QUALIFYING EXAMINATION
FALL 1992

PART I -- CLASSICAL MECHANICS

Work any 5 of the 6 problems.

1)

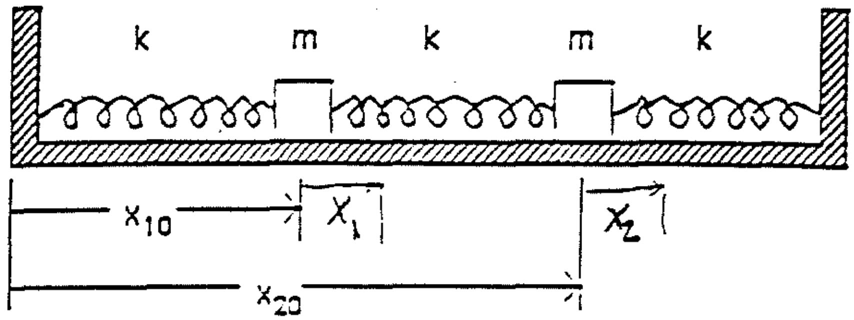
a) Find the value of the constant c such that the following force is conservative.

$$F = \hat{i}xy + \hat{j}cx^2 + \hat{k}z^3$$

b) If c has this value, find the potential energy function.

2) A scoop of mass m_1 is attached to an arm of length l and negligible weight. The arm is pivoted so that the scoop is free to swing in a vertical arc of radius l. At a distance l directly below the pivot is a pile of sand. The scoop is lifted until the arm is at a 45° angle with the vertical, and released. It swings down and scoops up a mass m_2 of sand. To what angle with the vertical does the arm of the scoop rise after picking up the sand? This problem is to be solved by considering carefully which conservation laws are applicable to each part of the swing of the scoop. Friction is to be neglected, except that required to keep the sand in the scoop.

3) Consider the combination of masses and springs shown in the following diagram. x_{10} and x_{20} are the equilibrium positions of the masses. All three springs are identical.



Find the equations of motion for this system.

4) A car rounds a banked curve. The radius of curvature of the road is R, the banking angle is θ, and the coefficient of static friction is μ. (a) Determine the range of speeds the car can have without slipping up or down the road. (b) Find the minimum value for μ such that the minimum speed is zero. (c) What is the range of speeds possible if $R = 100$ m, $\theta = 10°$, and $\mu = 0.1$ (slippery conditions)?

5) A block of mass m is given an initial velocity v_0 up an inclined plane that makes an angle θ with the horizontal. The coefficient of kinetic friction between the block and the plane is μ. The block goes up the plane a distance ℓ, then turns around and slides back down the plane.

How fast is the block going when it gets back to the bottom of the plane?

How much time does it take for the block to slide up the plane?

How much time does it take for the block to slide down the plane?

What is the relation between ℓ and μ?

6) A yo-yo of mass m is on a horizontal table. The radius of the yo-yo is a and the radius of its hub is b. A horizontal pull, F, is applied to the string of the yo-yo. What is its acceleration? Assume that the yo-yo has a moment of inertia given by $(1/2)ma^2$ and that there is enough friction that the yo-yo does not slide.

PART II - ELECTRICITY AND MAGNETISM

Do all 5 problems.

1) If empty space is filled with a charge density of the form

$$\rho(\vec{r}) = \rho_0 (b/r)^2 \, e^{-r/b}$$

with ρ_0 and b constant, what is the electric field everywhere in space?

2) If empty space is filled with a current density of the form

$$\vec{J}(\vec{r}) = j_0 \, (b/\rho) \, e^{-\rho/b} \, \hat{e}_z$$

what is the magnetic induction, $\vec{B}$, everywhere in space? Here j_0 and b are constant, ρ is the radial coordinate in the xy plane and e_z is the unit vector in the z direction.

3) An infinite wire carrying current I_1 in the direction of the positive z axis is a distance d away from a parallel wire carrying current I_2 in the same direction. What is the magnitude of the force per unit length exerted by the first wire on the second? Do the wires attract or repel?

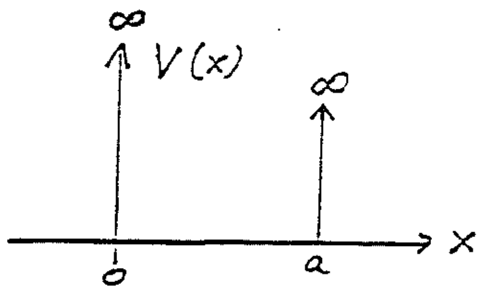
4) What is the capacitance of a system consisting of two concentric conducting spheres of radii R_1 and R_2 separated by a vacuum?

5) Write down Maxwell's equations in a vacuum. (MKS units) Solve these equations and show explicitly the solutions are traveling waves. What is the velocity of propagation of these waves?

PART III -- QUANTUM MECHANICS

Answer any four question.

1) Determine the energy levels and the normalized wave functions of a particle in a potential well. The potential energy V of the particle is:



$$V = \begin{cases} \infty & \text{for } x < 0 \text{ and } x > a \\ 0 & \text{for } 0 < x < a \end{cases}$$

2) The function

$$\Psi(r,\theta,\phi) = \frac{1}{8\sqrt{\pi}} \left(\frac{Z}{a_o}\right)^{3/2} \frac{Zr}{a_o} e^{-\frac{Zr}{2a_o}} \sin\theta \cdot e^{i\phi}$$

is the wave function for a particular quantum system written in spherical coordinates.

a) Is this an eigenfunction of the orbital angular momentum operator L_Z? If so, what is the eigenvalue for this operator?

b) Same as a) for the square of the total orbital angular momentum operator L^2.

3) A hydrogen atom is placed in a constant magnetic field. Use the Bohr model to compute the magnetic moment due to the orbital motion of the electron. From this result, find the relation between the magnetic moment and the angular momentum and find the new energy eigenvalues of hydrogen in a constant magnetic field (neglect spin). Now, using the known g-factor for the electron spin, include spin effects in your energy eigenvalues. Indicate how the energy levels of an electron in a p-state change in the presence of a magnetic field.

4) Two non-identical particles of mass m move in one dimension. Each experiences a linear restoring force corresponding to the harmonic oscillator potential $V(x) = 1/2\ kx^2$. In addition, there is a repulsive force acting between the two particles corresponding to the potential $- 1/2\ \lambda k(x_1 - x_2)^2$, where λ is a constant that is less than 1/2.

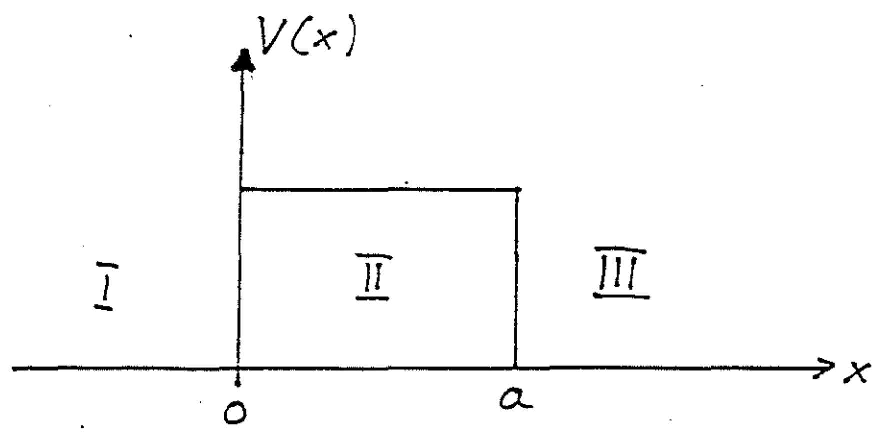
a) Write down the time-independent Schrodinger equation which governs the eigenfunctions $\Psi(x_1, x_2)$ of this system.

b) Rewrite the Schrodinger equation in terms of the reduced coordinate $r = (x_2 - x_1)$ and the center of mass coordinate $R = 1/2(x_2 + x_1)$.

c) Obtain the equations governing $\theta(r)$ and $\phi(R)$ where $\Psi(R,r) = \theta(r)\phi(R)$.

d) In terms of λ, k, and m, what are the energy eigenvalues of this system?

5) Determine the transmission coefficient of a particle (energy E) through a rectangular barrier (width a, height H).



PART IV - MIXED TOPICS

Do problems from 4 sections. Astrophysics may be chosen only in place of Electronics.

Do 6 of the 8 problems with at least one from each of the 4 chosen sections.

A: RELATIVITY

1) A particle of mass m_0 traveling at speed 0.98c collides head-on with a target particle of mass $3m_0$ initially at rest. If the target particle moves away from the collision at speed 0.89c, what is the velocity of the incident particle after the collision?

2) A beam of electrons is projected into a uniform magnetic field B, the beam perpendicular to B.
a) At what speed v will the radius of curvature of the electron beam in the magnetic field be twice that predicted by classical physics?
b) What is the kinetic energy (MeV) of the electrons in the beam?

B: THERMAL PHYSICS

1. An ideal gas of particles having a mass m is contained in a closed cavity at a temperature T. The probability of finding a particle with a speed between v and v + dv is given by the Maxwell-Boltzmann speed distribution

$$P(v)\ dv = A\ v^2\ e^{-mv^2/2kT}$$

where A is a constant chosen so that

$$\int_0^\infty P(v)\ dv = 1 \quad .$$

(A). Find the average energy of a particle inside the cavity.

(B). A small hole is made in a wall so that a small number of particles escapes. Find the probability of finding a particle with a speed between v and v + dv in the escaping beam.

(C). Show that the ratio of the average energy of particles in the beam to those inside is 4/3.

Given:

$$\int_0^\infty y^{2n}\ e^{-\beta y^2}\ dy = \frac{1\cdot 3\cdots(2n-1)}{2^{n+1}}\ [\pi/\beta^{2n+1}]^{1/2}$$

and

$$\int_0^\infty y^{2n+1}\ e^{-\beta y^2}\ dy = \frac{n!}{2\ \beta^{n+1}}$$

2. Consider a highly relativistic particle ($E \approx pc$) moving in a sphere of diameter L. The usual deBroglie relation

$$p = h/\lambda ,$$

where λ is the deBroglie wavelength holds.

a) Write down an expression for the allowable energies of the particle.

b) Find the partition function Z.

C: OPTICS

1) The electric field of an electromagnetic wave is given by

$$\mathbf{E} = \hat{i}E_x\cos(ky + \omega t) + \hat{k}E_z\cos(ky + \omega t + \pi/2)$$

where $\hat{i}$ is a unit vector in the x direction, $\hat{k}$ is a unit vector in the z direction, $\omega = 6\pi \times 10^{14}$ Hz and $k = 4\pi \times 10^{5}$ m^{-1}. In this problem $E_x = 4$ and $E_z = 3$ in arbitrary units.

a) In what direction is the wave traveling?

b) What is the wavelength of the wave?

c) What is the frequency of the wave?

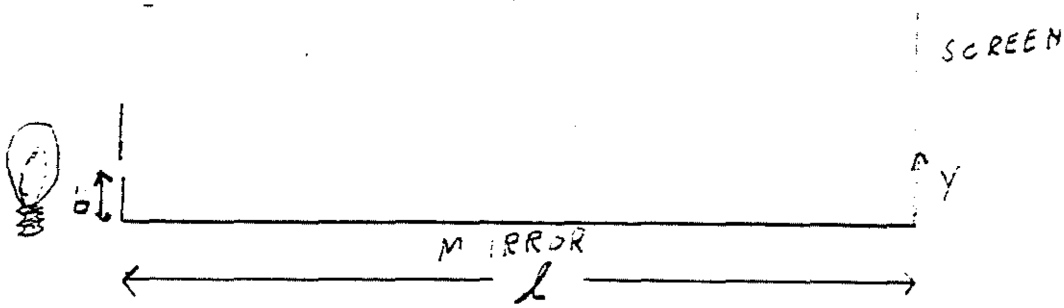
d) What is the index of refraction of the material that the wave is traveling in?

e) Write an expression for the magnetic field H for this wave. It can also be in arbitrary units.

f) What is the polarization state of the light?

g) What is the maximum value that E can have?

2) Consider a single slit whose width is large compared to the wavelength of light but small compared to all other dimensions in this problem. The slit is located a small distance b above a plane mirror, and a distance ℓ away there is a screen, as shown. Monochromatic light of wavelength λ passes through the slit. Assume $\ell >> b$.



a) Describe (in words) what is seen on the screen.

b) Where will the minima in intensity be located on the screen?

c) Is the intensity of light on the screen at y = 0 a maximum or a minimum?

d) Derive an expression for the relative intensity of light on the screen as a function of y, assuming that y stays much smaller than ℓ.

D: ASTROPHYSICS

1) Assume that some fraction, f, of the gravitational force is not balanced by the pressure force within the Sun. Solve for the length of time it takes the Sun to adjust to a small perturbation in its radius, δr, which is noticeable in terms of the Sun's radius (i.e. of order fRO). You may start with the unbalanced acceleration:

$$a = fG\frac{M_r}{r^2}$$

You may also take quantities out of integrals by assuming appropriate averages.

Since we know that the Sun is in hydrostatics equilibrium, what does this gravitational time-scale imply about the photon diffusion time (remember, the photons provide pressure to balance gravity)?

2) Given the Planck function:

$$B_\nu(T) = \frac{2h\nu^3/c^2}{\exp(h\nu/kT) - 1}$$

a) Derive the relation between B_ν and temperature, T, in the Rayleigh-Jeans approximation ($h\nu \ll kT$).

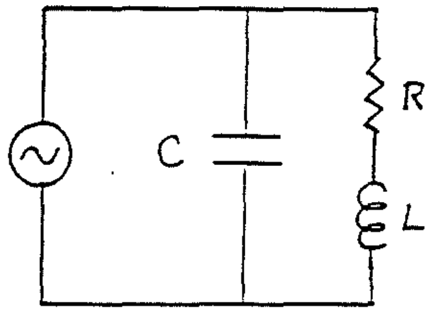
b) If a supernova remnant has an angular diameter $\theta = 3.2$ arc minutes and a flux at 100 MHz of $F = 1.0 \times 10^{-19}$ erg cm^{-2} s^{-1} Hz^{-1} what is the brightness temperature, T_b, of the source? Is the Rayleigh-Jeans approximation valid in this case?

c) The emitting region is actually more compact than indicated by the observed angular diameter. What effect does this have on the value of T_b?

d) At about what frequency will this object's radiation be a maximum, if the emission is blackbody? What radiation regime is this?

E: ELECTRONICS

1)



(a) Derive an expression for the complex impedance of the LCR combination in the circuit above.

(b) Derive an expression for the quality factor (Q) of the circuit and the resonant frequency (ω_o) assuming that $R << \omega_o L$.

(c) Calculate ω_o and Q for R = 100 Ω, L = 10 mH, and C = 0.01 μF.

2) Calculate the DC output voltage and the peak-to-peak ripple voltage for the power supply below.