Triplet doublet splitting, Proton Stability and an Extra Dimension:-

Kawamura' 2001 Arxiv:- 0012125

Motivation:-

i) Solving triplet doublet splitting without using fine tuning.

Setup:-

i) SUSY SU(5) Gut Model

ii) Orbifold $S^1/Z_1 \times Z_1'$ symmetry.

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SU(5) Model:-

1) Quarks & leptons are combined as mutiplets.

SM particle content

	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
q_L	3	2	1/6
u_L^c	3^*	1	$-2/3$
d_L^c	3^*	1	1/3
l_L^c	1	2	$-1/2$
e_L^c	1	1	1

$\underline{SU(5^*)}$

$$\psi_L \,|\, 5^* = \begin{pmatrix} d_L^r \\ d_L^g \\ d_L^b \\ \hdashline e_L \\ -\nu_L \end{pmatrix} \rightarrow 5^* \text{ plet of } SU(5) = \psi_5$$

$$5^* = \underbrace{(3^*, 1, 1/3)}_{d_L^c} \oplus \underbrace{(1, 2^*, -1/2)}_{L_L^c} \Rightarrow H_{\bar{5}} = \begin{pmatrix} H_c \\ H_d \end{pmatrix}$$

$$5 = \underbrace{(3, 1, -1/3)}_{\text{color Higgs}} \oplus \underbrace{(1, 2, 1/2)}_{\text{SM Higgs}} \Rightarrow H_5 = \begin{pmatrix} H_c \\ H_W \end{pmatrix}$$

$$(5 \times 5) \text{ antisymmetric} = \psi_{ij} = 10 \text{ plet} = \psi_{10}$$

$$= \underbrace{(3^*, 1, -2/3)}_{u_c^2} \oplus \underbrace{(3, 2, 1/6)}_{q_L} \oplus \underbrace{(1, 1, 1)}_{e_L^c}$$

Gauge bosons :– $N^2 - 1 = 5^2 - 1 = 24$ bosons

$$\left[\begin{array}{ccc|cc} & \text{Gluons} & & X_1^\dagger & Y_1^\dagger \\ & & & X_2^\dagger & Y_2^\dagger \\ & & & X_3^\dagger & Y_3^\dagger \\ \hline X_1 & X_2 & X_3 & \text{Weak} & \\ Y_1 & Y_2 & Y_3 & \text{bosons} & \end{array}\right] \Rightarrow \text{12 new gauge bosons.}$$

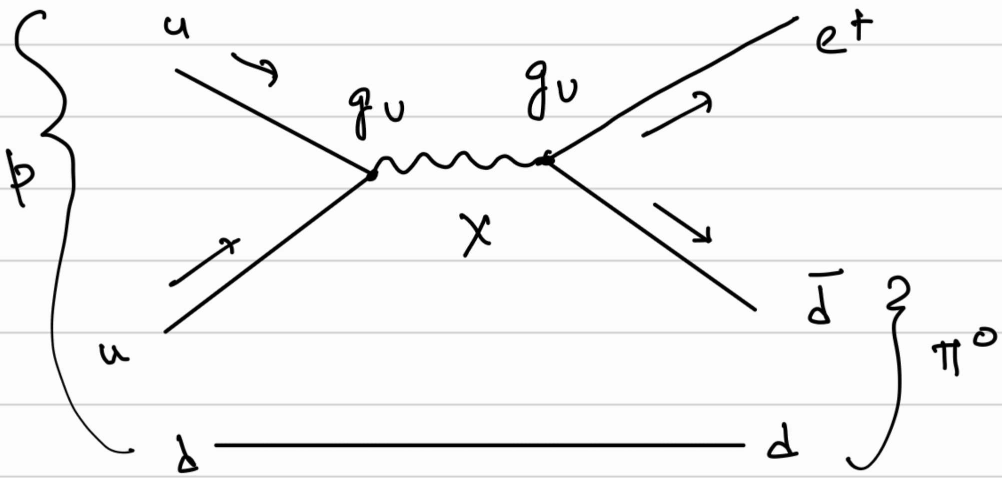
$$(5 \times 5^*) = 1 + 24$$

$$\supset (8, 1, 0) \oplus (1, 3, 0) \rightarrow \text{SM bosons}$$

$$\oplus (3, 2, -5/6) \oplus (3^*, 2, 5/6) \rightarrow \text{extra bosons.}$$

Problem in SU(5):—

i) X-Y mediated proton possible.



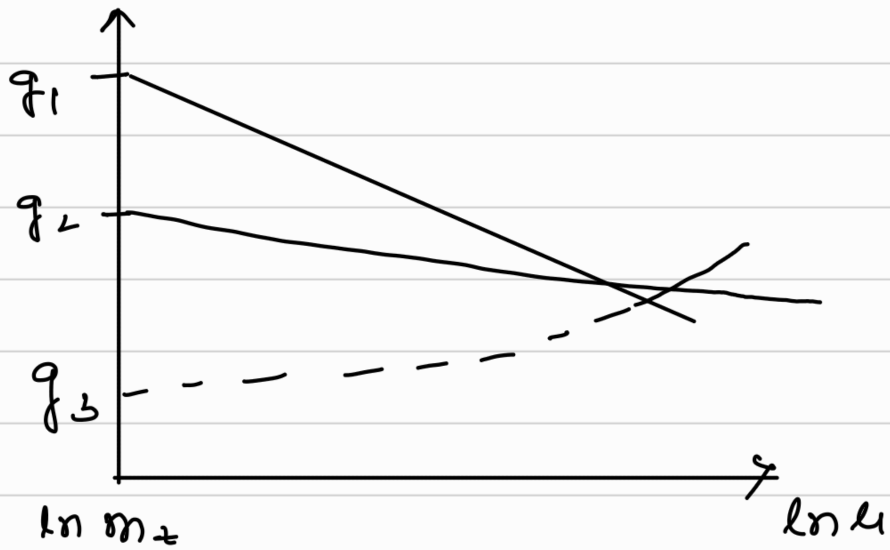
$p \to \pi^0 e^+$

$$\Gamma_{p \to \pi^0 e^+} \sim \frac{m_p^2}{m_X^4}$$

From experimentatal data of $\Gamma_{p \to \pi^0 e^+}$

$$m_X = v_{GUT} \gtrsim 10^{15} \text{ GeV.}$$

ii) Can't merge yukawa couplings g_1, g_2, g_3 at GUT scale.



∘ <u>**SUSY**</u> resolves the problem giving gauge coupling at $M_{susy} \sim 1\,\text{TeV}$.

So, for SUSY for each field we need one superpartner.

Super field	Particle
$\Phi_{\bar{5}}$	$(\phi_{\bar{5}}, \psi_{\bar{5}})$
H_5	(H_5, ψ_5)
$H_{\bar{5}}$	$(H_{\bar{5}}, \psi_{\bar{5}})$

• Fine tuning method for doublet-triplet method :-

GUT scale 10^{16} GeV

$$H_5 = \begin{pmatrix} H_3 \\ \hline H_u \end{pmatrix} \qquad H_{\bar 5} \begin{pmatrix} \bar{H}_3 \\ \hline H_d \end{pmatrix}$$

weak scale $\simeq 100$ GeV

• $f =$ GUT scale

$\mu, \lambda =$ fine tuning parameter

$$\Sigma = \mathrm{diag}\,(2, 2, 2, -3, -3)\, f$$

• Mass term in Lagrangian

$$= \int d^2\theta \; \lambda \cdot H_{\bar 5}\, \Sigma\, H_5 + \mu\, H_{\bar 5} H_5$$

$$= \int d^2\theta \left\{ (2f\lambda + \mu)\, \bar{H}_3 H_3 + \underbrace{(-3\lambda f + \mu)}_{\substack{\parallel \\ \text{weak scale mass} \\ \simeq\ 100\ \text{GeV}}} H_{\bar 2} H_2 \right\}$$

$$\therefore \quad M_{H_c} \simeq 10^{14}\, M_{H_2}$$

Q) Orbifold Symmetry $S^1/Z_1 \times Z_1'$

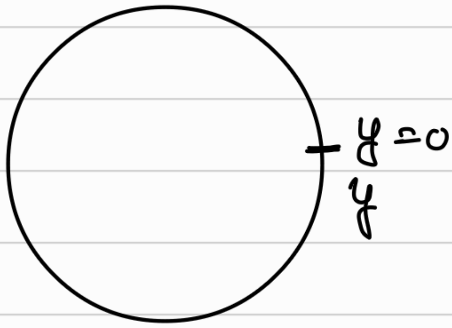
i) on 4+1 D SUSY $SU(5)$ GUT. $M^4 \times S^1/Z_1\times Z_1'$

Fields like $\phi(x^4, y)$

5th dimension

ii) Extra dimension curled up in S^1

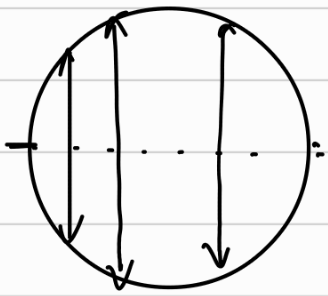
$y \to y + 2\pi R$



iii) For S^1/Z_1, opposite points identified.

$y \to y + \pi R$

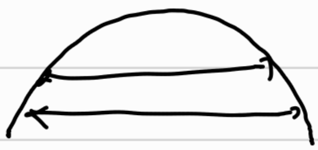
$$P\,\phi(x^4, y) = \phi(x^4, y+\pi R)$$
$$= \pm\phi(x^4, y)$$



Acts like partity operator

iv) For $S^1/Z_1 \times Z_1'$,

$y' \to y' + \pi R/2$



$$P'\,\phi(x^4, y') = \phi(x^4, y' + \pi R/2)$$

$$= \pm \ \phi(x^\mu, y')$$

v) Four Kaluza-Klein modes

$$\phi_{++}(x^\mu, y) = \sqrt{\frac{2}{\pi R}} \sum_{n=0}^{\infty} \phi_{++}^{(2n)}(x^\mu) \cos\frac{2ny}{R}$$

$$\phi_{+-}(x^\mu, y) = \sqrt{\frac{2}{\pi R}} \sum_{n=0}^{\infty} \phi_{+-}^{(2n+1)}(x^\mu) \cos\frac{(2n+1)y}{R}$$

$$\phi_{-+}(x^\mu, y) = \sqrt{\frac{2}{\pi R}} \sum_{n=0}^{\infty} \phi_{-+}^{(2n+1)}(x^\mu) \sin\frac{2n+1}{R}y$$

$$\phi_{--}(x^\mu, y) = \sqrt{\frac{2}{\pi R}} \sum_{n=0}^{\infty} \phi_{--}^{(2n+2)}(x^\mu) \sin\frac{2n+2}{R}y$$

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a Particle content in orbifold susy GUT:–

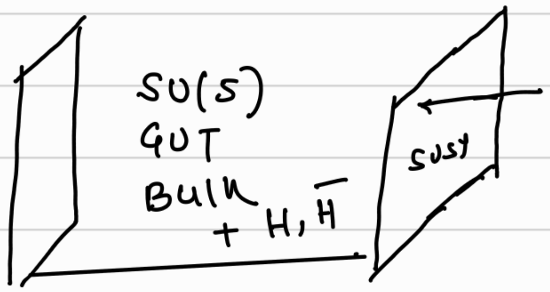
i) Vector multiplet:–

$$\{ \underbrace{A_m}_{\text{vector boson}}, \quad \underbrace{\lambda_L^i}_{\substack{2 \text{ bispinor} \\ i=1,2}}, \quad \underbrace{\Sigma}_{\text{real scalar}} \}$$

ii) 4 chiral multiplet / 2 hypemultiplet in susy

$$H_1 = \{ \underset{++}{H_5} = (H_1^1 \ \Psi_L^1), \ \underset{-+}{\hat{H}_{\bar{5}}} = (H_2^1 \ \bar{\Psi}_R^1) \}$$

$$H_2 = \{ \underset{--}{\hat{H}_5} = (H_1^2, \Psi_L^2), \ \underset{+-}{H_{\bar{5}}} = (H_2^2, \bar{\Psi}_R^2) \}$$



iii) $\Phi_{\bar{5}} = \{ \phi_{\bar{5}}, \psi_{\bar{5}} \}$

iv) $\Phi_{10} = \{ \phi_{10}, \psi_{10} \}$

$$S = \int \mathcal{L}^{(5)} d^5x + \frac{1}{2} \int \delta(y) \mathcal{L}^{(4)} d^5x$$

+ terms from brane at $y = -\pi R/2$

$$\mathcal{L}^{(5)} = \mathcal{L}^{(5)}_{YM} + \mathcal{L}^{(5)}_{H}$$

$$\mathcal{L}_{YM} = -\frac{1}{2} \mathrm{Tr}\, \underline{\underline{F_{MN}^2}} + \mathrm{Tr}\left[D_M \Sigma \right]^2 + \mathrm{Tr}\left[i \bar{\lambda}_i \gamma^M D_M \lambda^i \right]$$

$$- \mathrm{Tr}\left(\bar{\lambda}_i \left[\Sigma, \lambda^i \right] \right)$$

$$\mathcal{L}_H^{(5)} = |D_M H_i^s|^2 + i \bar{\psi}_s \gamma^M D_M \psi^s - \left(i \sqrt{2}\, g_{(5)} \bar{\psi}_s \lambda^i H_i^s + h.c. \right)$$

$$- \bar{\psi}_s \Sigma \psi^s - H_s^{i+} \Sigma^2 H_i^s - \frac{g_5^2}{2} \sum_{m,A} \left(H_s^{+i} (\sigma^m)_i^j T^A H_j^s \right)^2$$

$$\mathcal{L}^{(4)} = \sum_{3\ \text{fami}} \int d^2\bar{\theta}\, d^2\theta \left[\Phi_{\bar{5}}\, e^{2g_5 V^A T^A}\, \Phi_{\bar{5}} \right.$$

$$\left. + \Phi_{10}^{\dagger}\, e^{2g_5 V^A T^A}\, \Phi_{10} \right]$$

$$+ \sum_{3\ \text{fami}} \int d^2\theta \left(f_{U(5)}\, H_5\, \Phi_{10}\, \Phi_{10} + \hat{f}_{u(5)}\, \hat{H}_5\, \Phi_{10}\, \Phi_{10} \right.$$

$$\left. + f_{D(5)}\, H_{\bar{5}}\, \Phi_{10}\, \Phi_{\bar{5}} + \hat{f}_{D(5)}\, \hat{H}_{\bar{5}}\, \Phi_{10}\, \Phi_{\bar{5}} \right) + h.c.$$

- with:

→ $\lambda^i = \left(\lambda_L^i,\ \epsilon^{ij}\, \bar{\lambda}_{Lj} \right)^T$

→ $D_M = \partial_M - i g_5 A_M(x^\mu, y)$ → covariant derivative

→ g_5 5'D coupling const

→ f_U's, f_D's are yukawa coupling const

→ γ_5^M are 5D gamma matrices.

$$[\gamma^M, \gamma^N] = 2\eta^{MN}$$

Transformation rules:—

$\mathcal{L}$ is invariant

$$P A_\mu(x^\mu, y) P^{-1} = A_\mu(x^\mu, -y)$$

$$P A_5(x^\mu, y) P^{-1} = -A_5(x^\mu, -y)$$

$$P \lambda_L^1 (x^\mu, y)\, P^{-1} = -\lambda_L^1 (x^\mu, -y)$$

$$P \lambda_L^2 (x^\mu, y)\, P^{-1} = \lambda_L^2 (x^\mu, -y)$$

$$P\, \Sigma (x^\mu, y)\, P^{-1} = -\Sigma (x^\mu, -y)$$

$$P H_5 (x^\mu, y) = H_5 (x^\mu, -y)$$

$$P H_{\bar{5}} (x^\mu, y) = -\hat{H}_{\bar{5}} (x^\mu, -y)$$

$$P \hat{H}_5 (x^\mu, y) = -\hat{H}_5 (x^\mu, -y)$$

$$P H_{\bar{5}} (x^\mu, y) = H_{\bar{5}} (x^\mu, -y)$$

* <u>Doublet-triplet breaking of Higgs</u> :-

$$P = \mathrm{diag}(1, 1, 1, 1, 1)$$

$$P' = \mathrm{diag}(-1, -1, -1, 1, 1)$$

$$SU(5) \to G_{SM} = SU(3) \times SU(2) \times U(1)$$

$$P' T^a P'^{-1} = T^a , \quad P' \hat{T}^a P'^{-1} = -\hat{T}^a$$

T^a: Gauge generator of G_{SM}

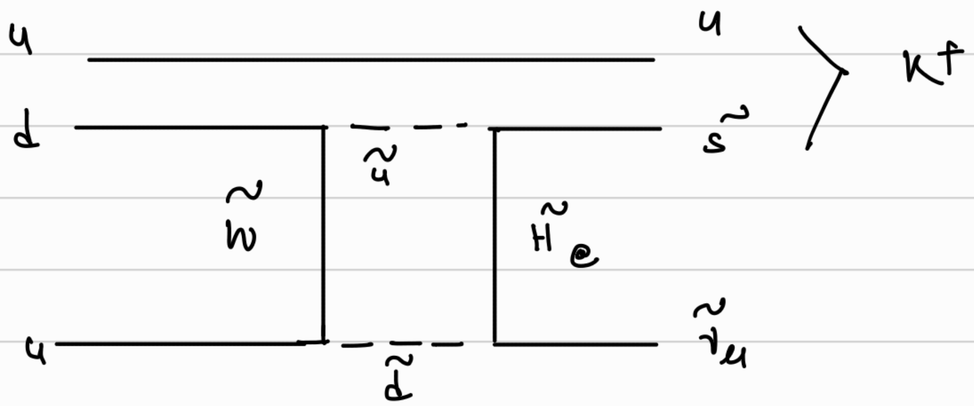
$\hat{T}^a$: Gauge generator of BSM

* $(A_\mu^0, \lambda^{2a(0)})$, $(H_u^{(0)}, H_d^{(0)})$ will give gauge multiplets & Higgs multiplets in MSSM.

→ From $SU(5)$ GUT prediction $g_1 = g_2 = g_3 = g_U$.

→ $f_d = f_e$

Proton Stability :—



$p \to K^+ \bar{\nu}_\mu$

$$\Gamma_{p \to K^+ \bar{\nu}_\mu} \cong \left[\frac{1}{m_{\tilde{H}_c}} \frac{1}{m_W}\right]^2 > \left[\frac{1}{m_{H_c}}\right]^4$$

⇒ GUT scale is higher for. then M_X.

Conclusion :—

i) Doublet - triplet higgs splitting done without fine tuning.

ii) Prediction obtained about GUT scale.