A Talk on

The Hierarchy Problem and New Dimensions at a Millimeter

November 5th, 2021

Contents:-

Motivation:-

- Solving Standard Model Hierarchy Problem
- New Theory different from Supersymmetry and Technicolor

Strategy

- Electroweak Interactions probed at distances $\frac{1}{m_{EW}}$
- Gravitational force probed upto 1 cm and based on assumption to be accurate upto Planck length 10^{-33} cm
- Only fundamental scale at $\frac{1}{m_{EW}}$ considered instead of two.
- Considering n extra dimension modified gravitational potential becomes:
  $V(r) = \frac{m_1 m_2}{M_{Pl(4+n)}^{n+2}} \frac{1}{r^{n+1}}$ for $(r << R)$,
  $V(r) = \frac{m_1 m_2}{M_{Pl(4+n)}^{n+2}} \frac{1}{rR^n}$, $(r >> R)$
- Effective 4D $M_{Pl}^2 = M_{Pl(4+n)}^{2+n} R^n$

Strategy Continued

- For $M_{Pl(4+n)} = m_{EW}$ demanding $R = 10^{\frac{30}{n}-17}$ cm $\times \frac{1 TeV}{m_{EW}}$
- n=1, $R \sim 10^{13}$cm shows deviations from Newtonian gravity over solar system distances
- n=2, $R \sim 100$ micrometer to 1 mm plausible range for gravity deviation.
- SM particles confined in 4D manifold, Gravitons can move in bulk, with couplings suppressed by m_{EW}.
- SM extensions leading to proton decay, neutral meson mixing are prohibited.
- New model shouldn't introduce large correction to EW observations.

Strategy Continued

- SM particles localization done using the concept of topological defect, where SM fields localized in throat of weak scale vortex of 6D theory.
- Supersymmetry not required, but for self consistency above m_{EW} scale the theory required is superstring theory.
- Gravity law changes from $\frac{1}{r^2}$ to $\frac{1}{r^4}$ in 100 micrometer to 1 cm range.
- For hard collisions $E_{esc} \geq m_{EW}$, SM particles can escape into 2 extra dimension, can be identified with transverse momentum loss in colliders.
- Single electric charge or any other conserved charge can't penetrate into the extra dimension for the conservation of charges

Astrophysical Constraints

- Considering multiplicity of gravitons to be $(ER)^n$ to be large, the combined emission rate for graviton is $\frac{1}{M_{Pl}^2}(\Delta ER)^n$, where ΔE is the energy available to the graviton
- Using $M_{Pl}^2 = M_{Pl(4+n)}^{(2+n)}$ this becomes $\frac{\Delta E^n}{m_{EW}^{2+n}}$
- Analogous to goldstone bosons , axions and neutrinos, gravitons carry bulk energy from stars and accelerate cooling of stars. Hence, Constraints comes from Sun, Red Giant and SN 1987A.
- Equating goldstone bosons emmission rate with that of gravitons
  $\frac{1}{F^2} = \frac{\Delta E^n}{m_{EW}^{2+n}}$, where F is decay constant of goldstone bosons.
- Using $\Delta E = 1$kev, $m_{EW} = 1$TeV, $n = 2$ gives $F = 10^{12}$GeV. observed one is 10^7 GeV
- or red giants with $\Delta E = 100$kev, $F = 10^{10}$GeV
- For SN1987A with $\Delta E = 20 - 70$Mev, $F = 10^8$GeV, while observed lower limit is 10^{10}GeV

Phenomenological Constraints

- 4. Among Phenomenological proton decay, $K - \bar{K}$ mixing are forbidden. Y epsilon decay branching ratio $= 10^{-8}$.
  $Z \to Z + graviton$ decay branching ratio becomes 10^{-5}.

Construction of a Realistic Model:

- 6D metric $g_{AB} = (-1, 1, 1, 1, 1, 1)$
- Two extra dimensions compactified in manifold with $R = 1mm$
- Two possible topologies possible are S^2 and T^2 of zero radius
- SM particles confines in small region of cutoff wavelength Λ^{-1} and can penetrate in the bulk in form of heavy mode of mass of order Λ
- Localization obtained considering zero modes trapped in the core of a four dimensional vortex. Topological defect, in its ground state, is independent of four coordinates and carves out 4D hypersurface, constituting our universe.

S^2:

- x_5 and x_6 compactified in S^2.
- 6D scalar field defined $\Phi = \phi_{bulk} \exp^{i\theta}$ with $0 \leq \theta \leq 2\pi$ obeying $U(1)_V$ symmetry and ϕ_{bulk} minimizes the potential.
- Nonezero VEV value Λ of Φbreaks $U(1)_V$ symmetry.
- Two zeros of the absolute value can be observed at the both sides of equator., can be placed in north and south poles, designated as vortex, anti vortex pair of thickness $^{-1}$. Size of the vortex much smaller than separation length 1 mm.vortex .
- Using Nielsen Olesen solution $\Phi = f(r) \exp(i\theta)$ with boundary conditions $f(0) = 0$ and $f(r)|_{r>>\Lambda^{-1}} \sim \phi_{bulk}$,$0 < r < 2\pi R$. The solution for anti vortex corresponding to $\theta \to -\theta$ and $r \to 2\pi R - r$

T^2:

- Stable compared to S^2

Localization of Fermions:

- Fermions can be trapped as zero mode in vortex.
- 1 pair 6D Weyl spinors defined Ψ and ζ
- Mass term formed out of vortex field coupling $h\Phi\Psi\zeta + h.c.$, with $h = O(\Lambda^{-1})$
- 6D Dirac equation in vortex background
  $\Gamma_A \delta^A \Psi^+ = h\phi_{bulk} \exp^{i\theta} \zeta$
  $\Gamma_A \delta^A \zeta + = h\phi_{bulk} \exp^{i\theta} \Psi$
- Using separation of variations $\Psi = \Psi(x_\mu)\beta(r)$ and $\zeta = \zeta(x_\mu)\beta(r)$ with $r = r(x_5, x_6)$
- Zero modes will satisfy
  $\Gamma_5 \exp^{i\theta(-\Gamma_5\Gamma_6)} \Psi^+ \delta^r \beta(r) = h\phi_{bulk} \exp^{i\theta} \zeta$
  $\Gamma_5 \; exp^{i\theta(-\Gamma_5\Gamma_6)} \zeta^+ \delta^r \beta(r) = h\phi_{bulk} \exp^{i\theta} \Psi$
- 5. Where $\beta(r) = \exp(-h \int_0^r f(r')dr')$, vortex supports single 4D chiral mode $\Psi_L +^+_R$
- Fermions get mass by $H\Psi\Psi'$ interaction

Localization of Higgs Scalars::

- For $h' > 0$ $h'|\Phi|^2|H|^2$ sort of interaction creates attractive potential for Higgs field localization.
- Negative effective mass at vortex throat achieved using $(h'|\Phi|^2 - m^2)HH^\dagger) + c(HH^\dagger)^2$ with $m^2, h', c > 0$
- $h'|\Phi|^2 - m^2 > 0$, $|H|$ zero in bulk.
- Small excitation in vortex background gives E.o.M. $-\delta^2_{5,6}H + (h'f - m^2)H = \omega^2 H$
- H being a doublet, gives rise to 3 goldstone bosons, which being eaten up by W, Z bosons makes them massive.

Localization of Gauge Fields:

- Localization using charged scalar interaction with vortex field makes photon massive.
- It involves SU(2) Yang Mills theory, a scalar field χ and vortex field breaking additional $U(1)_V$ symmetry and forming strings with Lagrangian
  $L = -\frac{1}{4g^2} TrG_{\mu\nu}G^{\mu\nu} + (D_\mu\chi)^2 - \lambda(\chi^2)^2 - \chi^2(h'|\Phi|^2 - M^2) + |\delta_\mu\Phi|^2 - \lambda'(|\Phi|^2 - \phi^2_{bulk})^2$
  where $M^2, h', \lambda, \lambda' > 0$
- $h'|\Phi|^2 - M^2 > 0$, hence χ zero in bulk.
- $SU(2)$ breaks to $U(1)_{EM}$, keeps phone massless and other gluons massive.
- Theory outside vortex being non-abelian makes photon massive.

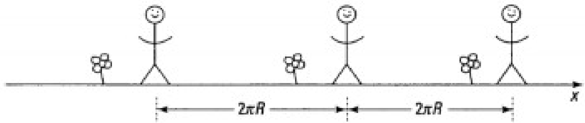
Realistic Theory:

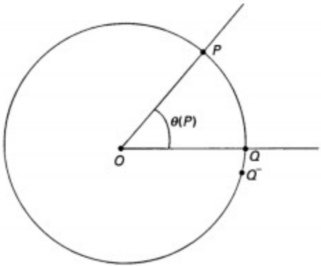
- SM embedded into Pati Salam Group G= $SU(4) \times SU(2)_R \times SU_R$
- $U(1)_V$ symmetry gives charge to vortex field Φ, which forms vortex of thickness of $O(\Delta^{-1})$ in compactified manifold (x_5, x_6).
- Gauge group spontaneously broken by 6D scalar fields $\chi = (15, 1, 1)$, $\chi' = (4, 2, 1)$, and $H = (1, 2, 2)$.
- Soft Hierachy assumed $\chi' = \chi = \Delta = 10H = m_{EW}$
- Unbroken $SU(3) \times U(1)_{EW}$ symmetry implies gluons and photon to be massive evrywhere in the vortex.
- Fermions generated from 6D chiral spinors $Q = (4, 1, 2), \bar{Q} = (\bar{4}, 1, 2), Q_c = (\bar{4}, 2, 1), \bar{Q}_c = (4, 2, 1)$
- These get bulk mass $h\Phi Q(\bar{Q}) + h'\Phi^* Q_c(\bar{Q}_c)$, keeping single chiral zero mode $Q_L + \bar{Q}_R^+$ and $Q_{cL}^+ + \bar{Q}_{cR}$
- Its gets mass from normal Higgs interaction $gHQQ_c + \bar{g}H\bar{Q}\bar{Q}_c$ in vortex

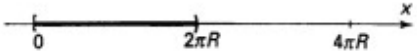
Conclusions:-

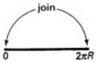
1. Fine tuning problem solved due to parameter constraints
2. There is no issue of ultraviolet sensitivity and radiative destabilizing issue of 1 mm scale

Extra Dimension Compactification:-









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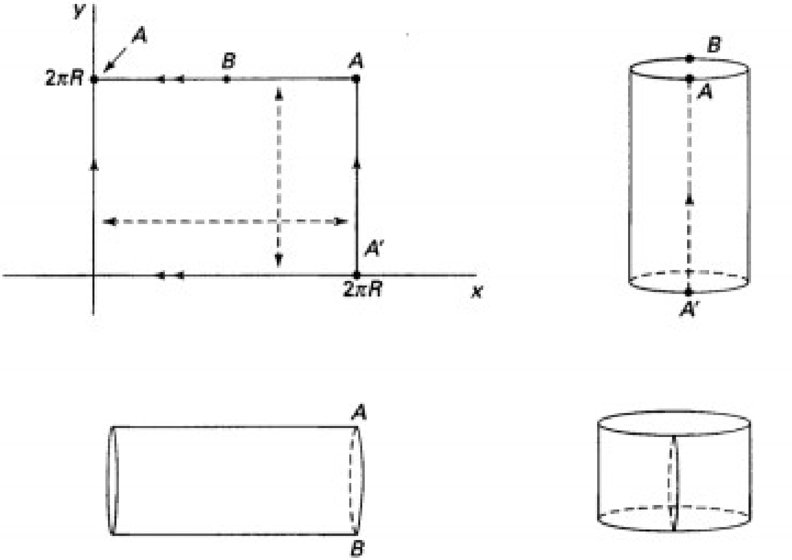

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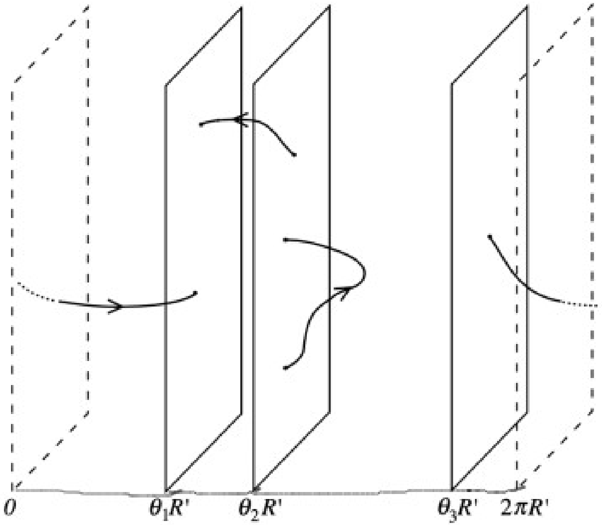

s^2 and T^2:-