~~Planckian Interacting Massive Particles (PI~~ as Dark Matter (PIDM) & Charged PIDM

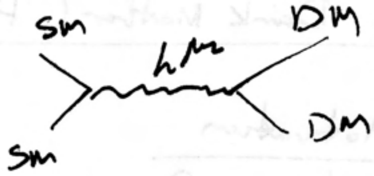
- Motivation
  - Lack of DM signals as of yet
  - "Naturalness" of DM abundance doesn't necessarily naturalness of any other DM properties
  - Why not DM at Planck scale, where we may expect new physics?

- How to get such heavy DM?
  - Only gravitationally coupled
  - Consequently, never in equilib with SM / dominant plasma => Freeze-in
  - (Such PIDM can produce DM in mass range 1 TeV < m_x < M_{GUT} if T_{RH} is high enough)
  - ~~Charged~~ Hidden charged PIDM, totally decoupled from SM (time allowing)

→ Goal: Determine what is required phenomenologically for such PIDM to be a viable model

- Calculational Setup

$$\mathcal{L} = \mathcal{L}_{SM} + \mathcal{L}_{DM} + \mathcal{L}_{EH} + \frac{\sqrt{8\pi}}{2m_p} h^{\mu\nu}\left(T^{SM}_{\mu\nu} + T^{DM}_{\mu\nu}\right)$$



- s-channel graviton exch.

- some global charge (model dependent) prevents DM-SM sector from coupling directly, forbidding DM decay.
- Non-self interacting DM (for simplicity).

$\Rightarrow$ 2-2 amplitude

$$\mathcal{M} = \frac{-i8\pi \langle p_1|T^{\mu\nu}_{SM}|p_2\rangle\langle k_a|T^{DM}_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T^{DM\alpha}{}_{\alpha}|k_b\rangle}{m_p^2 (k_a+k_b)^2}$$

w/ $(k_a+k_b)^{\mu}T_{\mu\nu} = 0$.

-> generalizes later to

$$\mathcal{M} = \frac{-i8\pi G}{s}\left(T^{\mu\nu}_{SM}T^{DM}_{\mu\nu} - \frac{1}{2}\underbrace{T_{SM}T_{DM}}_{\text{traces}}\right), G \equiv \frac{1}{m_p^2}$$

Boltzmann Eqns

$$\frac{\partial \rho_\phi}{\partial t} = -3H(1+w)\rho_\phi - S$$

$$\frac{\partial \rho_R}{\partial t} = -4H\rho_R + S + 2\langle\sigma v\rangle\langle E_x\rangle\left(n_x^2 - (n_x^{eq})^2\right)$$

$$\frac{\partial n_x}{\partial t} = -3Hn_x - \langle\sigma v\rangle\left(n_x^2 - (n_x^{eq})^2\right)$$

ρ_ϕ - inflaton energy density

ρ_R - radiation

n_x - DM number density

S - Describes inflaton decay to SM

- Generally $S(t)$, $w(t)$ have complicated t-dependence. Assume $S = \Gamma\rho_\phi$, Γ & w const.
- H_i - Hubble rate @ end of inflation.
  - ↳ Fast reheating if $\Gamma \to H_i$
  - ↳ pert. " if $\Gamma \ll H_i$

-> Define reheating Temp with $H = \Gamma$ by

$$T_{rh} = k_2\,\gamma\,(M_P H_i)^{1/2}$$

- This, in turn, defines

$$\gamma = \sqrt{\frac{\Gamma}{H_i}} = \left(\frac{g_i}{g_{RH}}\right)^{1/4} e^{-\frac{3}{4}N_{RH}(1+w)}$$

~~paramet~~ parametrizes RH efficiency.

$\gamma \in (0,1)$; $\gamma = 1 \to$ perfectly instant. RH.

- For Freeze-in scenario to work, need $\gamma \sim 1$

-> Possible even in pert. RH scenarios. In this case, $\Gamma = g^2 m_\phi / 8\pi$, and $\gamma \sim 0.2\,g\sqrt{\frac{m_\phi}{H_i}}$. This works if g & m_ϕ appropriately large.

- PIDM prod. occurs @ highest energy scales after inflation:
  - end of inflation
  - reheating
  - post-reheating
- Reheating contribution dominates, so use reheating dynamics eqns at left.

- $k_2 = \left(\frac{45}{(4\pi^3\, g_{RH})}\right)^{1/4} \sim 0.25$

  g_{RH} = SM d.o.f @ r.h.

- N_{RH} = duration of reheating in e-folds

- g is SM - ϕ (inflaton) coupling.

- For inflaton, assuming it dominates, ~~$\rho_\varphi \propto a^{-3(1+w)}$~~
$\rho_\varphi \propto a^{-3(1+w)}$. This holds until RH stops & thermal plasma dominates.

$$\to H \simeq H_i \begin{cases} (a/a_i)^{-3(1+w)/2} & a < a_{rh} \\ (a/a_{rh})^{-2}\ \gamma^2 & a > a_{rh} \end{cases}$$

$a_i = a$ when RH starts
$a_{rh} = a_i \gamma^{-\frac{4}{3(1+w)}}$ at end of RH.

- The 2nd eqn, when solved, yields $T(a)$, → During RH!

$$T(a) = \frac{K_1 (\gamma m_p H_i)^{1/2}}{(1+3/5w)^{1/4}} \left(a^{-\frac{3(1-w)}{2}} - a^{-4}\right)^{1/4}$$

$$K_1 = \left(\frac{9}{2\pi^3 g_{max}}\right)^{1/4} \simeq 0.2$$

needed for Abundance calculations.

Post-RH, $T(a) = T_{RH} \frac{a_{RH}}{a}$

- 3rd eqn requires $n_x^{eq} = \frac{g}{2\pi^2} m_x^2 T K_2(m/T)$

$K_2(x)$ is a modified Bessel fn.

- As for bounds on H_i, just $H_i < H_{CMB}$ is enough, and yields
$$H_i < 6.6\times10^{-6}\ m_p\left(\frac{r}{0.1}\right)^{1/2}$$
for the PIDM scenario.

from CMB bound on tensor waves, $r < 0.07$ (in this paper)

Abundance Calc.

~~Redefine~~ Define dimensionless abundance $X \equiv n_x a^3/T_{rh}^3$

$$\hookrightarrow \frac{dX}{da} = \frac{a^2}{T_{rh}^3 H(a)} \langle\sigma v\rangle (n_x^{eq})^2$$

Valid when $n_x \ll n_{eq}$ & inverse annihilation is dominant.

Direct integration possible (if initial abundance vanishes)

$$X_f = \frac{1}{T_{rh}^3}\int_{a_i}^{a_f} da\ \frac{a^2}{H(a)} \langle\sigma v\rangle (n_x^{eq})^2$$

$a_f \to \infty$ allowed, since production exp. supp. in this region

- Yields number density in terms of m_x, H_i, γ, & w.

Resultant abundance is related to present-day abundance:

$$\Omega_X h^2 = Q\,\gamma^{\frac{4}{1+w}}\frac{m_X}{m_p}X_f, \quad Q = \frac{1}{8}\frac{T_{rh}^3\, m_p s_0}{s_{rh}\,\rho_c}$$

- For definiteness, consider $\boxed{\text{Scalar PIDM}}$

- Generally, $\langle\sigma v\rangle = N_0\langle\sigma v\rangle_0 + N_{\frac12}\langle\sigma v\rangle_{\frac12} + N_1\langle\sigma v\rangle_1$,

$N_0 = 4,\ N_{\frac12} = 45,\ N_1 = 12$ in SM

As usual

$$\langle\sigma v\rangle = \frac{1}{8 m_X^4 T K_2(m_X/T)^2}\int_{4m_X^2}^{\infty} ds\,\sqrt{s}\,(s-4m_X^2)\,\sigma(s) \times K_1(\sqrt{s}/T)$$

$$\sigma(s) = \frac{-1}{16\pi s\,(s-4m_X^2)}\int_{t_+}^{t_-} dt\,|M|^2$$

with $|M|$ depending on PIDM spin.

$$M = \frac{-i\,8\pi G}{s}\left(T^{\mu\nu}_{SM}T^{DM}_{\mu\nu} - \frac12 T_{SM}T_{DM}\right)$$

$G \equiv \frac{1}{m_p^2}$, $T_{SM,DM}$ - traces

- Skipping explicit / long calculations,

$$\langle\sigma v\rangle_0 = \frac{\pi m_X^2}{8 m_p^4}\left[\frac35\frac{K_1^2}{K_2^2} + \frac25 + \frac45\frac{T}{m_X}\frac{K_1}{K_2} + \frac85\frac{T^2}{m_X^2}\right]$$

$$\langle\sigma v\rangle_{\frac12,1} = \frac{4\pi T^2}{m_p^{4}}\left[\frac{2}{15}\left(\frac{m_X^2}{T^2}\left(\frac{K_1^2}{K_2^2}-1\right) + \frac{3m_X}{T}\frac{K_1}{K_2} + 6\right)\right]$$

- Letting $m_X \gg T$, $w = 0$, abundance can be calculated:

$$X_f = \frac{N_0 m_X^5}{8\pi^2 m_p^4 T_{rh}^3 H_i}\left[T_1^3\int_1^{a_{rh}} da\,a^{19/8}\,e^{-\frac{2m_X}{T_1}a^{3/8}} + \frac{a_{rh}}{\gamma^2}T_{rh}^3\int_{a_{rh}}^{a_f} da\,\frac1a\,e^{-\frac{2m_X}{T_{rh}}\frac{a}{a_{rh}}}\right]$$

$a_{rh} \gg 1$, $a_f \gg a_{rh}$:

$$X_f = \frac{N_0 m_X^4}{8\pi^2 m_p^4 T_{rh}^3 H_i}\left[\frac43 T_1^4 e^{-\frac{2m_X}{T_1}} + \frac12\frac{a_{rh}}{\gamma^2}T_{rh}^4 e^{-\frac{2m_X}{T_{rh}}}\right]$$

$Q \simeq 9.2\times 10^{24}$

s - entropy densities

ρ_c - crit. density

$\frac18$ - factor accounting for continued entropy production after RH.

- Subscripts denote spin of SM particles, N_i are d.o.f. of i in SM.

$t_\pm = -\left(\sqrt{s/4 - m_X^2} \mp \sqrt{s/4}\right)^2$

⊛ Quantities in brackets asymptote to 1 for $m_X \gg T$

$T_1 = \left(\frac{1728}{3125}\right)^{1/20} T_{max} = 0.97\,T_{max}$

T_{max} is max value of $T(a)$ from before

- For $\gamma \ll 1$, $T_{max} \gg T_{rh}$, so second term can be ignored, and we can solve for $H_i(m_x)$:

$$H_i(m) = \frac{4 m_x^2}{\kappa_1^2 \gamma m_p} W_{-1}\left(-\lambda \gamma^{-7/2} \frac{m_p^4}{m^4_x}\right)^{-2}$$

=> If argument of $W_{-1}(x)$ is less than $-1/e$, no real sol^n exists. This leads to a restriction on model parameters:

$$\boxed{\gamma^{7/8} m_x > 2.5\times10^{-6}\, m_p}$$

=> for small m_x, heavy PIDM approx breaks down

=> for large masses, we find a lower bound on RH efficiency γ needed for this production mechanism to work

=> For large mass, ProductLog ~ log, and

$$H_i \propto \frac{m^2}{\gamma \log(\gamma^{7/8} m_x)^2}$$

=> Scalar vs. Fermion vs. Vector DM

- $\langle\sigma v\rangle$ for all 3 cases are very similar, ~~only~~ differing mainly in terms of prefactors or a single power of DM mass m_x. So numerical results don't vary greatly.

Nonminimally Coupled PIDM

$$\mathcal{L}_{nm} = \frac{1}{2}(\xi_\phi \phi^2 + \xi_x X^2)R \quad , \phi - \text{SM scalar}$$

Calculating $\langle\sigma v\rangle$, in the relativistic limit

$$\langle\sigma v\rangle \to \frac{G^2 m^2 \pi}{8}(1+4\xi_x)^2(1+6\xi_\phi)^2$$

effectively $m \to m(1+4\xi_x)(1+6\xi_\phi)$

- for small couplings, previous results still hold

W_{-1} - Lambert ProductLog P^2

$$\lambda = 12\pi^2 \Omega_x h^2 \frac{\kappa_2^3}{(\kappa_1^4 Q)}$$

$\sim 1.4\times10^{-23}$

-> see exact results. plots?

Point: PIDM needs a high inflation scale & fast reheating

- Possible generaliz. to f(R) gravitational models?

→ From numerical results (scalar), m_χ can span a large range. ($\sim 10^{-10}$ to $10^{-2} m_p$)

- Very sensitive to RH efficiency γ.
  If $\gamma \le 10^{-3}$ ($N_{rh} \gtrsim \frac{10}{\Gamma_{rw}}$ efolds), PIDM freeze-in is impossible.
- Higher PIDM masses are more favorable; for large γ, $m_\chi \gtrsim M_{GUT}$ possible.
- for given bound on r,

$$m_{max} = 0.023\, \gamma^{1/2} r^{1/4}\, m_p$$

↳ for $r < 0.07$, $m_{max} = 0.013 m_p$, decreasing for smaller γ.

↳ if CMB experiments exclude tensor modes to $r \sim 10^{-4}$ or less, PIDM ~~or~~ viability only significantly below the natural cutoff scale.

note/reminder:

used std. reheating setup w/ constant Γ & w. Results could change for more general cases.

• Charged PIDM

- In light of similar results b/w scalar/fermion/vec, we can consider fermionic DM with only gravitational ~~com~~ interactions w/ SM. Also with a dark U(1) gauge symmetry; gauge boson γ_D.

$$\mathcal{L}_{DM} = -\frac{1}{4} V_{\mu\nu} V^{\mu\nu} + \bar{\chi}\, i \not{D} \chi - m_\chi \chi \bar{\chi}$$

$$D_\mu = \partial_\mu - i g_D V_\mu$$

$\Rightarrow 4\pi \alpha_D = g_D^2$ defines dark fine-struct. constant α_D.

- Possible SM-DM couplings from Quantum corrections. Loops involve gravitons, and it turns out any contributions from these diagrams that contribute to kinetic mixing go to 0.

→ ~~Bo~~ A few motivations for Self-interacting DM like this are solutions for "small scale problems" of DM (discrepancies b/w simulations of structure formation in collisionless CDM & galactic / sub-galactic scale observations)

NOTE: This requires small (light) m_x. Later will discuss what happens for $m_x \gg T_{rh}$.

→ For charged PIDM, abundance calc split into two parts: ① Around reheating, Dark sector populated by freeze-in, predominantly
② Subsequent evolution, dependent on whether ~~a~~ Dark sector thermalizes. This may affect PIDM abundance, depending on α_D.

① This calculation is similar to scalar case, just with appropriate changes for fermions, and also accounting for Dark Photon γ_D density X_{γ_D}.

- Doing the integrals for X_x & X_{γ_D}, we can find the ~~abundances~~ "initial" densities produced from freeze-in @ end of inflation:

$$n_{i,x} \sim 0.27 \frac{T_{rh}^6}{m_p^3}, \quad n_{i,\gamma_D} \sim 0.65 \frac{T_{rh}^6}{m_p^3}$$

~~→ These are smaller by a factor of $(T_{rh}/m_p)^3$ than would be for an equilibrium dist. at the typical energy scale T_{SM}~~

→ Freeze-in gives non-thermal dist of particles in dark sector with momentum dist. $f_{x,\gamma_D}(p)$ peaked around T_{SM}.
But init. densities are smaller by $(T_{rh}/m_p)^3$ ⇒ Freeze-in produces an underpop. dist.

~~Weak undampop~~ Dark Sector won't come to thermal eq. with SM, but might equilibrate with itself.

Relativistic Regime $T_{SM} \gg m_X \to$ freeze-in prod.

- Omit detailed analysis. $\exists$ a critical values of α_D which determines Dark sector pheno.

$$\alpha_D^{crit,mel(a)} \simeq 2\times10^{-3}\left(\frac{m_X}{100\,GeV}\right)^{2/5}\left(\frac{10^{-4}m_p}{T_{rh}}\right)^{9/10}$$

$$\alpha_D^{crit,mel(b)} \simeq 5\times10^{-4}\left(\frac{m_X}{100\,GeV}\right)^{1/2}\left(\frac{10^{-4}m_p}{T_{rh}}\right)^{9/8}$$

leads to thermalization w/in Dark sector

- $\alpha_D \gg \alpha_{crit} \equiv \max\left(\alpha_D^{crit,mel(a)}, \alpha_D^{crit,mel(b)}\right)$

  $\hookrightarrow$ Thermaliz $\to$ Dark Sector equilib.

Non-Rel. Regime

- Even if $\alpha_D \ll \alpha_{crit}$, & no Thermaliz, PIDM self-scatting processes ("dark Coulomb Scatting") can bring PIDM alone into kinetic eq.

=> Overall, Dark Sector can evolve to an eq. dist. in both weak & strong α_D.

$\alpha_D \ll \alpha_D^{crit}$ (weak):

DM abundance set mainly by Freeze-in

$$\Omega_X h^2 \simeq 0.12\left(\frac{m_X}{340\,GeV}\right)\left(\frac{T_{rh}}{6\times10^{-4}m_p}\right)^3$$

$r < 0.064$ bound $\to$ $T_{rh}/m_p \lesssim 6\times10^{-4}$.

Thus, plus $\Omega_X h^2 \sim 0.120$ => $m_X \gtrsim 400\,GeV$

(~~w/o residual~~ if instant ~~transition~~ to rad. ~~dominated~~ domination w/o residual entropy production, $\Omega_X h^2 \to 8\Omega_X h^2$, $m_X \gtrsim 50\,GeV$)

$\alpha_D \gg \alpha_D^{crit}$ (strong):

$\to$ Freeze-out in dark sector

$\xi = T_D/T_{SM}$

$x \equiv m_X/T_D$

- A bit more complicated, but essentially normal freeze-out except with T_D (dark sector temp.)
- freeze-out happens before kinetic decoupling for allowed T_{rh}.
- Relic density leads to a bound on x_f (at freeze-out):

$$x_f < 3\times10^3\left(\frac{\alpha_D}{10^{-2}}\right)\left(\frac{100\,GeV}{m_X}\right)^{1/2}\left(\frac{T_{rh}}{10^{-4}m_p}\right)^{3/4}$$

& $x_f \lesssim 15$

- as an example, if $\xi = 0.5$, $\Omega_\chi h^2$ is obtained for $\alpha_D \sim 0.001$, $m_\chi \sim 100\,\text{GeV}$

↳ More specifically, $\xi \propto \left(\frac{T_{rh}}{m_p}\right)^{3/4} \ll 10^{-2}$ and $\alpha_D \gg \alpha_{D,crit}$ means $\Omega h^2 \sim 0.12$ achievable only for $m_\chi \gtrsim 10^4\,\text{GeV}$.

-> For cleaner/more in depth treatment, see reference

=> What about Charged GUT scale PIDM? (now at $m_\chi \sim 10^{-3} m_p$, $T_{rh} \sim 10^{-4} m_p$ scales) (m_χ close to T_{rh})

- Assume self-interacting DM not necessary to explain simulation/observational issues.
- PIDM @ this mass are already non-rela. when produced by SM plasma.

$$\langle \sigma v \rangle_{\chi\bar\chi \to sm\, sm'} = \frac{230\pi\, m_\chi T}{m_p^4} \bar S_{ann}\left(\frac{m_\chi}{T}\right)$$

however, only if $\pi \alpha_D \gtrsim \sqrt{T_{rh}/m_\chi}$. else, $\bar S_{ann} \sim O(1)$.

-> $T \ll m_\chi$ limit

-> must account for Sommerfeld enchancement

$\bar S_{ann} \sim O(1)$ for $T_{rh} \ll m_\chi$

=> $\alpha_D \gtrsim \frac{1}{\pi}\sqrt{T_{rh}/m_p}$ not satisfied

- Solving the usual way yields

$$n_{i,\chi} \simeq 0.18 \frac{m_\chi^3 T_{rh}^3}{m_p^3} \exp\left[-2m_\chi/T_{rh}\right]$$

exp. suppressed compared to γ_D density.

-> No freeze out, but possible $\gamma_D\gamma_D \to \chi\bar\chi$ production. Comparing the rates

$$\Gamma_{\gamma\gamma \to \chi\bar\chi} \simeq \frac{1}{n_\chi}\left(\frac{n_{\gamma_D}}{n_{\gamma_D}^{eq}}\right)^2 (n_{\gamma_D}^{eq})^2 \frac{\pi \alpha_D^2}{m_\chi^2} \bar S_{ann}\left(\frac{m_\chi}{T}\right)$$

$$\Gamma_{grav} \simeq \frac{1}{n_\chi}(n_\chi^{eq})^2 \frac{230\pi\, m_\chi T}{m_p^4} \bar S_{ann}\left(\frac{m_\chi}{T}\right)$$

$$\Rightarrow \frac{\Gamma_{\gamma\gamma\to\chi\bar\chi}}{\Gamma_{grav}} \sim \left(\frac{n_{\gamma_D}}{n_{\gamma_D}^{eq}}\right)^2 \frac{\alpha_D^2 m_p^4}{230 m_\chi^3 T} \sim 0.06 \frac{\alpha_D^2 T_{rh}^5}{m_\chi^3 m_p^2} \ll 1$$

↳ $\gamma\gamma \to \chi\bar\chi$ enhanced vs. grav, but $\left(\frac{n_{\gamma_D}}{n_{\gamma_D}^{eq}}\right)^2 \sim \left(\frac{T_{rh}}{m_p}\right)^6 \ll 1$.

=> U(1) interaction only affects GUT scale PIDM production via Somm. enhancement of grav. production for very large α_D: $\pi\alpha_D \gg \sqrt{T_{rh}/m_\chi}$

• In Situ PIDM

- A few scenarios/models in which PIDM can easily or naturally be incorporated

① Monodromy Inflaton + PIDM (effective description)

- 4D monodromy potential obtained from compactif. of 11D SUGRA via mixing of an axion(-like) particle w/ a 4-form from effective action:

$$S_{infl.} = \int d^4x \sqrt{-g}\left[\frac{1}{16\pi^2 m_p^2}R - \frac{1}{2\cdot 4!}F_{\mu\nu\lambda\rho}F^{\mu\nu\lambda\rho} - \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) + \frac{\mu}{4!}\phi\,\epsilon^{\mu\nu\lambda\rho}F_{\mu\nu\lambda\rho}\right] + \frac{1}{c}\int d^4x\sqrt{-g}\,\nabla_\mu\left[F^{\mu\nu\lambda\sigma}A_{\nu\lambda\sigma} - \mu\phi\frac{\epsilon^{\mu\nu\lambda\rho}}{\sqrt{-g}}A_{\nu\lambda\rho}\right]$$

→ axion mixing

→ can just show $S_{infl} \supset \frac{\mu}{4}\phi\,\epsilon^{\mu\nu\lambda\rho}F_{\mu\nu\lambda\rho}$

- PIDM incorporated by adding scalar d.o.f. with a mass $M \sim \mathcal{O}(M_{11})$

M_{11} - 11D Planck mass

$$S_{PIDM} = -\frac{1}{2}\int d^4x\sqrt{-g}\left[(\partial_\mu\sigma)(\partial^\mu\sigma) + M^2\sigma^2\right]$$

and RH mechanism via

$$S_{RH} = \int d^4x\sqrt{-g}\,\frac{\phi}{f_\phi}G_{\mu\nu}\tilde{G}^{\mu\nu}$$

- inflaton ϕ coupled to an SM gauge field strength

$$S_{EFF} = S_{infl} + S_{PIDM} + S_{RH}$$

- In the model, the 4-form describes a membrane moving in 11D SUGRA. 4-form background breaks shift symm., providing quadratic potential for ϕ:

$$V_{eff} = \frac{1}{2}(q + \mu\phi)^2$$

valid for $m_p \ll \phi \leq M_{11}^2/\mu$ → allows large-field inflation to occur.

$$\left(\mu \sim \mathcal{O}(10^3)\left(\frac{M_{11}}{m_p}\right)^2 M_{11} \ll M_{11}\right)$$

↳ amplitude of density fluct. $\delta\rho/\rho \sim 10^{-5}$

$\Rightarrow \mu \sim 10^{13} \Rightarrow M_{11} \sim M_{GUT} = 10^{16}\,\text{GeV}$

So $m_X \sim M_{11} \sim M_{GUT}$ naturally.

- Efficient PIDM from grav. scattering requires high T_{RH}.

- Inflation ends when inflaton BG value $\frac{a}{\mu}+\varphi < M_{pl}$ & inflaton begins oscillating.

  - Inflaton - SM coupling with $\mu \lesssim f_\varphi \lesssim M_{11}$ needed to determine decay of inflaton to SM gauge sector:

$$\Gamma = \frac{\mu^3}{8\pi f_\varphi^2} \Rightarrow T_{RH} = \frac{K_2}{(8\pi^2)^{1/4}} \frac{\mu^{3/2} m_p^{1/2}}{f_\varphi}$$

  $\Rightarrow$ Thus, RH efficiency param

$$\gamma = \frac{1}{\sqrt{8\pi}} \frac{\mu}{f_\varphi}$$

- From prev. parts, $m_X \sim M_{11} \sim M_{GUT}$ means $\gamma \gtrsim 0.1 \Rightarrow f_\varphi \sim \mu \sim 10^{13}$ GeV.

  (alternatively, can lower m_X to $M \sim \mu$, in turn requiring $f_\varphi \sim M_{11} \sim M_{GUT}$)

② Higgs Inflation PIDM

- Higgs (SM) as the inflaton.

$$\mathcal{L} = \left(\frac{1}{16\pi^2} m_p^2 + \xi H^\dagger H\right) R + g^{\mu\nu}(D_\mu H)(D^\mu H) - \lambda\left((H^\dagger H) - \frac{v^2}{2}\right)^2$$

  ↪ also add the action S_{PIDM} from monodromy case.

- Critical vs. noncritical Higgs infl:

Non-Crit: inflation occurs on plateau of Starobinsky infl. form

$$V_{Stm} \approx \frac{\lambda m_p^4}{256\pi^4 \xi^2}\left(1 + e^{-4\pi\phi/\sqrt{3}m_p}\right)^{-2}$$ ← Einstein frame

ϕ - canonically renorm. Higgs field acting as inflaton.

$\rightarrow$ for $\lambda \sim O(1)$, it turns out $\xi \sim 10^4$, and

$r = 16\epsilon$, $\epsilon = 3/4N_*^2$

- $N_* = \#$ e-folds ~~at CMB~~ @ horizon exit.

$\rightarrow$ Pert. RH $\rightarrow T_{RH} \sim 10^{14}$.

with $r \sim 3\times10^{-3} \Rightarrow \gamma \sim 10^{-2}$

$\Rightarrow m_X \sim 10^{-5} m_p$

- Authors ask if $m_X \sim 10^{-5} m_p$ warrants a consideration/connection to leptogenesis, as $m_X \sim$ RHN mass scale

Criti: m_h & m_t are finely tuned s.t. second vacuum vanishes & becomes an inflection point of potential that can be used for inflation.

- No infl. @ plateau
- $\epsilon \sim 1/N_*^2$ broken
- r large is possible for $\xi \sim 10$ and $T_{rh} \lesssim$ GUT scale
  ↳ $m_X \sim M_{GUT}$ PIDM possible.