{\mu_0} \frac{\chi}{1+\chi} B$$

So

$$\nabla \times B = \mu_0 (1+\chi) J_{\rm ext} =: \mu_m J_{\rm ext}$$

χ : magnetic susceptibility
μ_m : magnetic const.

The magnetic const reps. a medium effect. For the bulk 5-dim case, the medium is $\mathcal{N} = 4$ SYM and magnetic const comes from there.

Now include the supercurrent as well. w/ a delta-fn source,

$$q^2 \mathcal{Y} = -e^2(q^2 + 2I)\mathcal{Y} + e^2 \qquad I = \int_0^1 du\, |\varphi_0|^2$$

$$\mathcal{Y} \propto \frac{1}{(1+e^2)q^2 + 2e^2 I} \to e^{-x/\lambda}$$

w/ the magnetic penetration length:

$$\lambda^2 = \frac{1}{2\mu_m I} = \frac{1+e^2}{2e^2 I}$$

cf. GL:

$$\lambda_{\mathrm{GL}}^2 = \frac{1}{2e^2 |\psi|^2}$$

- At weak coupling e<<1, holographic result reduces to the GL result.
- In the limit e→∞, GL implies λ=0. Strong Meissner. "extreme type I"
- For HSC, λ remains finite. "extreme type I" cannot be reached.

GL parameter

SC has 2 characteristic length scales:

- Magnetic penetration length λ (gauge field A_i mass) → W-boson mass $\lambda_{\rm GL}^2 = \frac{1}{2e^2|\psi|^2} = \frac{1}{2e^2}\frac{b}{|T-T_c|}$
- Correlation length ξ (order parameter ψ mass) → Higgs mass $\xi_{\rm GL}^2 = \frac{1}{|T-T_c|}$

Then, SC is characterized by a dimensionless parameter, GL parameter:

$$\kappa^2 = \frac{\lambda^2}{\xi^2} \qquad \kappa_{\rm GL}^2 = \frac{b}{2e^2}$$

Whether SC is type I or II depends on κ:

$\kappa^2 < 1/2$: Type I

$\kappa^2 > 1/2$: Type II→longer λ so magnetic penetration becomes possible

In the bulk 4-dim, the analytic expression is not possible for κ, but in the bulk 5-dim, an analytic expression is possible.

Plan

- Review: Superconductivity & Ginzburg-Landau theory
- Review: Holographic superconductors
- Holographic Meissner effect (bulk 4-dim)
- The bulk 5-dim. case and the dual GL theory

SAdS5 case

In bulk 5-dim, there exits a simple analytic solution for scalar mass m[2]=-4

BF bound

$$\varphi_0 = \sqrt{\frac{24(\mu-\mu_c)}{\pi T}}\frac{u}{1+u^2}$$

mean-field

w/ critical temperature $(\mu/T)_c = 2\pi$

Herzog, 1003.3278
Natsuume - Okamura, 1801.03154

The solution is a special case of a 1-parameter family of analytic solutions for "holographic Lifshitz superconductors."

Then, you can compute everything explicitly.
I omit the technical details and quote only the results.

SAdS5 case

In this case, one can evaluate I:

$$I = \int_0^1 du \frac{|\varphi_0|^2}{u} = \frac{6(\mu - \mu_c)}{\pi T}$$

$$\to \lambda^2 = \frac{1}{2\mu_m (\pi T)^2 I} = \frac{1}{12\mu_m} \frac{1}{(\mu - \mu_c)\pi T}$$

In bulk 5-dim., magnetic const:

$$\mu_m = \frac{e^2}{1 - e^2 \ln(\pi T)}$$

The correlation length:

$$\xi^2 = \frac{1}{2(\mu - \mu_c)\pi T}$$

from scalar QNM computation

Then GL parameter:

$$\kappa^2 = \frac{\lambda^2}{\xi^2} = \frac{1}{6\mu_m} = \frac{1 - e^2 \ln(\pi T)}{6e^2}$$

$$\kappa^2 = \frac{\lambda^2}{\xi^2} = \frac{1}{6\mu_m} = \frac{1 - e^2 \ln(\pi T)}{6e^2}$$

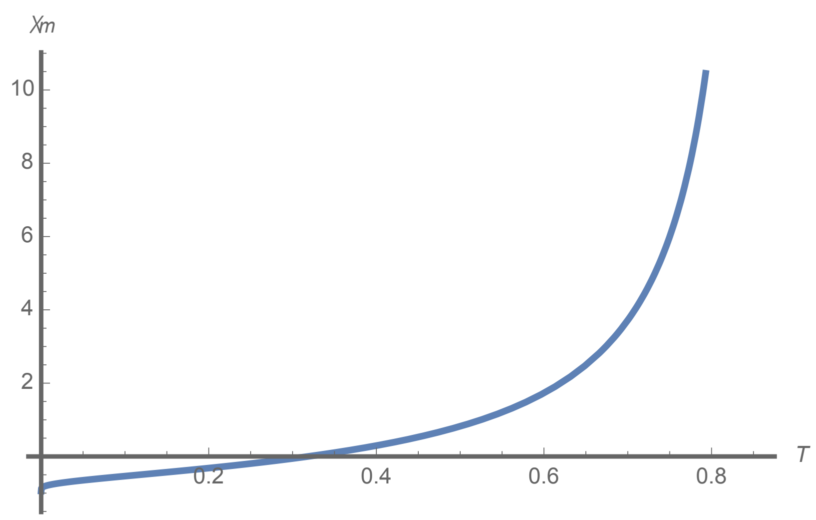
cf. GL: $$\kappa_{\mathrm{GL}}^2 = \frac{b}{2e^2}$$ $$\frac{b}{2} |\psi|^4$$

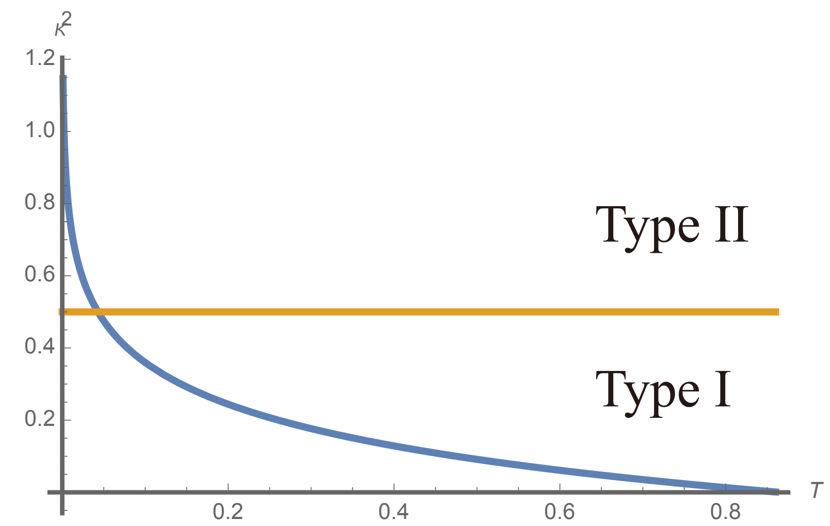
GL parameter of HSC is determined analytically for the first time.

- Focus on $\pi T \ll 1$. At weak coupling $e \ll 1$, holographic result reduces to the GL result.
- In the limit $e \to \infty$, GL implies $\kappa_{\mathrm{GL}}=0$. Strong Meissner. "extreme type I." For HSC, κ remains finite even in the limit. "extreme type I" cannot be reached.
- κ depends on T. As one increases T, κ decreases. In general, type I or type II depends on temperature.

$$\kappa^2 = \frac{\lambda^2}{\xi^2} = \frac{1}{6\mu_m} = \frac{1 - e^2 \ln(\pi T)}{6e^2}$$

$e / g = 1$





diamagnetic paramagnetic

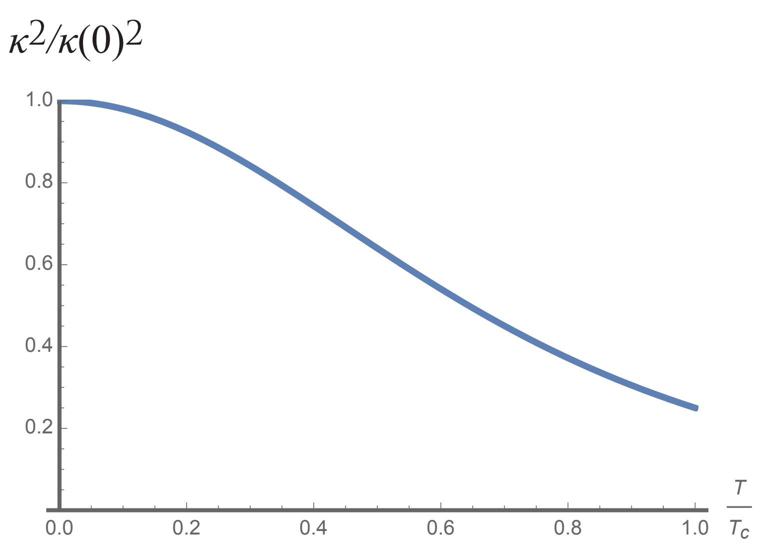
As one increases T, Type II→Type I

Temperature dependence of GL parameter

Interestingly, many superconducting materials (including high-Tc) show a similar behavior:
As one increases T, κ decreases.

e.g. Tinkham, "*Introduction to superconductivity*" gives an empirical rough estimate (Sect. 4.2)
"Of course, this is only a rough approximation…"

$$\kappa^2 \propto \left(\frac{1}{1+t^2}\right)^2 \qquad t = T/T_c$$



This does not imply the same physics. In the GL theory, this comes from T-dependence on "b."

Dual GL theory

In the bulk 5-dims, analytic results are possible.

w/ all information we have, one can write down the dual GL theory:

Natsuume - Okamura, 1801.03154

$$F = \int d^3x \, \frac{1}{4} |D_i\phi|^2 - \frac{1}{2}(\mu - \mu_c)|\phi|^2 + \frac{1}{96}|\phi|^4 - (\phi J^\dagger + \phi^\dagger J) + \frac{1}{4\mu_m}\mathscr{F}_{ij}^2$$

$$D_i = \partial_i - i\mathscr{A}_i$$

derivative terms: only the ones which contribute to linear perturb probs

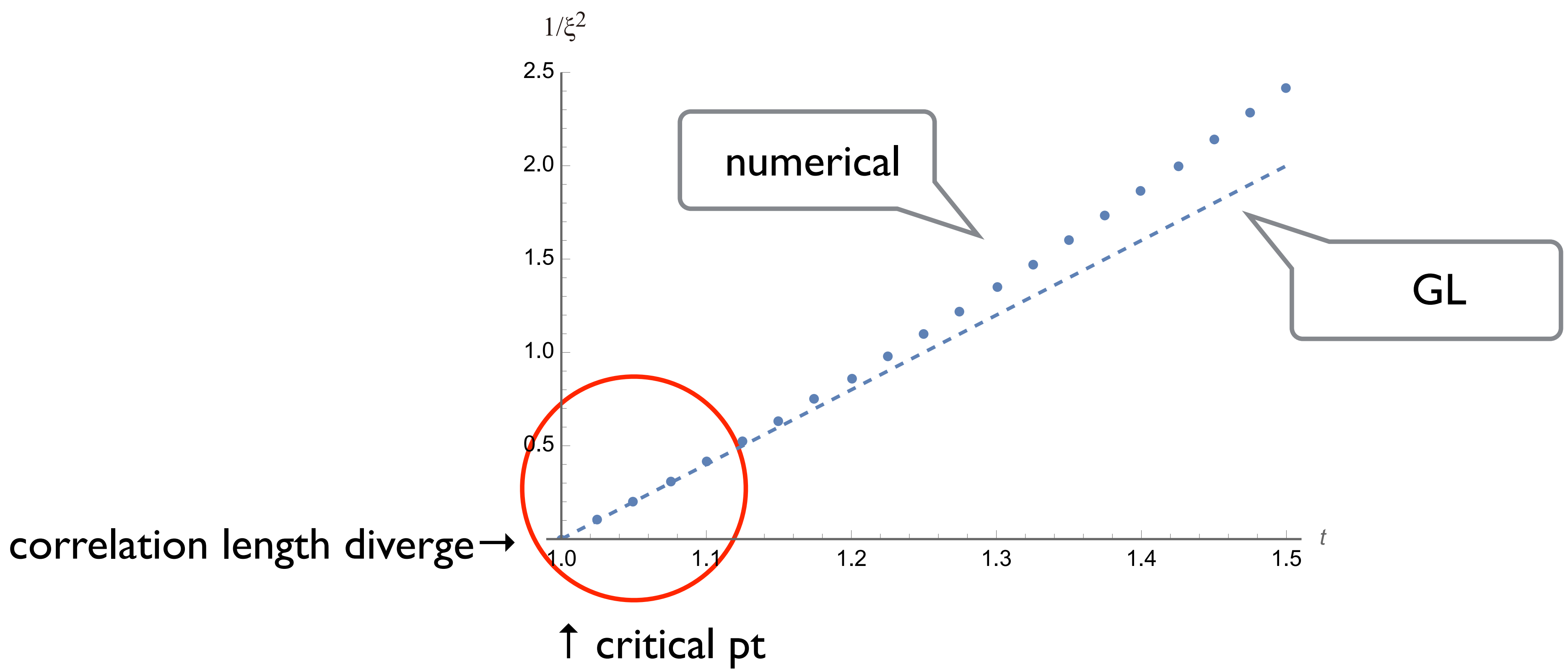
Just like GL, this should be regarded as an effective theory, i.e., leading terms near the critical pt. (small-ϕ)

We use various bulk computations and they are all consistent.

How good is the GL theory?

Comparison w/ numerical results

Here is the correlation length at high temp. (both analytically & numerically).



$t = T/T_c$

It agrees well near the critical pt, but it deviates as we are away from the critical pt.

The reason is clear: the GL theory is an effective theory and it is just leading terms.

Summary

- In most applications of AdS/CFT, the Maxwell field is added as an external source.
- One can make it dynamical by changing the BC on the AdS bdy ("holographic semiclassical eq").
- As an application, we study the holographic Meissner effect.
  - In standard HSCs, there is no Meissner effect because the Maxwell field is nondynamical.
  - We show the Meissner effect analytically.
- We also identify the dual GL theory w/ first-order corrections. HSC is described by GL theory very well. I hope to report the complete analysis in near future.

Backup

Some suggestions

- The Neumann-like BC should be possible even for SAdS5.
- Confirm Type I SC.
- Take backreaction into account (but analytic computation may not be possible).
- Extend our argument to the other analytic solutions (only a few, e.g. holographic Lifshitz SC.)
- Apply "holographic semiclassical eqs." to the other problems.

Previous studies: example

Domenech-Montull-Pomarol-Salvio-Silva, 1005.1776

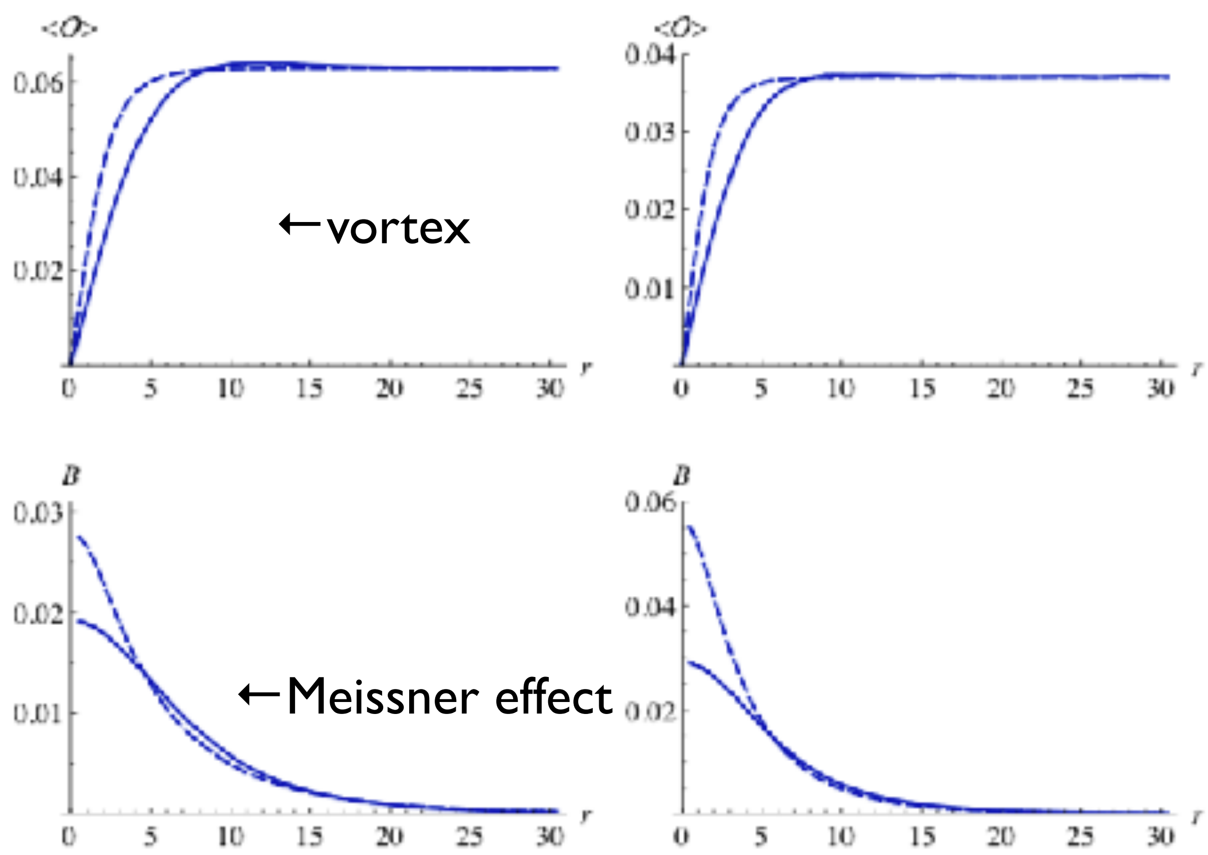


Figure 2: The modulus of $\langle\mathcal{O}\rangle$ (up to a factor L^{d-3}/g^2) and B as functions of r from our holographic model in the $n=1$ superconductor vortex solution for $d=2+1$ (solid lines on the left) and $d=3+1$ (solid lines on the right). The dashed lines are the corresponding profiles in the GL theory. Presented in units of $\mu=1$.

Probe limit, bulk 4-dim (Neumann BC) & 5-dim (holographic semiclassical eq.)

So, the Meissner effect has been shown, but it is desirable to show the effect more clearly.

Neumann BC

In previous studies, one typically imposes Neumann BC or J=0.

$$A_i \sim \mathcal{A}_i + \left\langle J^i \right\rangle u + \cdots \quad (u \to 0)$$

fixed: Dirichlet ↓ ↓fixed: Neumann

In our language, Neumann BC corresponds to the e→∞ limit since the kinetic term is gone, but the nontrivial result even in the limit, so the BC is possible.

$$\partial_j \mathcal{F}^{ij} = e^2 \left\langle J^i \right\rangle$$

What happens is

- J=0 does not mean no supercurrent because J consists of normal current as well as supercurrent J=Jn+Js.

$$q^2 \mathcal{Y} = -e^2 (q^2 + 2l) \mathcal{Y}$$

- The normal current has "induced" kinetic term.

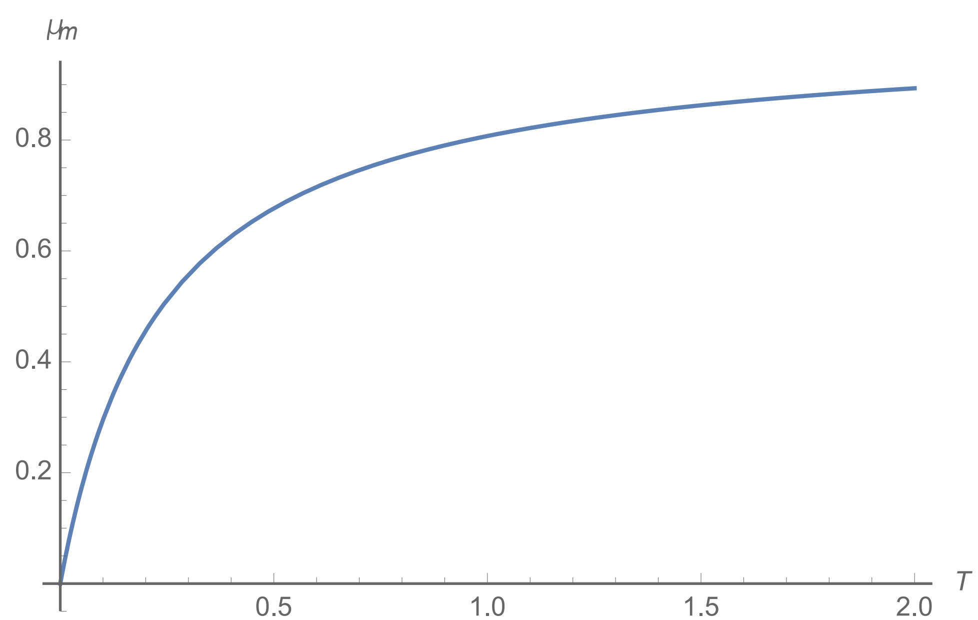
SAdS4 case

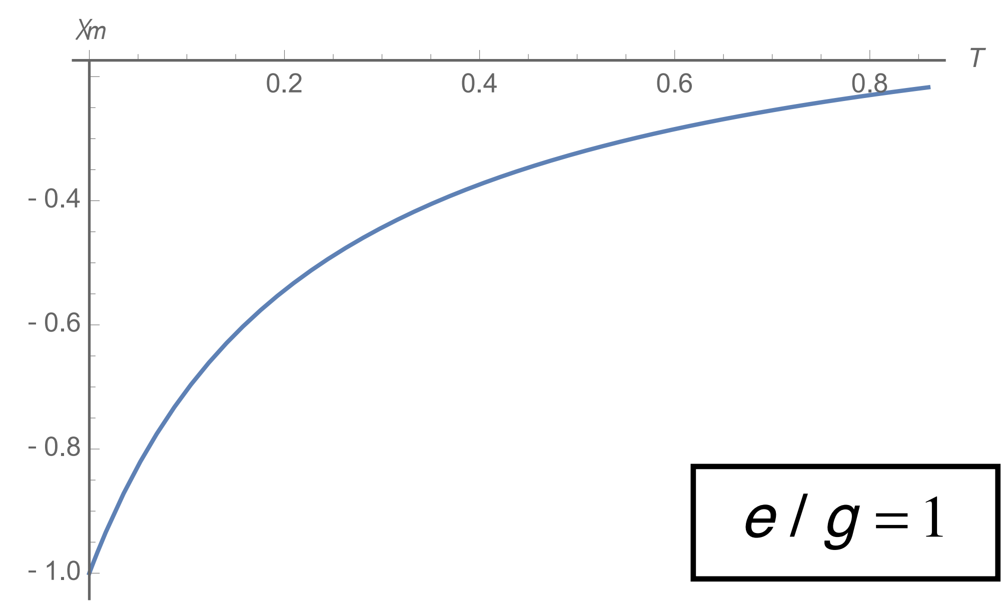
Restoring T, one gets

$$\mu_m = \frac{e^2}{1 + e^2/r_0}$$

$$\chi_m = -\frac{e^2/r_0}{1 + e^2/r_0}$$

$$\lambda^2 = \frac{1 + e^2/r_0}{2e^2 I}\frac{1}{r_0}$$





$e/g = 1$

SAdS5: details

In the bulk 5-dimensions,

$$\langle J_y \rangle = \frac{1}{u}\partial_u Y - q^2 Y \ln\epsilon \Big|_{u=\epsilon}$$

log divergent↓ ↓counterterm

from the CT action:

$$S_{CT} = -\int d^4x \frac{1}{4g^2}\sqrt{-\gamma}\gamma^{\mu\nu}\gamma^{\rho\sigma}F_{\mu\rho}F_{\nu\sigma} \times \ln u$$

$\gamma_{\mu\nu}$: bdy metric

ϵ : UV cutoff

In the bulk 5-dimensions,

$$Y = \mathcal{Y}\left\{1 - \int_0^u \frac{u'du'}{f(u')}\int_{u'}^{u_0} du''\frac{1}{u''}(q^2 + 2|\varphi_0|^2) + \cdots\right\} \qquad u_0 = 1/r_0$$

$$\langle J_y \rangle = \frac{1}{u}\partial_u Y - q^2 Y \ln\epsilon\Big|_{u=\epsilon}$$

$$= \mathcal{Y}\left\{-\int_\epsilon^{u_0} du\frac{1}{u}(q^2 + 2|\varphi_0|^2) + \cdots\right\} - q^2\mathcal{Y}\ln\epsilon$$

$$= \mathcal{Y}\left\{-q^2(\ln u_0 - \ln\epsilon) - \int_0^{u_0} du\frac{2}{u}|\varphi_0|^2 - q^2\ln\epsilon\right\}$$

$$= \mathcal{Y}\left\{-q^2\ln u_0 - \int_0^{u_0} du\frac{2}{u}|\varphi_0|^2\right\}$$

Holographic semiclassical eq then gives (when no supercurrent)

$$q^2\mathcal{Y} = -e^2 q^2 \ln u_0 \times \mathcal{Y} + e^2 J_{\text{ext}}$$

$$= \frac{e^2}{\underbrace{1 + e^2\ln u_0}_{\mu_m}} J_{\text{ext}}$$

Dual GL theory (time-dependent)

In the bulk 5-dims, analytic results are possible.

Dynamic case: time-dependent Ginzburg-Landau eq at linear level (from QNM computation of scalar at high temp)

$$\Gamma^{-1}\partial_t\phi = \frac{1}{4}\partial_i^2\phi + a\phi + \cdots$$

$$\Gamma = \frac{2}{5}(1+3i)$$

It takes the form of diffusion eq. $\partial_t\rho = D\partial_x^2\rho$, so it damps, but Γ is complex, so damped oscillation.

Magnetic susceptibility: QCD

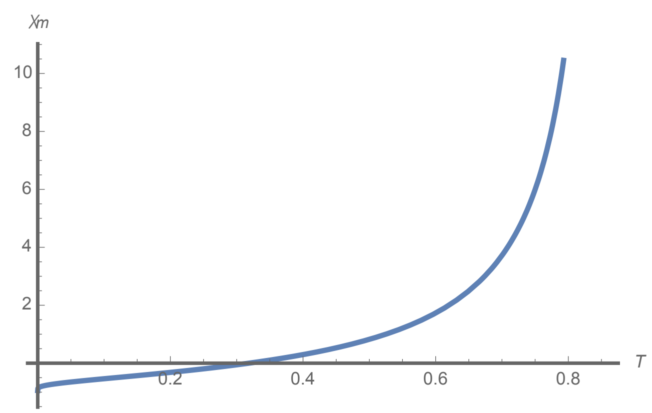
Putting aside HSC, consider Einstein-Maxwell theory.
The dual theory is $\mathcal{N}$=4 SYM + U(1).
Our computation of μ and χ themselves is valid there (in probe limit).

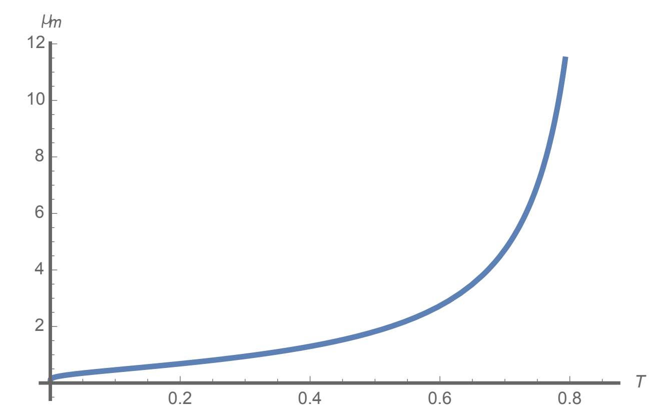
Just for fun, let us compare w/ QCD.

At high temp., paramagnetic like our case

But more careful comparison is necessary.

Magnetic susceptibility: comparison





diamagnetic paramagnetic

Bali et al., 1406.0269 [hep-lat]

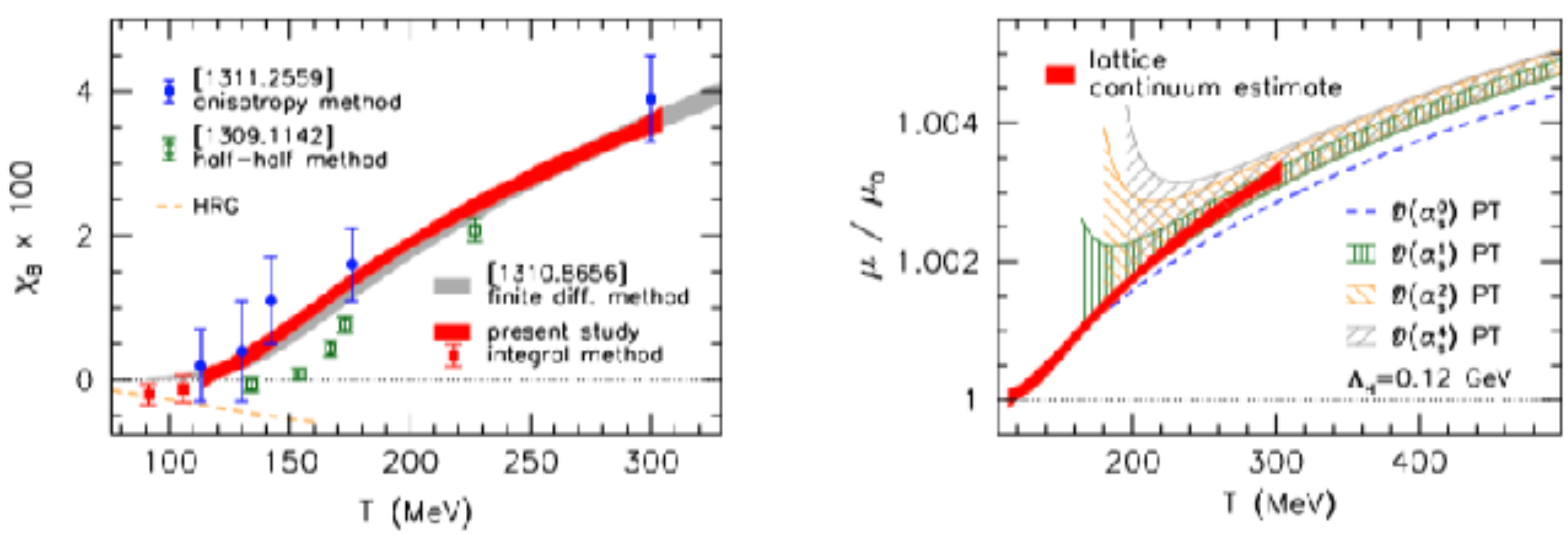


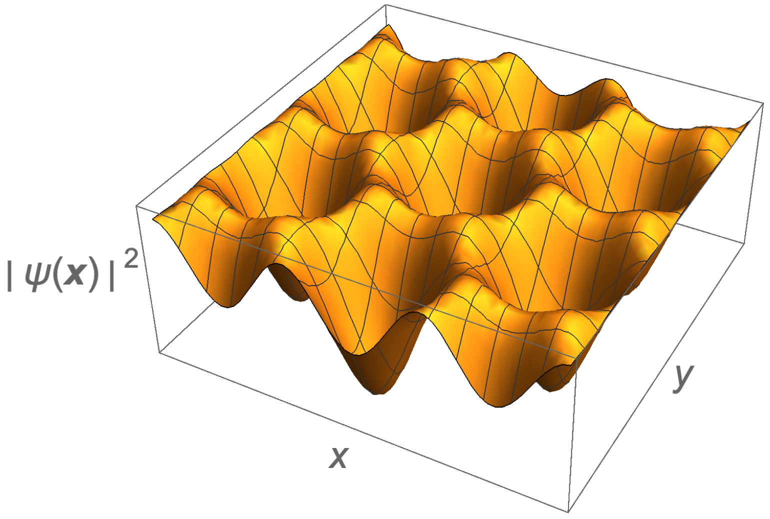
Figure 9. Left panel: magnetic susceptibility of QCD as a function of the temperature. Results with different lattice approaches are collected. Right panel: QCD magnetic permeability in units of the vacuum permeability μ_0, and a comparison to perturbation theory, truncated at various orders of the strong coupling.

Near upper critical magnetic field

We consider a small magnetic field.

As one increases the magnetic field, more and more vortices are created, and they form a vortex lattice.

Eventually, SC state is completely broken at the upper critical magnetic field H_{c2}.



According to the GL theory,

$$B = H - e^2 |\psi|^2$$

B reduces by the amount $|\psi|^2$ which implies Meissner effect.

Holographic semiclassical eq case

Impose the holographic semiclassical eq. I only quote the final result.

For the bulk 4-dim (in the hydro limit q→0)

$$B = H - \frac{e^2}{1+e^2}(\text{numerical factors})\left|\langle \mathcal{O} \rangle\right|^2$$

cf. GL:

$$B = H - e^2 |\psi|^2$$

Once again,

- At weak coupling e<<1, holographic result reduces to the GL result.
- There is a nontrivial e→∞ limit unlike the GL theory.
- This comes from the nontrivial magnetic const. The magnetic const. obtained here agrees w/ the small magnetic field case.

Critical magnetic field

- Upper critical magnetic field: $H_{c2} = a = \sqrt{2}\kappa H_c$
  SC state is completely destroyed.

$$\kappa = \sqrt{\frac{b}{2e^2}}$$

- Critical magnetic field: $H_c = e\frac{a}{\sqrt{b}}$

  Uniform |ψ| thermodynamically favored.

- Lower critical magnetic field: H_{c1}
  Vortex begins to form.

For Type II or $\kappa > 1/\sqrt{2}$, $H_c < H_{c2}$

For Type I, $H_c > H_{c2}$
As one lowers H, ψ=0 remains as the supercooled state, and vortex is formed for $H < H_{c2}$.

Wikipedia