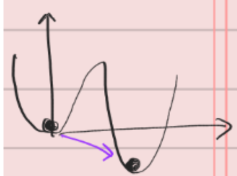
Gravitational waves from 1st order PT

- An alternative/additional search method for BSM physics.
  - As is, SM does not have a 1st order PT; the QCD & EW phase transitions are so-called "crossovers."



1st order – order param. of a system discontinuously jumps
2nd order – " " changes in a cts., but non-analytic way.

On the other hand, crossover transitions are generically "smooth." This appears to be the case for the SM.

- BSM physics may include/introduce sectors/features that introduce a 1st order PT in the larger theory.
  - Naturally, BSM also grants possibility of solving/addressing unanswered questions (DM, baryogenesis, origin of EWSB)

- PT itself can induce or create GW, so there may be hope for detecting BSM physics tied to GW production.

- GW from PT between a higher-degree symmetry phase down to the broken EW symmetry. If PT is first order, then bubbles of "true vacuum" would be nucleated within the "false vacuum."
  - Expansion, collision, & merging of bubbles will create a stochastic GW background.

(Note: GWs can also originate from primordial universe & cosmic defects (cosmic strings / domain walls). These are not discussed here.)

a) Bubble nucleation (Dynamics of PT)
b) " Expansion
c) " Percolation

d) GW

a) Nucleation

- Vacuum decay → barrier penetration from false to true vacuum.
  ↳ field config. captured in "bounce equation"
- "Bounce eqn" describes the true vacuum bubbles, with a prob/time of the form:

$$\Gamma = A \exp(-B/\hbar)\left[1 + \mathcal{O}(\hbar)\right]$$

In flat spacetime;

ϕ a single scalar field with no derivative interactions.

$$\Gamma(T) \simeq T^4 \left(\frac{S_3[\phi_B(r), T]}{2\pi T} \right)^{3/2} \exp\left(- \frac{S_3[\phi_B(r), T]}{T} \right)$$

where $S_3[\phi(r), T]$ is the Euclidean action

$$S_3[\phi(r)] = 4\pi \int_0^\infty dr \; r^2 \left[\frac{1}{2}\left(\frac{d\phi}{dr}\right)^2 + V_{eff}(\phi, T) \right]$$

is estimated at the Bounce Profile $\phi_B(r)$, which is a sol$^{\underline{n}}$ of the eqn. of motion $\frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} = \frac{\partial V_{eff}}{\partial \phi}$.

- $V_{eff}(\phi, T)$ is the effective potential. This usually includes tree-level, 1-loop, & finite temp. contributions (Daisy resummation?). Model dependent.

b) Bubble Expansion

- After nucleation, true vacuum bubbles will expand until reaching c or until collision w/other bubbles.
  - BG full of relativistic particles ⇒ frictional effects.

⇒ The Goal is to find out how

bubble wall velocity: v_w
friction from plasma: η
Strength of PT: α (measures released vac. energy density to radiation energy density)
efficiency factors: κ_ϕ, κ_v (ϕ: vac. energy → wall expansion)

(v: " " → bulk fluid motion)

are inter connected). This serves as interface b/w

c) Bubble Percolation

- True vacuum bubbles collide until PT is complete.
- PT duration can be estimated by mean bubble size at collision, characterized by parameter β.

This requires potential barrier tunneling

↳ both β and α evaluated at the Nucleation Temperature T_N.

T_N: Temp. at which # of generated bubbles per unit time per Hubble vol. ~ 1.

- Percolation has 3 main sources:

(i) Bubble collisions
- Collisions are violent
- QCD PTs might have detectable GW from collisions (Witten); EW too (Hogan)
- GWs from collisions depends mainly on α & β params.

(ii) Turbulent Magnetohydrodynamics (MHD)
- From turbulent motion of bulk fluid

(iii) Sound Waves
- Overlapping sound waves in bulk fluid.

⇒ All can source GW.

d) Gravitational Waves

- Bubble nucleation requires a potential barrier for tunneling from false vacuum → true vacuum.
- Expansion needs fast-enough moving bubble walls for strong signals.
- Percolation needs efficient collisions so vacuum energy is dispersed into bulk fluid motions.

⇒ So the question is:

Which models of physics have 1st order PT so that we have detectable GW?

A quick list:

- Additional Scalar Sectors
  - Gauge singlet ext. (measure Higgs 3x coupling), Charged scalar (2HDM)
- Higher-dim. operators
  - Simplest to add a cubic term (expected SUSY extensions) to create 1st order PT.
- SUSY extensions
  - Naturally exhibit PTs is MSSM & NMSSM.
- Hidden Dark Sectors
  - GWs originating from Dark sector dynamics.
- Other BSM extensions
  - Extra dim., Pecci-Quinn PT, non-linear EWPT, QCD PT.

→ We'll focus mostly on Gauge Singlet scalar extension, and take a short look at a more complex model.

Singlet Scalar extension of SM [1611.01617]

- Simple case, introduce a new real scalar singlet ϕ.

$$V_0(H,\phi) = \mu^2 |H|^2 + \frac{\lambda}{2}|H|^4 + \frac{1}{2}M_\phi^2\phi^2 + \frac{1}{3}\kappa\phi^3 + \frac{\lambda_\phi}{2}\phi^4$$
$$+ \kappa_1 \phi |H|^2 + \frac{1}{2}\kappa_2 \phi^2 |H|^2$$

($m^3\phi$ removed by shifting ϕ WLOG)

as the tree-level potential.

- We will also consider a one-loop effective potential with finite-temperature corrections (i.e., a free energy)

- Imposing standard low-energy phenomenology (Higgs properties, stability) requires:
  1. H, ϕ in a true, stable vacuum at origin at high T.
  2. At the bubble nucleation temp $T \approx T_N \approx T_c \in (10^7, 10^8)$ GeV, ϕ acquires a vev in a strong 1^{st} order PT, generating GW.
     (T_N chosen to coincide with aLIGO sensitivity peak)
  3. At low T, H acquires vev $v_h \approx 246$ GeV.

- Explicit contributions to V_{eff} are:

Thermal / Finite Temp:

$$\boxed{\Delta V_T = \frac{T^4}{2\pi^2}\left[\sum_b J_B\left(\frac{m_b^2}{T^2}\right) + \sum_f J_F\left(\frac{m_f^2}{T^2}\right)\right]}$$

$$J_{B/F}\left(\frac{m_i^2}{T^2}\right) \equiv \int_0^\infty dk\, k^2 \ln\left[1 \mp \exp\left(-\sqrt{\frac{k^2+m_i^2}{T^2}}\right)\right]$$

↳ thermal bosonic/fermionic fns

summed over field-dependent B/F mass eigenvalues.

0-Temp. 1-loop Coleman-Weinberg corrections

$$\boxed{\Delta V_{CW} = \sum_i \frac{g_i m_i^2}{64\pi^2}\left[\ln\left(\frac{m_i^2}{\mu^2}\right) - n_i\right]}$$, i over massive particles

g_i = # of d.o.f. of i
$n_i = \frac{3}{2}$ (scalars fermions) $\frac{5}{6}$ (massive gauge bosons)

Debye Masses (not enough explanation in reference)

- Come from bare mass terms in $\mathcal{L}$ getting corrections like $\Delta m_T^2 \propto T^2$ (Finite-Temp F.T. corrections) (self-energy corrections?)

$$\Rightarrow \boxed{V_{eff} = V_0 + \Delta V_D + \Delta V_T + \Delta V_{cw}}$$

- To have a strongly 1st order PT with GW, we need a few more things:

$\gamma > 5 \to$ PT cannot dominate Universe
$\gamma < 2.3 \to$ GW amp too low for aLIGO

① At least two minima $\left.\frac{\partial V}{\partial \phi}\right|_F = \left.\frac{\partial V}{\partial \phi}\right|_T = 0$
(false vacuum) (true vacuum)

why?

② $\exists$ critical temp. T_c where $V|_F = V|_T$

③ Order param. $\gamma \equiv \frac{\langle \phi \rangle}{T_c}$ must be $\sim O(1)$ at T_c

④ Bubble nucleation, growth, collision.

(↳ in this ref., authors choose to fix T_c & γ, and solve for $\mathcal{L}$ parameters at high scales s.t. these conditions hold.)

- Peak freq & Amp. of GW are controlled by T_N.
  ↳ this is also Temp at which a 1/e Volume fraction of the Universe is in the true vacua.

  Occurs approximately at

$$\boxed{p(t)\, t^4 \approx 1}$$

↳ prob. per unit time per unit vol. that a critical bubble forms.

$$\boxed{p(T) \approx T^4 \exp\left[\frac{-S_E(T; S_b(r,T))}{T}\right]} \quad \text{(as a fn of } T\text{)}$$

S_E here is the same Euclidean action evaluated at a bounce solution, as before:

$$S_E = 4\pi \int_0^\infty dr\, r^2 \left[\left(\frac{\partial \phi(r)}{\partial r}\right)^2 + V_{eff}(\phi, T) \right]$$

where ϕ satisfies

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{2}{r}\frac{\partial \phi}{\partial r} = \frac{\partial V(\phi, T)}{\partial \phi}, \quad \phi'(0) = 0, \quad \phi(\infty) = 0$$

- In a radiation-dominated universe, Temp <-> time relation is
$$\boxed{T^2 t = \sqrt{\frac{45}{16\pi^3}}\frac{M_P}{\sqrt{g_*}}}$$

Combining the last 3 boxed equations, there is a relation
$$\boxed{\frac{S_E(T_N;\phi_b(r;T_N))}{T_N} \approx 170 - 4\ln\left(\frac{T_N}{1\,\mathrm{GeV}}\right) - 2\ln(g_*)}$$
which can be solved numerically for T_N (by bisection)

- GW amplitude from PT depends on bubble wall velocity v_w, latent heat released in transition b/w vacua $\Delta\rho$, efficiency of conversion of latent heat to GW, and duration of transition.
  - Duration parameterized by
    $$\beta = -\frac{dS_4}{dt}\Big|_{t_N} = H_N\left[\frac{\partial \ln(S_E)/T}{\partial \ln(T)}\right]\frac{S_E}{T}\Big|_{T_N}$$
    $$(S_4 = S_E/T\ ,\quad H = -\dot{T}/T)$$
    characteristic timescale is then $1/\beta$.
    - Timescale can be approx. as $\frac{\beta}{H_N} \approx \frac{S_E(T_N)}{T_N}$ to $O(1)$
  - Latent heat param. by
    $$\alpha \equiv \frac{\Delta\rho}{\rho_N}\ ,\quad \rho_N = \frac{\pi^2 g_* T_N^4}{30}$$
    where ρ_N is energy density of false vacuum, g_* = # of rela. d.o.f.
    $$\Delta\rho = \left[V - \frac{\partial V}{\partial T}T_N\right]_F - \left[V - \frac{\partial V}{\partial T}T_N\right]_T$$
    is latent heat in transition from $F \to T$
  - v_w influences amp. of GW; slowed by friction terms arising from particle interactions. So, less interactions means less friction. For high-scale SSM, very few friction terms, so $v_w \sim 1$.
  - Conversion efficiency $\epsilon \neq 1$, so $\epsilon = 1$ is used as $\gamma \approx 1.75$ is used.

- All factors combined into the envelope approximation to find peak amplitude:

$$\boxed{\Omega_{GW} \simeq 10^{-9} \left(\frac{31.6\, H_N}{\beta}\right) \left(\frac{\alpha}{\alpha+1}\right)^2 \epsilon^2 \left(\frac{4v_w}{0.43+v_w^2}\right)\left(\frac{100}{g_*}\right)^{1/3}}$$

$g_* = 107.75$ for SSM

- Factors are $\mathcal{O}(1)$ for a PT with $T_N \in (10^7, 10^8)$ GeV, $\boxed{\Omega_{GW} \sim \mathcal{O}(10^{-9})$, if $\alpha \simeq 1$ and $\gamma \sim 2.}$
  ↳ aLIGO would be sensitive to $\Omega_{GW} \gtrsim 5\times10^{-10}$ at about $\mathcal{O}(10) \sim \mathcal{O}(100)$ Hz

- Peak observable amp today at peak freq.

$$\boxed{\begin{gathered} f_0 \simeq 16.5\ \text{Hz} \left(\frac{f_N}{H_N}\right)\left(\frac{T_N}{10^8\,\text{GeV}}\right)\left(\frac{g_*}{100}\right)^{1/6} \\ f_N = \frac{0.62\beta}{1.8 - 0.1v_w + v_w^2} \end{gathered}}$$

  ↳ f_0 coincides with aLIGO's max sensitivity of ~ 20 Hz if $T_N \in (10^7, 10^8)$ GeV.

- Additional discussion: Vacuum stability
  - at large field values, $V_{eff}(h) = \frac{1}{2}\lambda(\mu \simeq h)\, h^4$ for SM V_{eff}.

    Instability of SM Higgs potential.
  - Instability can be remedied with SSM via modified β-function for quartic coupling or negative Higgs mass corrections.
  - SSM can have additional stability corrections → consider $K_2 > 0$

$$\lambda \geq 0, \quad \lambda_\phi \geq 0 \quad \text{and} \quad K_2 \geq -2\sqrt{\lambda_\phi \lambda} \quad (*)$$

    from considerations in $H=0, \phi=0, \lambda H^4 = \lambda_\phi \phi^4$ field space.
  - Insure that $H-\phi$ mixing angle is small so model agrees with experimental constraints for SM Higgs.

$$\tan(\theta) \approx -\frac{K_1 + K_2 v_\phi}{4\lambda_\phi v_\phi + K}\left(\frac{v}{v_\phi}\right) + \mathcal{O}\left(\frac{v^3}{v_\phi^3}\right)$$

  - There is a remaining residual threshold correction to λ, the SM quartic coupling (negative contribution to Higgs mass)

$$m_h^2 = \left(\lambda - \frac{(K_1 + K_2 v_\phi)^2}{v_\phi(4\lambda_\phi v_\phi + K)}\right) v^2 \leq \lambda v^2$$

$\Rightarrow \lambda$ must be greater than in SM

$$\Delta\lambda = \frac{(k_1 + k_2 v_\phi)^2}{v_\phi(4\lambda_\phi v_\phi + k} \geq 0$$

- Conditions in ⊛ are necessary, but insufficient for stability. For a $\mathbb{Z}_2$ sym. potential and RN scales $\mu \lesssim M_\phi$ and $k_2 > 0$,

$$\lambda_{SM} \equiv \lambda - \Delta\lambda \geq 0$$

is required to avoid deeper minima in $\phi = 0$ dir$^{\underline{n}}$.

overall, need

$$\mu \lesssim M_S \lesssim \Lambda_I$$

→ SM instability scale

↳ Even if ∃ a scale at which this condition is broken, vacuum can still be stable.

- SSM with Baryogenesis [1702.06124]

- Near-identical setup ($-\mu^2 |H|^2$ instead ?)
- Depending on the sign of μ_ϕ^2 (or M_ϕ^2), EW PT can occur in two ways:

① $\underline{\mu_\phi^2 > 0}$: happens at large m_ϕ, small λ_ϕ. Potential grows as going away from $\phi = 0$; one-step PT from origin to EW minimum.

② $\underline{\mu_\phi^2 < 0}$: small m_ϕ, large λ_ϕ; origin → ϕ min, then ϕ_{min} → EW min.

- Dynamics of PT are basically the same as before.
- For BG, necessary condition is decoupling of Sphaleron processes after EWPT (not main focus)

↳ provide B violation necessary for asymmetry

- DM signals
  - ϕ can be treated as DM candidate, as it is stable
  - Follow standard analysis, using Boltzmn eqn

$$\boxed{\frac{\partial Y}{\partial x} = \frac{2\pi}{45}\frac{m_\phi^3}{x^4 H}\left(h_{eff} + \frac{T}{3}\frac{\partial h_{eff}}{\partial T}\right)\langle\sigma v\rangle\left(Y_{eq}^2 - Y^2\right)}$$

$Y \equiv n/s$, $x = m_\phi/T$, $\langle\sigma v\rangle =$ Therm. avg annihil. cross-sec, $h_{eff} =$ eff. # of entropy d.o.f.

  - Solve to find n_0, obtain

$$\Omega_\phi h^2 = \frac{m_\phi Y_0 s_0}{3 M_P^2 H_0^2} \sim m_\phi Y_0 \times 2.76 \times 10^8$$

$$\Omega_{DM} h^2 = 0.120 \quad (\text{Planck } 2018)$$

↳ imposing direct detection limits, ϕ cannot be a single-component DM; in fact, ϕ cannot be all of DM if it is the only hidden sector particle.

  - More in-depth discussion + cosmological modification consideration in the reference paper.

- Asymmetric DM from GW (2209.04788)
  - A more complex BSM physics model featuring potentially detectable GW from 1st order PT (also from domain walls).
  - Gauge group

$$SU(3)_c \times SU(2)_L \times U(1)_Y \times SU(2)_\ell$$

↳ quarks are $SU(2)_\ell$ singlets

leptons are upper components of $SU(2)_\ell$ doublets

$$(l_L \;\; \tilde{l}_L)^T \equiv \hat{l}_L = (1, 2, -\tfrac{1}{2}, 2) \qquad l_R' = (1, 2, -\tfrac{1}{2}, 1)$$

$$(e_R \;\; \tilde{e}_R)^T \equiv \hat{e}_R = (1, 1, -1, 2) \qquad e_L' = (1, 1, -1, 1)$$

$$(\nu_R \;\; \tilde{\nu}_R)^T \equiv \hat{\nu}_R = (1, 1, 0, 2) \qquad \nu_L' = (1, 1, 0, 1)$$

- Break $SU(2)_\ell$ spontaneously by introducing two complex $SU(2)_\ell$

scalar doublets, Φ_1 & Φ_2 (this also provides a mechanism for BG)

- Scalar potential

$$V(\Phi_1, \Phi_2) = m_1^2 |\Phi_1|^2 + m_2^2 |\Phi_2|^2 - (m_{12}^2 \Phi_1^\dagger \Phi_2 + h.c.)$$
$$+ \lambda_1 |\Phi_1|^4 + \lambda_2 |\Phi_2|^4 + \lambda_3 |\Phi_1|^2 |\Phi_2|^2 + \lambda_4 |\Phi_1 \Phi_2|^2$$
$$+ \left[\left(\tilde{\lambda}_5 |\Phi_1|^2 + \tilde{\lambda}_6 |\Phi_2|^2 + \tilde{\lambda}_7 \Phi_1^\dagger \Phi_2 \right) \Phi_1^\dagger \Phi_2 + h.c. \right]$$

$m_{12}^2, \tilde{\lambda}_5, \tilde{\lambda}_6, \tilde{\lambda}_7 \in \mathbb{C}$

- $\langle \Phi_1 \rangle = v_1$, $\langle \Phi_2 \rangle = v_2$ breaks $SU(2)_\ell$ symm, leaving the SM Gauge group.

$$\Phi_j = \begin{pmatrix} c_{1j} + i c_{2j} \\ \frac{1}{\sqrt{2}}(v_j + \rho_j + i a_j) \end{pmatrix} \qquad c_{1j}, c_{2j}, \rho_j, a_j \text{ real fields}$$

- After $SU(2)_\ell$ breaking, there are 6 electrically charged and 6 neutral new fermionic states. A remnant $U(1)_\ell$ symmetry prevents decay to SM particles.
  - ↳ lightest of these could be DM candidate!
- 3 new gauge bosons Z', W_1', W_2', masses
  $m_{Z', W'_{1,2}} = \frac{1}{2} g_\ell v_\ell$
- Scalars $P_{1,2}$ (CP-even), A (CP-odd), $C_{1,2}$ (complex conj.)

- $V_{eff}(\phi_1, \phi_2, T) = V_{tree}(\phi_1, \phi_2) + V_{1\text{-loop}}(\phi_1, \phi_2) + V_T(\phi_1, \phi_2, T)$, very similar to SSM cases.

=> As before, potential shape evolves/develops new minima as T drops to T_c & T_N $(\phi_1, \phi_2)_{true1}$
- In this case, multiple (4) minima develop, related by approximate $\mathbb{Z}_2$ symm. of the potential, two pairs related by a gauge symmetry.
  - i.e., only two physically distinct true vacua:
    $(\phi_1, \phi_2)_{true1}$, $(\phi_1, -\phi_2)_{true2}$
    - ↳ energy densities of each may differ slightly; if splitting is small, domain walls may form.

- 1st order PT GW analysis just as before, one can obtain estimates for peak amplitude & freq (Ωh^2, f) numerically.

Classically Conformal U(1) extension (tie-in to current work)

- Impose classical conformal symmetry to forbid explicit
  - mass^2 terms
- Extend SM gauge group to $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_H$, introducing new Higgs-like scalar Φ in new hidden sector.
  ↳ $U(1)_H$ symmetry broken by CW mechanism at v_ϕ, from 1-loop corrections via $V_{loop}(\phi)$.
  $\Rightarrow V = \frac{1}{4}\lambda |H|^4 + \frac{1}{4}\lambda_\phi |\phi|^4 + \lambda_{mix}|H|^2|\phi|^2 + V_{loop}$
  (− ?)

  $-\mu^2$ term induced by Higgs-ϕ mixing term.
  ⇒ EWSB induced by CW mechanism symmetry breaking.
  ↳ $\boxed{\text{perhaps } \exists \text{ a } 1^{st} \text{ order PT between } v_\phi \text{ minimum and EW minimum}}$

- How does the conformal symmetry affect the dynamics of the PT?
- Will other sources of GW be more dominant than bubble collisions?
- What happens if we take Z'_μ (or another particle) as DM candidate?
- What if we use $SU(2)_H$ as the extension?