Computationally waves from 1st order PT

- An alternative/additional search method for BSM physics.
- As is, SM does not have a 1st order PT; the GCD & EW phase transitions are so-called "crossovers."
  
  1st order - order parameter discontinuously jumps
  2nd order - "soft" changes in a cts., but non-analytic way

On the other hand, crossover transitions are generally "smooth."
This appears to be the case for the SM.

- BSM physics may include/introduce seeds that introduce a 1st order PT in the larger theory.
- Naturally, BSM also grants possibility of solving/addressing unanswered questions (DM, baryogenesis, org. of EWSB)

- PT itself can induce or create GW, so there may be hope for detecting BSM physics tied to GW production.

- GW from PT between a higher-degree symmetry phase down to the broken EW symmetry. If PT is 1st order, then bubbles of "true vacuum" would be nucleated within the "false vacuum."

- Expansion, collision, & merging of bubbles will create a stochastic GW background.

  (Note: GWs can also originate from primordial universe & cosmic defects (Cosmic strings/domain walls).)

  These are not discussed here.

  a) Bubble nucleation (Dynamics of PT)
  b) " Expansion
  c) " Percolation
a) Nucleation
- Vacuum decay -> barrier penetration from false to true vacuums.
- Resulting tunneling captured in "bounce equation"
- "Bounce eqn" describes the true vacuum bubbles, with a
  prob/time of the form:
  \[ \Gamma = A \exp \left( -\frac{\beta}{4\hbar} \right) \left[ 1 + O(\hbar) \right] \]
  In flat spacetime:
  \[ \Gamma(T) \approx T^{\frac{1}{4}} \left( \frac{S_{e}[\phi_{B}(r), T]}{2\pi T} \right)^{\frac{3}{2}} \exp \left( -\frac{S_{e}[\phi_{B}(r), T]}{T} \right) \]
  where \( S_{e}[\phi(r), T] \) is the Euclidean action

\[ S_{e}[\phi(r)] = 4\pi \int_{0}^{\infty} dr \, r^{2} [\frac{1}{2} (\frac{d\phi}{dr})^{2} + V_{\text{eff}}(\phi, T)] \]

is estimated at the bounce profile \( \phi_{B}(r) \), which is
a \( \phi^{4} \) field in the eqn. of motion \( \frac{d^{2}\phi}{dr^{2}} + \frac{2}{r} \frac{d\phi}{dr} = \frac{2\beta}{\hbar} \).

- \( V_{\text{eff}}(\phi, T) \) is the effective potential. This usually
  includes tree-level, 1-loop, 2-loop temp. contributions
  (Daisy resummation?). Model dependent.

b) Bubble Expansion
- After nucleation, true vacuum bubbles will expand and
  reach \( c \) or until collision within bubbles.
- BGR full of relativistic particles \( \Rightarrow \) fractional exponents.
  \( \Rightarrow \) The goal is to find out how
  bubble wall velocity: \( v \)
  friction from plasma: \( \eta \)
  Strength of \( PT \): \( \alpha \) (measures released vac. energy)
  Efficiency reaches \( \eta_{p} \) ky, ky \( \left( \frac{\text{vac. energy}}{\text{wall expansion}} \right) \)
c) Bubble Porculation
- True vacuum bubbles collide until PT is complete.
- PT duration can be estimated by mean bubble size at collision, characterized by parameter \( \beta \).
- Both \( \beta \) and \( \propto \) evaluated at the Nucleation Temperature \( T_0 \).

\[ T_0 : \text{Temp. at which } \propto \text{ of generated bubbles per unit time per Hubble vol. } = 1. \]

- Porculation has 3 main sources:
  
  (i) Bubble collisions
  - Collisions are violent
  - QCD PTs might have detectable GW from collisions (Witten), EW too (Hogan)
  - GWs from collisions depend mainly on \( \propto \) & \( \beta \) parameters.

  (ii) Turbulent Magnetohydrodynamics (MHD)
  - From turbulent motion of bulk fluid

  (iii) Sound Waves
  - Over-killing sound waves in bulk fluid.
  
  \[ \Rightarrow \text{All can source GW.} \]

d) Gravitational Waves
- Bubble nucleation requires a potential barrier for tunneling from false vacuum to true vacuum.
- Expansion needs fast-enough moving bubble walls for strong signals.
- Porculation needs efficient collisions so vacuum energy is dispersed into bulk fluid motions.

\[ \Rightarrow \text{So the question is:} \]

Which models of physics have 1st order PT so that we have detectable GW?
A quick list:

• **Additional Scalar Scalars**
  - (Gauge singlet ext. (weak Higgs & coupling), change scale (2ndHM))

• **Higher-dim. operators**
  - Simplest to add a cubic term (expected SUSY extensions)
    to create 1st order PT.

• **SUSY extensions**
  - Naturally exhibit PTs is MSSM & NMSSM.

• **Hidden Dark Sectors**
  - GWs originating from Dark sector dynamics.

• **Other RSM extensions**
  - Extra dim., Ricci–Gum PT, non-linear EWPT, QCD PT.

→ we'll focus mostly on Gauge Singlet scalar extension,
and take a short look at a more complex model.
Singlet Scaler extension of SM [1611.01617]

- Simple case, introduce a new real scalar singlet \( \phi \).

\[
V_c(H, \phi) = \mu^2 |H|^2 + \frac{\lambda}{2} |H|^4 + \frac{1}{2} M_q^2 \phi^2 + \frac{1}{2} \lambda_\phi^2 \phi^4 + \frac{\lambda_\phi}{2} |\phi|^4
+ \frac{\lambda_\phi}{2} |H|^2 + \frac{1}{2} \lambda_{\phi H} |\phi|^2 |H|^2
\]

\( m^2 \phi \) removed by shifting \( \phi \) WLOG.

- We will also consider a one-loop effective potential with finite-temperature corrections (i.e., a free energy).

- Imposing standard low-energy phenomenology (Higgs properties, stability) requires:

  1. \( H, \phi \) in a true, stable vacuum at origin at high \( T \).

  2. At the bubble nucleation temp \( T \approx T_N \approx T_c \approx (10^3, 10^5) \text{ GeV} \),

  \( \phi \) acquires a vev in a strong 1st order PT, giving CWeW.

  (\( T_N \) chosen to coincide with a 1600 TeV stringy peak)

  3. At low \( T \), \( H \) acquires vev \( V_H \approx 246 \text{ GeV} \).

- Explicit contributions to \( V_{\text{eff}} \) are:

\[ \Delta V_{\phi} = \frac{1}{2 \pi^2} \left[ \sum_{\text{all } F} J_{\phi F} \left( \frac{m^2_{F}}{T^2} \right) + \frac{Z_F}{Z_\phi} J_{\phi F} \left( \frac{m^2_{F}}{T^2} \right) \right] \]

\[ J_{\phi F} \left( \frac{m^2_{F}}{T^2} \right) = \int_0^x \text{ d}k \ k^4 \ln \left[ 1 + \exp \left( - \frac{k^2 + m^2_{F}}{T^2} \right) \right] \]

Sums over field-dependent BIF mass eigenvalues.

\( \Delta V_{\text{CW}} = \sum_{i} \frac{\alpha_i m_i^4}{24 \pi^2} \left[ \ln \left( \frac{m_i^2}{\mu^2} \right) - \frac{1}{2} \right] \)

\( \mu \) over-mass particles.
\[ \phi_i = \# \text{ d.o.f. of } \phi_i \]

\[ n_i = \frac{3}{2} \text{ (order parameters)} \]

\[ \frac{5}{2} \text{ (in vacua and near vacua)} \]

\textbf{Debye Masses}  (not enough explanation in reference)

- Come from bare mass terms in \( Z \) getting corrections
  like \( \Delta M_i^2 \propto T^2 \)  (Fermi-Dirac F.T. corrections)

\( \Rightarrow \begin{align*}
  \mathcal{V}_{\text{eff}} = \mathcal{V}_0 + \Delta V_D + \Delta V_T + \Delta V_{\text{ew}}
\end{align*} \)

- To have a strongly 1st order PT with GW, we need

  \begin{enumerate}
    \item At least two minima  \( \frac{\mathcal{V}_1}{\mathcal{V}_1'} = \frac{\mathcal{V}_T}{\mathcal{V}_T'} = 0 \)
      (false vacua)  (true vacua)
    \item Exist critical temp \( T_c \) where \( \mathcal{V}_1 = \mathcal{V}_1' \)
    \item Order param. \( \gamma = \frac{\langle \phi^2 \rangle}{\langle \phi \rangle^2} \) must be \( \approx O(1) \) at \( T_c \)
    \item Bubble nucleation, growth, collision.
  \end{enumerate}

(\( \Rightarrow \) in this ref, authors choose \( T_c \), \( \mathcal{V}_1 \) & \( \gamma \), and solve for \( \mathcal{Z} \)

parameters at high scales s.t. these conditions hold)

- Peak freq & Amp. of GW are controlled by \( T_c \).
  \( \Rightarrow \) this is also Temp at which a \( \gamma_c \) Volume fraction of the

  

\textit{Universe is in the true vacua.}

Occurs approximately at

\[ p(\ell) \propto \ell^{4+1} \]

\( \propto \) prob per unit time per unit vol. that a

\textbf{critical bubble forms.}

\[ p(\ell) \propto T^{-4} \exp \left[ -\frac{\langle \phi^2 \rangle (\mathcal{Z})}{T} \right] \]

(\( \propto \phi^2 \))

\( \propto \phi^2 \)

\textbf{So here is the same Euclidean action evaluated at a}

\textit{brane solution, as before:}

\[ S_E = \int d^4x \sqrt{g} \left[ \left( \frac{\partial \phi}{\partial x} \right)^2 + \mathcal{V}_{\text{eff}}(\phi, \dot{\phi}) \right] \]

\[ \text{where } \phi \text{ satisfies}
\]

\[ \frac{\partial^2 \phi}{\partial x^2} + \frac{2}{r^2} \frac{\partial \phi}{\partial x} = 2\nu(\phi, T) \]

\[ \phi(0) = 0, \phi(\infty) = 0 \]
- In a radiation-dominated universe, Temp <-> time relation is
\[ T^2 = -\frac{45}{16\pi^2} \frac{1}{\rho} \left( \frac{\dot{T}}{T} \right) \]

Combining the last 3 boxed equations, there is a relation
\[ SE(TW; \epsilon_s(c,TW)) \approx 170 - 4 \ln \left( \frac{T}{10^3} \right) - 2 \ln(a) \]
which can be solved numerically for \( T \) (by bisection).

- GW amplitude \( A_g \), \( PT \) depends on bulk wall velocity \( v_w \), latent heat released in transition b/w vacuum \( H \), efficiency of conversion of latent heat to GW, and duration of transition.

- Duration parameterized by
\[ \beta = -\frac{\dot{S}_T}{\dot{S}_U} \left| \delta V \right| = H \left( \frac{\partial H(c,eT)}{\partial \ln(T)} \right) \frac{SE}{T} \]
\[ \left( S_T = \frac{SE}{T}, \quad H = -\frac{\dot{T}}{T} \right) \]

Characteristic timescale is \( \tau \approx \frac{1}{\beta} \).

- Time-scale can be approx. as \( \frac{1}{H} \approx \frac{SE(TW)}{T} \) 

- Latent heat param, by
\[ \Delta_c = \frac{\Delta p}{f_n}, \quad f_n = \frac{V_0}{\Delta c} \]

where \( f_n \) is energy density of false vacuum, \( \Delta c = \#d\text{-wave cond} \).

\[ \Delta p = \left[ V_0 \frac{\partial f_n}{\partial T} \right] _E - \left[ V_0 \frac{\partial f_n}{\partial T} \right] _T \]

is latent heat in transition from \( E \rightarrow T \)

- \( v_w \) influences amp. \( P \) GW; slowed by friction terms arising from particle interactions. \( P \), less interacting means less friction. For high-scale SSU, very few friction terms, so \( v_w \approx 1 \).

- Conversion efficiency \( \epsilon \approx 1 \), so \( \epsilon = 1 \) is used as \( \gamma = 1.75 \) (is used).

- All factors combined into the envelope approximation to

\[ (A_g)^2 \approx 1 \]

- GW peak amplitude:
\[ \Omega_{2\text{ew}} \propto 10^{-4} \left( \frac{31.6 \text{ GeV}}{\beta} \right) \left( \frac{1.18}{\alpha + i} \right)^2 e^{\left( \frac{4 \times 10^{-8}}{0.45 \text{ GeV}^2} \right) \left( \frac{10^7}{3} \right)^{\frac{1}{3}}} \]

\[ \alpha = 10.77 \text{ GeV} \quad \text{in SSM} \]

- Factors as \( O(1) \) for a PT with \( T = (10^3, 10^8) \text{ GeV} \),

\[ \Omega_{2\text{ew}} \propto O(10^{-4}) \quad \text{if } x \approx 1 \text{ and } y \approx 2. \]

\( \text{If } \) aLIGO would be sensitive to \( \Omega_{2\text{ew}} \geq 5 \times 10^{-10} \)

\( \text{at about } O(100) \text{ Hz} \)

- Peak observable amp today at peak freq.

\[ f_0 \approx \frac{10.5 \text{ Hz}}{(\frac{4\pi}{T_0}) \left( \frac{T_n}{10^8 \text{ GeV}} \right)^{\frac{1}{16}}} \]

\[ f_n = \frac{0.005}{1.5 - 0.4m + m^2} \]

\( \Rightarrow f_0 \) coincides with aLIGO's max sensitivity \( f \approx 20 \text{ Hz} \)

\( \frac{T_n}{10^8 \text{ GeV}} \)

- Additional discussion: Vacuum stability

- at large field values, \( V_{eV}(n) = \frac{1}{2} \lambda (\phi \psi \phi) \psi^4 \)

for SM VEVs.

- Instability of SM Higgs potential.

- Instability can be remedied with SSM via modified \( \beta \)-function for quartic coupling or negative Higgs mass corrections.

- SSM can have additional stability constraints

\[ \lambda > 0, \quad \phi > 0 \quad \text{and} \quad K_2 > -2 \sqrt{\lambda} \]

from constraints in \( H = 0, \phi \geq 0, \lambda \psi^4 \) field space.

- Insure that \( H - \phi \) mixing angle is small so model agrees with experimental constraints for SM Higgs.

\[ \tan(\theta) \approx \frac{K_1 + K_2 \psi^2}{4 \lambda \psi^2 + K} \left( \frac{V}{V^0} \right) + O\left( \frac{V}{V^3} \right) \]

- There is a remaining residual \( \phi \)-modulated contribution to \( \lambda \), the SM quartic coupling (negative contribution to Higgs mass)

\[ m_h^2 = \left( \lambda - \frac{(K_1 + K_2 \psi^2)}{4 \lambda \psi^2 + K} V \right)^2 \leq V^2 \]

\( \Rightarrow \lambda \) must be greater than in SM.
\[ \Delta \lambda = \frac{(k_1 + \bar{k}_1 \lambda)}{k_2} \geq 0 \]

- Conditions in (4) are necessary, but not sufficient for stability. For a $\mathbb{Z}_2$ singlet potential and RN scales $\mu_2 \leq \lambda_2$ and $k_2 > 0$,

\[ \lambda_{SM} = \lambda - \Delta \lambda \geq 0 \]

is required to avoid deeper minima in $\phi = 0$ dirac.

Overall, need

\[ \mu_2 \leq \lambda_2 \leq \lambda_1 \]

6. Even if it is a scale at which this condition is broken, vacuum can still be stable.

SSM with Baryogenesis [1702.06124]

- Near-identical setup (- $\mu_2 \mid H|^2$ instead ? )

- Depending on the sign of $\mu_2$ (or $\lambda_2$), EW PT can occur in two ways:

1. $\mu_2 > 0$: happens at large $\lambda_2$, small $\lambda_2$.

   Potential grows as going away from $\phi = 0$, one-step PT from origin to EW min.

2. $\mu_2 < 0$: Small $\lambda_2$, large $\lambda_2$, origin $\rightarrow$ $\phi_{min}$, origin $\rightarrow$ EW min.

- Dynamics & PT are basically the same as before.

- For BG, necessary condition is decoupling of sphaleron processes after EWPT (not main focus)

   $\rightarrow$ provide B violation necessary for asymmetry.
- DM signals
  - $\phi$ can be treated as DM candidate, as it is stable.
  - Follow standard analysis using Boltzmann eqn.

$$\frac{\partial Y}{\partial x} = \frac{2\pi N_c}{4\pi} \left( \frac{m_\phi^2}{x^2} \right) \left( \frac{\text{thermal}}{5} \right) <\text{X}\hat{\text{X}}> \left( Y_{\text{th}} - Y_{\text{e}} \right)$$

$Y = \frac{W}{S}$, $x = \frac{m_\phi}{T}$, $<\text{X}\hat{\text{X}}>$. Thermo. avg. mass. Cross section, heat = energy. Dep.

- Solve 1 to find $n_{\phi, L}$ obtained.

$$\begin{align*}
\sum q_{\phi L}^2 &= \frac{N_c}{3} \frac{M_\phi^2}{M_\phi^2} \sim m_\phi^2 \times 2.76 \times 10^{-9} \\
\sum p_{\phi L}^2 &= 0.14 \quad (\text{Planck 2018})
\end{align*}$$

- Improving direct detection limits, $\phi$ cannot be a single-component DM; in fact, $\phi$ cannot be all of DM. It is the only hidden sector particle.

- More in-depth discussion on cosmological modification consideration in the reference paper.

- Asymmetric DM from GW (2209.04788)
  - A more complex BSM physics model featuring potentially detectable GW from 1st order PT (also Prim. vacuum walls).
  - Gauge group

$$SU(3)_c \times SU(2)_L \times U(1)_y \times SU(2)_e$$

- Quarks are $SU(2)_c$ singlets
- Leptons are upper components of $SU(2)_e$ doublets

$$\begin{align*}
(l_1, \bar{l}_1)^T &= (1, 1, \frac{1}{2}, 1) \\
(l_2, \bar{l}_2)^T &= (1, 1, -\frac{1}{2}, 1) \\
(l_3, \bar{l}_3)^T &= (1, 1, -1, 1)
\end{align*}$$

$$\begin{align*}
(\nu_1, \bar{\nu}_1)^T &= (1, 1, 0, 1) \\
(\nu_2, \bar{\nu}_2)^T &= (1, 1, 0, 1)
\end{align*}$$

- Break $SU(2)_c$ spontaneously by introducing two complex doublets.
scalar doublets, $\Phi_1$ and $\Phi_2$ (this also provides a mechanism for $B-L$)

- Scalar potential

$$V(\Phi_1, \Phi_2) = m_{\psi_1}^2 |\Phi_1|^2 + m_{\psi_2}^2 |\Phi_2|^2 - (m_{\psi_{12}} \Phi_1^\dagger \Phi_2 + \text{h.c.})$$

$$\quad + \lambda_1 |\Phi_1|^4 + \lambda_2 |\Phi_2|^4 + \lambda_3 |\Phi_1|^2 |\Phi_2|^2 + \lambda_{12} \Phi_1^\dagger \Phi_2^\dagger$$

$$\quad + \left[ \left( \tilde{\lambda}_1 |\Phi_1|^2 + \tilde{\lambda}_2 |\Phi_2|^2 + \tilde{\lambda}_{12} \Phi_1^\dagger \Phi_2^\dagger \right) \Phi_1^\dagger \Phi_2 + \text{h.c.} \right]$$

$$m_{\psi_1}^2, \lambda_1, \lambda_2, \lambda_3 \in \mathbb{C}$$

- $\langle \Phi_1 \rangle = v_1$, $\langle \Phi_2 \rangle = v_2$, breaks $SU(2)_L$ symmetry, leaving $\mathbb{R}^4$ in the SM Gauge group.

$$\Phi_1 = \begin{pmatrix} c_\theta + ic_{\psi_1} \\ i\hat{c}_\theta + p_{\psi_1} + i\alpha_{\psi_1} \end{pmatrix}$$

- After $SU(2)_L$ breaking, there are 6 electrically charged and 6 neutral new fermionic stables. A remnant $U(1)_e$ symmetry prevents decay to SM particles.

1. highest of these could be DM candidate!

- 3 new gauge bosons $Z'$, $\omega'$, $\omega''$, masses

$$m_{Z',\omega',\omega''} = \frac{g_1}{2} v_1$$

- Scalars $\Phi_{1,2}$ (CP even), $A$ (CP odd), $\chi_{1,2}$ (complex)

$$
\text{Veff}(\Phi_1, \Phi_2, T) = V_{\text{tree}}(\Phi_1, \Phi_2) + V_{\text{loop}}(\Phi_1, \Phi_2) + V_T(\Phi_1, \Phi_2, T),
$$

very similar to SUSM cases.

$\Rightarrow$ As before, potential shape evolves/develops new minima as $T$

drops to Te & TN ($\Phi_1, \Phi_2$ stable)

- In this case, multiple (4) minima develop, related

by approximate $Z_2$ symmetry of the potential, two pairs related by a gauge symmetry,

i.e., only two physically distinct true vacua:

$$(\Phi_1, \Phi_2)_{\text{true}} \quad (\Phi_1, -\Phi_2)_{\text{true}}$$

$\Rightarrow$ energy density of each may differ slightly;
if splitting is small, domain walls may form.

- 1st order PT GW analysis just as before, one
Classically Conformal U(1) extension (tie-in to current work)

- Impose classical conformal symmetry to forbid explicit
  - mass^2 terms
- Extend SM gauge group to SU(3) x SU(2)_L x U(1)_Y x U(1)_H,
  introducing new Higgs-like scalar \( H \) in new hidden sector.
  - U(1)_H symmetry broken by CW mechanism at \( V \), from
    1-loop corrections via \( V_{\text{loop}}(\phi) \).
  \[
  \Rightarrow V = \frac{1}{2} \chi H H' + \frac{1}{4} \phi \phi H H' + \alpha \chi H H' H H' + V_{\text{loop}}(\phi)
  \]
  - \( \mu^2 \) term induced by Higgs-\( H \) mixing term.
  \Rightarrow EWSB induced by CW mechanism symmetry breaking.
  \( \Rightarrow \) perhaps \( H \) a 1st order PT between
  \( V \phi \) minimum and EW minimum

- How does the conformal symmetry affect the dynamics of the PT?
- Will other sources of GW be more dominant than bubble collisions?
- What happens if we take \( Z' \) (or another particle) as DM candidate?
- What if we use SU(2)_H as the extension?