QUALIFYING EXAMINATION

FALL 1995

General Instructions: No reference materials (other than a calculator) are permitted. Do all work in your answer booklet. Turn in the questions for each part with the answer booklet. You may finish Part I early, turn it in and start on Part II.

PART I: Classical Mechanics

Work any 5 of the 6 problems.

1. Assume the earth to be a solid sphere of uniform density. A hole is drilled through the earth, passing through its center, and a ball is dropped into the hole. Neglect friction and:

   a) calculate the time for the ball to return to the release point. Express the answer in terms of g (the acceleration of gravity at the surface of the earth) and R (the radius of the earth);

   b) compare the result of part a) to the time required for the ball to complete a circular orbit of radius R in the same gravitational field.

2. A uniform disk of radius R is pivoted to oscillate about a horizontal axis which is perpendicular to the disk and located at the perimeter of the disk. Find the frequency of oscillation assuming small amplitudes.

3. A 70 lb. ladder of uniform construction leans against a wall as shown below. The wall is perfectly smooth and the coefficient of static friction between the ladder and the floor is $\mu = 0.4$. How far can a 160 lb. man climb up the ladder before it slips?

4. Use the Lagrangian method to determine the equations of motion of a spherical pendulum (a simple pendulum with a massless, rigid rod free to swing through the entire solid angle about a point). What are the constants of the motion?

5. A billiard ball moving at 5 m/s strikes a stationary ball of the same mass. After the collision, the first ball moves at 4.33 m/s at an angle of 30° with respect to the original line of motion. Assume that rotational effects and friction can be neglected.

   a) Find the magnitude and direction of the struck ball's velocity.

   b) Is the collision elastic or inelastic? Explain.

6. A point mass m is located a distance x from the center and along the axis of a homogeneous ring of mass M and radius R as shown below.

   a) What is the acceleration of the mass m due to its gravitational attraction with the ring?

   b) If the mass m is released from rest, find the speed of the mass when it passes through the center of the ring. (Assume $M>>m$.)

PART II: Electricity and Magnetism

Work any 5 out of 6 problems.

1. Consider two charges, - 4.0 μC and + 8.0 μC, separated by 6 cm.

a) What is the magnitude and direction of the electric field at a point midway between the charges?

b) Find the point where the electric field is zero.

2. The terminals of a battery having an emf of 2.5 V are connected together by a copper wire of length 314 m and a diameter of 1 mm. The current (at room temperature) in the circuit is 75 mA.

a) Find the resistivity of copper at room temperature.

b) After 10 minutes, the temperature of the wire has increased by 100°C. Find the new resistance of the copper wire. The temperature coefficient of resistivity for copper is

$$\alpha_{cu} = 0.0068\ (°C)^{-1}$$

3. Consider the spherically symmetric potential

$$V(\vec{r}) = V(r) = \frac{Q}{4\pi\varepsilon_0} e^{-\alpha r}$$

where α is a constant having dimensions of m^{-1}.

a) Find the electric field $\vec{E}(r)$.

b) Find the charge volume charge density $\rho(r)$.

4. Consider a solid sphere having a radius of R_2. The charge density is given by

$$\rho(r) = \begin{cases} \rho_1, & \text{if } 0 < r < R_1 \\ \rho_2, & \text{if } R_1 < r < R_2 \end{cases}$$

where ρ_1 and ρ_2 are constants.

a) Find the electric field, $\vec{E}(r)$ at a point r such that $R_1<r<R_2$.

b) Find the potential V(r) at a point r such that $R_1 < r < R_2$.

5. Consider an infinite straight wire having a circular cross section of radius R. The current $\bar{I}$ is uniformly distributed within the wire.

a) Find the $\bar{B}$ field in the wire at a distance r from the central axis where $r < R$.

b) Find the $\bar{B}$ field outside of the wire at a distance r from the central axis.

6. Consider the two vectors

$$\vec{v}_1 = \alpha[(x + 2y + 4z)\,\hat{i} + (2x - 3y - z)\,\hat{j} + (4x - y + 2z)\,\hat{k}] \text{ and}$$

$$\vec{v}_2 = \beta[(xy)\,\hat{i} + (2yz)\,\hat{j} + (3xz)\,\hat{k}]$$

where α and β are dimensional constants. One of these vectors represents an electric field and the other a vector potential for a non zero magnetic field. Which is which?

PART III: Quantum Mechanics

Work any 5 out of 6 problems

1. The ground state wavefunction of the Hydrogen atom is

$$\psi = A \exp \frac{-r}{a_o}$$

where r is the radial coordinate.

a) Determine the normalization constant A in terms of a_o

b) Determine the value of r at which the electron is most likely found.

2. What is the deBroglie wavelength of a:

a) 1 MeV photon?

b) 1 MeV electron?

Use $h = 4.14 \times 10^{-15}$ eV-s and $m_o c^2 = .511$ MeV, m_o being the rest mass of the electron.

3. a) What are the electronic quantum numbers for the hydrogen atom? What values can they take? What physical significance does each quantum number have?

b) In spectroscopic notation the state $^2S_{3/2}$ means $l = 0$, $j = 3/2$ and $s = 1/2$. Is this state possible? Explain.

4. a) For which component(s) of the momentum operator $\vec{P} = (P_x, P_y, P_z)$ is

$$\Psi(x, y, z) = e^{iax} \cos(by) \sinh(cy)$$

an eigenfunction? (a, b and c are constants).
What is (are) the associated eigenvalue(s)?

b) Show that

$$\Psi(x, y, z) = y + iz$$

is an eigenfunction of the angular momentum operator L_x. What is its associated eigenvalue?

5. Starting with the time independent Schrödinger equation, derive the eigenfunctions and allowed energy levels of a particle in an infinitely deep square well. Sketch the first three eigenfunctions and probability functions.

6. A particle with energy $E > V_o$ is incident upon a potential barrier, M. The potential given by

$$V = \begin{cases} 0, & x < 0 \\ V_o, & x > 0. \end{cases}$$

Write down the expression for the reflection coefficient as a function only of the ratio V_o/E.

PART IV: Mixed Topics

Do problems form 4 of the 5 sections. Astrophysics may be chosen only in place of electronics. Do 6 of the 8 problems with at least one from each of the 4 chosen sections. Use a different answer book for each lettered section.

A: Relativity

1. a) Write down the equations for the Lorentz transformation from an inertial frame [with coordinates (x, y, z, t)] to another inertial frame [with coordinates (x′,y′,z′,t′)] traveling in the position x-direction with speed V relative to the former frame.

   c) Show that if $v_x = 0$, $v_y = 0$, and $v_z = c$, then the magnitude of the velocity ($\bar{v}'$) in the (x′,y′,z′,t′) system is c.

2. A space ship is to travel to the star α-Centauri, a distance of 4.4 light years from the earth. Assuming that the space ship travels at a constant velocity for all but a negligible portion of the trip and that, according to a clock on the earth, the time for the trip is 6 years, find the time as experienced by the space travelers and the distance traveled in their reference frame.

B: Thermodynamics

1. Two systems A and B with specific heat constants c_A and c_B and masses m_A and m_B initially at temperatures T_A and T_B, respectively, are brought into contact with each other. After the systems reach a common final temperature T_f, find the change in entropy of the entire system. Show explicitly that this change in entropy is never negative.

2. Air at a temperature $T = 20°$ C and a pressure $p = 760$ mm Hg is carried by a wind to the top of a mountain, thus reducing the pressure to $p' = 600$ mm Hg. Assuming that the change can be considered as a continuous adiabatic process and treating air as an ideal diatomic gas with constant heat capacity c_v per mole, find its temperature T' in ^{0}C at the top of the mountain.

1. An object is placed 15 cm to the left of a convergent lens ($f_1 = 10$ cm). A divergent lens ($f_2 = -20$ cm) is placed 20 cm to the right of the converging lens.

a) Draw a careful ray diagram on the grid provided below, and use it to locate and determine the characteristics of the final image formed by the two lenses. Is it real or virtual, upright or inverted, enlarged or reduced?

b) Use the thin lens formula to calculate the position and magnification of the final image formed by the two lenses. Compare with the results of part a) and explaine any discrepancies.

2. Vertically polarized light is passed through three successive polaroid filters.

a) If the transmission axes are at 30°, 60°, and 90° from the vertical, what percentage of the incident intensity is transmitted?

b) If the first and last polaroids are reoriented at 0° and 90° respectively and the center polaroid is free to rotate, to what angle should it be turned so that the transmitted intensity is maximized?

c) To what angle should it be turned to minimize the transmitted intensity?

D: Astrophysics

1. a) The radio luminosity of Cygnus A is $L_r \approx 10^{41}$ erg s^{-1}. Let us assume that this active galaxy is powered by an accretion disk around a massive black hole. If the potential energy of infall is given by the potential energy at the Schwartzschild radius, $R_{bh} = 2\ Gm_{bh}/c^2$, and the fraction that can be converted into radio emission is $\chi = 0.001$, what is the amount of material in solar masses/ year that must fall into the black hole?

b) With this value for $\frac{dM}{dt}$ in units of solar masses/year, use the following expression

$L_T = 2.4\ \frac{dM}{dt}$ x 10^{46} erg s^{-1} to obtain the total luminosity of Cygnus A (neglecting stellar emission).

c) Finally, assuming that emission from the accretion disk is primarily in the UV portion of the spectrum ($\lambda = 1000$ Å) use the Wien law $\lambda_{max} = 2.9 \times 10^7$ Å/T (kelvins) and

$$T = 2.3 \times 10^7\ [L_T\ (\text{erg s}^{-1})/10^{38}]^{1/4}\ R_{bh}(\text{km})^{-1/2}\ \text{kelvins}$$

to estimate the radius R_{bh} in kilometers and the mass M_{bh} in solar mass units of the black hole.

$G = 6.67 \times 10^{-8}$dyn cm^{2}gm^{-1}, solar mass $= 2 \times 10^{33}$gm, $c = 3 \times 10^{10}$cm s^{-1})

2. In 1979 the central core in the Seyfert galaxy NGC 4151 flared. Thirteen days later the carbon IV line at a rest wavelength of 5801 Å flared. Assume that the carbon IV line comes from clouds orbiting the central core and the core and line emitting region is unresolved by the telescope. The line has a gaussian profile with full width at half maximum of $\Delta\lambda = 542$ Å. How far are the clouds from the central core? What is the approximate orbital velocity of the clouds relative to the core? How much mass is in the central core in solar mass units?

($G = 6.67 \times 10^{-8}$dyn cm^{2}gm^{-1}, solar mass $= 2 \times 10^{33}$gm, $c = 3 \times 10^{10}$cm s^{-1})

E: Electronics

1. The circuit below includes a standard npn transistor with a current gain $h_{FE}=\beta=100$ and a simple 12 V, 100 mA Light Emitting Diode (LED).

   a) When the switch S is open, what are the currents I_B, I_C and I_E? Does the LED light?

   b) When the switch S is closed, what are the currents I_B, I_C and I_E? Does the LED light?

   c) Resistor R1 is replaced with a different value of R1'=1 kΩ and switch S is closed. What are the currents I_B, I_C and I_E? Does the LED light?

2. Consider the circuit shown below.

   a) State Kirchoff's Laws.

   b) What Conservation Laws are they derived from?

   c) Find the currents I_1 and I_2. Assume the directions of the currents are as indicated.