② Nonlinear Realization of CCWZ

- Amplitudes of soft NG boson processes are uniquely determined by the low-energy theorems.
  ↳ Since Low-Energy thms apply pretty universally to systems with G symmetry broken to H, a low energy effective $\mathcal{L}$ realizing this symmetry can make it easier to read off soft NG boson amplitudes.
  - A popular approach is the Nonlinear Sigma Model, in which $\mathcal{L}_{eff}$ is constructed in terms of nonlinearly transforming NG bosons alone.

This is supposed to be easier than deriving low-energy theorems the conventional way.

• Model based on manifold G/H.

• These authors introduce the nonlinear σ model in a different way than CCWZ (check reference)

- G & H simple compact groups. G spontaneously broken to the subgroup H.
- Let T^A be the generators of G. Split into two parts:

$$\{T^A\} = \{S^\alpha \in \mathcal{H},\ X^a \in \mathcal{G}\}$$

satisfying $Tr(T^A T^B) = \frac{1}{2}\delta^{AB}$, $Tr(S^\alpha X^a) = 0$

• Script letters denote the algebra of the groups.

- ∃ NG bosons with number equal to $\dim(G/H) = \dim G - \dim H$.
  ↳ NG bosons transform linearly under H, but not under G.

- Let $\xi(\pi)$ be "representatives" of the left coset space G/H:

$$\xi(\pi) = e^{i\pi(x)/f_\pi}, \quad \pi(x) \equiv \sum_{a\in\mathcal{G}-\mathcal{H}} \pi^a(x) T^a$$

• Unitary Matrix rep. of G, parametrized in terms of NG bosons $\pi(x)$. f_π - decay const.

  ↳ Right multiplication by $g^\dagger \in G$ yields $\xi(\pi)g^\dagger$, which can be uniquely decomposed into a coset part & an unbroken part $h^\dagger \in H$:

$$\xi(\pi)g^\dagger = h^\dagger(\pi, g)\,\xi(\pi')$$

Thus, the nonlinear transf. of $\pi(x)$ (or $\xi(\pi)$) under $g \in G$ is

$$\xi(\pi) \to \xi(\pi') = h(\pi, g)\,\xi(\pi)\,g^\dagger, \quad g \in G$$

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– A fundamental obj. to be used is the Maurer-Cartan 1-form constructed from $\xi(x) \in G/H$:

$$\alpha_\mu(\pi) = \frac{1}{i} \partial_\mu \xi(\pi) \cdot \xi^{-1}(\pi)$$

It is expandable in terms of $\{T^a\}$, so we can define the $\parallel$ and $\perp$ components of $\alpha_\mu(\pi)$ to $\mathcal{H}$:

$$\alpha_{\mu\parallel}(\pi) = (2\ \mathrm{tr}\ S^a \alpha_\mu(\pi)) S^a \in \mathcal{H}$$
$$\alpha_{\mu\perp}(\pi) = (2\ \mathrm{tr}\ X^a \alpha_\mu(\pi)) X^a \in \mathcal{G} - \mathcal{H}$$

↳ The transf. law of $\alpha_\mu(\pi)$ can be found from that of $\xi(\pi)$:

$$\alpha_\mu(\pi) \to \alpha_\mu(\pi') = h(\pi, g)\, \alpha_\mu(\pi)\, h^{+}(\pi, g) + \frac{1}{i} \partial_\mu h(\pi, g) \cdot h^{+}(\pi, g)$$

As $\partial_\mu h \cdot h^{+} \in \mathcal{H}$, we get

$$\alpha_{\mu\parallel}(\pi) \to \alpha_{\mu\parallel}(\pi') = h(\pi, g)\, \alpha_{\mu\parallel}(\pi)\, h^{+}(\pi, g) + \frac{1}{i} \partial_\mu h(\pi, g) \cdot h^{+}(\pi, g)$$
$$\alpha_{\mu\perp}(\pi) \to \alpha_{\mu\perp}(\pi') = h(\pi, g)\, \alpha_{\mu\perp}(\pi)\, h^{+}(\pi, g)$$

Since only $\alpha_{\mu\perp}(\pi)$ transforms homogeneously, we can construct G-invariants from $\alpha_{\mu\perp}(\pi)$ alone: $\mathrm{tr}(\alpha_{\mu\perp}(\pi))^2$

Thus, the most general $\mathcal{L}$ made up of $\xi(\pi)$ with the smallest # of derivative is

$$\mathcal{L}_{CCWZ} = f_\pi^2\ \mathrm{tr}\,(\alpha_{\mu\perp}(\pi))^2$$

• Same $\mathcal{L}$ as derived by CCWZ.
f_π^2 serves to normalize $\pi(x)$ kinetic terms.

• CCWZ call $f_\pi\, \alpha_{\mu\perp}(\pi)$ the "Covariant Derivative".

- Goal: Write $\mathcal{L}_{CCWZ}$ in terms of NG fields $\pi(x)$.
  Authors use the formula:

$$i f_\pi \left(e^{\frac{i\pi(x)}{f_\pi}} \partial_\mu e^{-\frac{i\pi(x)}{f_\pi}} \right) = \left[\frac{e^{\Delta_{i\pi/f_\pi}} - 1}{-\Delta_{i\pi/f_\pi}} \right] (\partial_\mu \pi)$$

$$= \partial_\mu \pi + \frac{1}{2!} \frac{i}{f_\pi} [\pi, \partial_\mu \pi] + \frac{1}{3!} \left(\frac{i}{f_\pi}\right)^2 [\pi, [\pi, \partial_\mu \pi]] + \dots$$

$\Delta_{i\pi/f_\pi} \circ X \equiv [\frac{i\pi}{f_\pi}, X] \quad \forall X \in \mathcal{G}$

(?)

if G/H is a symmetric space, then for the terms in the expansion:

even $\longleftrightarrow$ belong to $\mathcal{H}$

odd $\longleftrightarrow$ belong to $\mathcal{G} - \mathcal{H}$

& the perp. part becomes

$$f_\pi \, d_{\mu\perp}(\pi) = \frac{f_\pi}{2i} \left[\partial_\mu \xi(\pi) \cdot \xi^\dagger(\pi) - \partial_\mu \xi^\dagger(\pi) \cdot \xi(\pi) \right]$$

$$= \partial_\mu \pi + \frac{1}{3!} \left(\frac{i}{f_\pi}\right) [\pi, [\pi, \partial_\mu \pi]] + \dots$$

$$\Rightarrow \mathcal{L}_{CCWZ} = \mathrm{tr}(\text{above})^2$$

$$= \mathrm{tr}(\partial_\mu \pi)^2 - \frac{1}{3 f_\pi^2} \mathrm{tr}\left(\partial^\mu \pi \, [\pi, [\pi, \partial_\mu \pi]]\right) + \dots$$

- To incorporate matter fields:

  $X(x)$ introduced as a linear rep. of H:

$$X \to X' = \rho_0(h) X \qquad \text{under } h \in H$$

  Define the transform. of $X(x)$ under G as

$$X(x) \to X'(x) = \rho_0(h(\pi, g)) X(x)$$

$h(\pi, g)$ from before, G-transf. is nonlinear

  ↳ we can convert $X(x)$ into a linear representation field $\psi(x)$ by using any representation ρ of G whose restriction to H contains ρ_0.

$$\psi(x) = \rho(\xi^\dagger) X(x)$$

  Note linearity of ψ transformation:

$$\psi \xrightarrow{g} \psi'(x) = \rho(\xi'^\dagger) X'(x) = \rho(g \xi^\dagger h^\dagger(\pi, g)) \rho_0(h(\pi, g)) X(x)$$

$$= \rho(g) \rho(\xi^\dagger) X(x)$$

$$= \rho(g) \psi(x)$$

- From this, we can construct G-invariants for matter fields. From the transformation laws of $\alpha_{\mu\parallel}(\pi)$ and $\alpha_{\mu\perp}(\pi)$, one finds the following fermion bilinears to be G-inv.:

$$\bar{\chi}\chi = \bar{\psi}\psi$$
$$\bar{\chi}\gamma^{\mu}\alpha_{\mu\perp}(\pi)\chi = \bar{\psi}\gamma^{\mu}\xi^{+}(\pi)\alpha_{\mu\perp}(\pi)\xi(\pi)\psi$$
$$\bar{\chi}i\gamma^{\mu}[\partial_{\mu} - i\alpha_{\mu\parallel}(\pi)]\chi = \bar{\psi}i\gamma^{\mu}\partial_{\mu}\psi$$

& the fermion matter $\mathcal{L}$ is given then by

$$\mathcal{L}_{matt} = \bar{\chi}i\gamma^{\mu}[\partial_{\mu} - i\alpha_{\mu\parallel}(\pi)]\chi - m\bar{\chi}\chi + \lambda\bar{\chi}\gamma^{\mu}\alpha_{\mu\perp}(\pi)\chi$$

• Assume $\chi(x)$ & $\psi(x)$ belong to the fund. reps. of H & G: $\rho_0(h) = h$, $\rho(g^{+}) = g^{+}$

- External Gauge Fields:

Ext. gauge field $\mathcal{V}_{\mu}(x)$, gauge group $I \subset G$. Accounting for this requires replacing $\partial_{\mu}\xi(\pi)$ in the Maurer-Cartan 1-form with the covariant derivative $D_{\mu}\xi(\pi) = \partial_{\mu}\xi(\pi) + i\xi(\pi)\mathcal{V}_{\mu}(x)$

• $\mathcal{V}_{\mu}(x) = \mathcal{V}_{\mu}^{i}(x)Q^{i}$, Q^{i} generators of I

$$\Rightarrow \hat{\alpha}_{\mu}(\pi) = \frac{1}{i}D_{\mu}\xi(\pi)\cdot\xi^{+}(\pi) = \alpha_{\mu}(\pi) + \hat{\mathcal{V}}_{\mu}(x)$$
$$\hat{\mathcal{V}}_{\mu} \equiv \xi(\pi)\mathcal{V}_{\mu}(x)\xi^{+}(\pi)$$
$$= \mathcal{V}_{\mu}(x) + \frac{i}{f_{\pi}}[\pi(x), \mathcal{V}_{\mu}(x)] + \frac{1}{2!}\left(\frac{i}{f_{\pi}}\right)^{2}[\pi,[\pi, \mathcal{V}_{\mu}(x)]] + \dots$$

we then replace the $\parallel$ & $\perp$ terms from before with:

$$\hat{\alpha}_{\mu\parallel}(\pi) = \alpha_{\mu\parallel}(\pi) + \hat{\mathcal{V}}_{\mu\parallel}(x), \quad \hat{\alpha}_{\mu\perp}(\pi) = \alpha_{\mu\perp}(\pi) + \hat{\mathcal{V}}_{\mu\perp}(x)$$
$$\hat{\mathcal{V}}_{\mu\parallel} = [2\,\mathrm{tr}\,S^{a}\hat{\mathcal{V}}_{\mu}(x)]S^{a} \in \mathcal{H}$$
$$\hat{\mathcal{V}}_{\mu\perp} = [2\,\mathrm{tr}\,X^{a}\hat{\mathcal{V}}_{\mu}(x)]X^{a} \in \mathcal{G} - \mathcal{H}$$

(*) If I is larger than unbroken subgroup H, (i.e. ∃ nonzero $\mathcal{V}_{\mu\perp}(x) \in \mathcal{I} - \mathcal{H}$) those perp. components become massive.

→ If G/H is symmetric, the components of $\hat{\mathcal{V}}_{\mu}$ can be expanded & split into even and odd "parity" parts. Then $\mathcal{L}_{kin}$ reads:

$$\mathcal{L}_{kin} = f_{\pi}^{2}\,\mathrm{tr}[\hat{\alpha}_{\mu\perp}(\pi)]^{2} = \mathrm{tr}[\partial_{\mu}\pi(x) + f_{\pi}\mathcal{V}_{\mu\perp}(x)]^{2} + \dots$$

4) Hidden Local Symmetry

(*) Any Nonlinear Sigma Model based on G/H is gauge equivalent to a "linear" model with symmetry $G_{global} \times H_{local}$.

• Can be extended to $G_{global} \times G_{local}$, too.

- Let $\xi(x)$ take the value of a unitary matrix rep. of G which transforms under the group $G_{global} \times H_{local}$ as

$$\xi(x) \to \xi'(x) = h(x)\,\xi(x)\,g^\dagger, \quad g \in G_{global},\ h(x) \in H_{local}$$

$$\xi(x) = \xi(\sigma)\,\xi(\pi) = e^{i\sigma(x)/f_\sigma}\, e^{i\pi(x)/f_\pi}$$

$$\pi(x) \equiv \pi^a(x) X^a, \quad \sigma(x) \equiv \sigma^\alpha(x) S^\alpha$$

• $\sigma(x)$ type-terms are referred to as "compensators"

Define a Maurer-Cartan 1-form & its transformation

$$\alpha_\mu(x) = \frac{1}{i}\partial_\mu \xi(x) \cdot \xi^\dagger(x)$$

$$\alpha_\mu(x) \to \alpha'_\mu(x) = h(x)\,\alpha_\mu(x)\,h^\dagger(x) + \frac{1}{i}\partial_\mu h(x) \cdot h^\dagger(x)$$

With projections $\perp$ & $\parallel$:

$$\alpha_{\mu\parallel}(x) \to \alpha'_{\mu\parallel}(x) = h(x)\,\alpha_{\mu\parallel}(x)\,h^\dagger(x) + \frac{1}{i}\partial_\mu h(x) \cdot h^\dagger(x)$$

$$\alpha_{\mu\perp}(x) \to \alpha'_{\mu\perp}(x) = h(x)\,\alpha_{\mu\perp}(x)\,h^\dagger(x)$$

Define a covariant derivative

$$D_\mu \xi(x) = \partial_\mu \xi(x) - i V_\mu(x)\,\xi(x)$$

and $V_\mu(x) \to V'_\mu(x) = i\,h(x)\,\partial_\mu h^\dagger(x) + h(x)\,V_\mu(x)\,h^\dagger(x)$

• $V_\mu(x) \equiv V^\alpha_\mu(x) S^\alpha$ is the gauge field corresponding to H_{local}

So the covariantized 1-form of $\alpha_\mu(x)$ is now

$$\hat\alpha_\mu(x) = \frac{1}{i} D_\mu \xi(x) \cdot \xi^\dagger(x) = \alpha_\mu(x) - V_\mu(x)$$

Transforming as

$$\hat\alpha_\mu(x) \to \hat\alpha'_\mu(x) = h(x)\,\hat\alpha_\mu(x)\,h^\dagger(x)$$

With these, two invariants can be written:

$$\mathcal{L}_V = f_\pi^2\, \mathrm{tr}\left(\hat\alpha_{\mu\parallel}(x)\right)^2 = f_\pi^2\, \mathrm{tr}\left(V_\mu(x) - \alpha_{\mu\parallel}(x)\right)^2$$

$$\mathcal{L}_A = f_\pi^2\, \mathrm{tr}\left(\hat\alpha_{\mu\perp}(x)\right)^2 = f_\pi^2\, \mathrm{tr}\left(\alpha_{\mu\perp}(x)\right)^2$$

And the most general $\mathcal{L}$ made of ξ & $D_\mu\xi$ with fewest derivatives is

$$\mathcal{L} = \mathcal{L}_A + a\,\mathcal{L}_V$$

- How is the $\mathcal{L}$ with arbitr. a equivalent to the nonlinear sigma model one?

↳ Using the E.O.M. solution for V_μ:

$$V_\mu^a = 2\,tr(S^a \alpha_{\mu\parallel}(x)) = 2i\,tr(S^a \xi(x)\partial_\mu \xi^\dagger(x))$$

$\Rightarrow \mathcal{L}_1 \to 0$

Can also fix a gauge by setting $\sigma(x) \to 0$:

$$\xi(x) = e^{i\sigma(x)/f_\pi} e^{i\pi(x)/f_\pi} = e^{i\pi(x)/f_\pi} = \xi(\pi)$$

Then

$$\mathcal{L} = \mathcal{L}_A = f_\pi^2\, tr(\alpha_{\mu\perp}(x))^2 = -2f_\pi^2 \sum_{X^a \in G-H} [tr(X^a \partial_\mu \xi(\pi)\cdot \xi^\dagger(\pi))]^2 = \mathcal{L}_{CCWZ}$$

↳ This gauge fixing eliminates H_{local} symm. (since $\sigma(x) = \sigma^a S^a$, $S^a \in H$). But $\sigma(x) = 0$ is not generally preserved under G_{global} transformations either.

$$G_{gl}: \xi(x) \to \xi'(x) = \xi(x) g^\dagger = e^{i\sigma'(\pi(x),g)/f_\pi} e^{i\pi'(x)/f_\pi}$$

• i.e., σ is "regenerated" after a G_{global} transform.

If, however, we also do an H_{local} gauge transf. with (simultaneously)

$$H_{local}: h(x) = e^{-i\sigma'(\pi(x),g)/f_\pi} = h(\pi(x),g)$$

Then global G symmetry is preserved:

$$G: \xi(x) \to \xi'(x) = h(\pi(x),g)\,\xi(x)\,g^\dagger = \xi(\pi'(x))$$

• i.e., The local transf. $h(\pi(x),g)$ kills the field created by g. Physically, the Vector fields "eat" the scalar field σ' and acquire longitudinal components, via a transformation $V_\mu \to V_\mu - \frac{1}{gf}\partial_\mu \sigma'$

$\Rightarrow$ now $\mathcal{L}$ & the transf. law are the same as the nonlinear sigma model one. The Gauge Equivalence holds with the addition of matter & ext. Gauge fields.

note: matter & Gauge fields are treated similarly to before. Result: two arbitrary parameters from matter inclusion, and

$$\mathcal{L}_V = f_\pi^2\, tr(V_\mu(x) - \alpha_{\mu\parallel}(x) - \mathcal{V}_\mu(x))^2$$

$$\mathcal{L}_A = f_\pi^2\, tr(\alpha_{\mu\perp}(x) + \mathcal{V}_\mu(x))^2$$

• Generalized Hidden Local Symmetry

- In some cases, more bosons are needed than afforded by Hlocal in Gglobal × Hlocal (this is true for QCD). Maybe we can get a Gglobal × Glocal model.

Process: Start with Gglobal × Hlocal × Glocal linear model, then gauge fix Hlocal and use EOM to eliminate the corresponding boson, leaving us with Gglobal × Glocal.

↳ $\xi_{1,2}(x)$ dynamical vars of $G_g \times H_l \times G_l$,

$$\xi_1 \to \xi_1'(x) = h(x)\,\xi_1(x)\,\tilde{g}^\dagger(x), \quad \tilde{g} \in G_{local}$$
$$\xi_2 \to \xi_2'(x) = h(x)\,\xi_2(x)\,g^\dagger(x), \quad g \in G_{global}$$

$h(x) \in H_{local}$

Correspondingly, we'll now have four covariant Maurer-Cartan 1-form components

$$\hat{\alpha}^{(1)}_{\mu\parallel}(x) = \left(\tfrac{1}{i} D_\mu \xi_1(x) \cdot \xi_1^\dagger(x)\right)_\parallel = \alpha^{(1)}_{\mu\parallel}(x) - \tilde{V}_\mu(x) + [\xi_1(x) V_\mu(x) \xi_1^\dagger(x)]_\parallel$$
$$\hat{\alpha}^{(1)}_{\mu\perp}(x) = \alpha^{(1)}_{\mu\perp}(x) + [\xi_1(x) V_\mu(x) \xi_1^\dagger(x)]_\perp$$
$$\hat{\alpha}^{(2)}_{\mu\parallel}(x) = \alpha^{(2)}_{\mu\parallel}(x) - \tilde{V}_\mu(x) + [\xi_2(x) \mathcal{V}_\mu(x) \xi_2^\dagger(x)]_\parallel$$
$$\hat{\alpha}^{(2)}_{\mu\perp}(x) = \alpha^{(2)}_{\mu\perp}(x) + [\xi_2(x) \mathcal{V}_\mu(x) \xi_2^\dagger(x)]_\perp$$

• $\alpha^{(i)}_\mu = \frac{1}{i}\partial_\mu \xi_i(x) \cdot \xi_i^\dagger(x)$

$\tilde{V}_\mu$ - Hlocal gauge fields

V_μ - Glocal gauge fields

$\mathcal{V}_\mu(x)$ - ext. gauge fields from Gglobal gauging

From this, there are 6 $G_g \times H_l \times G_l$ invariants:

$$\mathcal{L}_{(V,A)} = f_\pi^2\, \mathrm{tr}\left(\hat{\alpha}^{(1)}_{\mu(\parallel,\perp)} - \hat{\alpha}^{(2)}_{\mu(\parallel,\perp)}\right)^2$$
$$\mathcal{L}_{(M,\pi)} = f_\pi^2\, \mathrm{tr}\left(\hat{\alpha}^{(1,2)}_{\mu\perp}(x)\right)^2$$
$$\mathcal{L}_{(V1,V2)} = f_\pi^2\, \mathrm{tr}\left(\hat{\alpha}^{(1,2)}_{\mu\parallel}(x)\right)^2$$

$$\Rightarrow \mathcal{L} = a'\mathcal{L}_V + b\mathcal{L}_A + c\mathcal{L}_M + d\mathcal{L}_\pi + e\mathcal{L}_{V1} + f\mathcal{L}_{V2} + \mathcal{L}_{kinematic}(\mathcal{V}_\mu)$$

is the most general $\mathcal{L}$

• Then, solve out $\tilde{V}_\mu(x)$ with EOM to get

$$\mathcal{L} = a\mathcal{L}_V + b\mathcal{L}_A + c\mathcal{L}_M + d\mathcal{L}_\pi + \mathcal{L}_{kin}(\mathcal{V}_\mu),$$
$$a = a' + \frac{ef}{(e+f)}$$

Now parametrize $\xi_{1,2}(x)$ by

$$\xi_1(x) = \xi(s)\,\xi(p) = e^{is\cdot S}\,e^{ip\cdot X}$$
$$\xi_2(x) = \xi(\sigma)\,\xi(\pi) = e^{i\sigma\cdot S}\,e^{i\pi\cdot X}$$

fixing the H_{local} gauge to $\xi(s) = 1$, the last 3 variables transform under the residual $G_{global} \times G_{local}$ as

$$\xi(p) \to \xi(p') = h(p(x), \tilde{g})\,\xi(p)\,\tilde{g}^{\dagger}(x)$$
$$\xi(\sigma) \to \xi(\sigma') = h(p(x), \tilde{g})\,\xi(\sigma)\,h^{\dagger}(\pi(x), g)$$
$$\xi(\pi) \to \xi(\pi') = h(\pi(x), g)\,\xi(\pi)\,g^{\dagger}(x)$$

← Recheck this

These + the $\mathcal{L}$ from before are the same as the $G_{global} \times G_{local}$ model.

(*) Check the ref. for this, if you can find it. ← elaborate?

→ So $G_g \times H_l \times G_l$ is gauge equiv. to $G_g \times G_l$, which in turn is " " to $G_g \times H_l$ (seen by fixing $\xi(p) = 1$), which can also be gauge-fixed to G/H model.

↳ The more compensators (like $\sigma(x)$) we introduce, the larger hidden local symmetries that can be present.

- Compensators are redundant variables here.

- The gauge group of hidden local symmetry can be made as large as possible using methods like above. But dynamics of the system determine whether further introduced local symmetries become physical or just gauge freedoms to eliminate the compensators.

⑦ Dynamical Gauge Bosons in QCD

- Consider low-energy effective $\mathcal{L}$ of QCD $SU(2)_L \times SU(2)_R / SU(2)_V$ ($\sim G/H$).

$G_{global} = [SU(2)_L \times SU(2)_R]_{global}$

$H_{local} = [SU(2)_V]_{local}$

$SU(2)_L \times SU(2)_R$ global symmetry is spontaneously broken to the diag. subgroup $SU(2)_V$.

(note: this is for massless 2-flavor QCD. It can be generalized to $U(3)_L \times U(3)_R / U(3)_V$ for massless 3-flavor QCD.)

↳ This model is built with two SU(2)-matrix valued variables:

$$\xi_L(x),\ \xi_R(x) : U(x) = e^{2i\pi(x)/f_\pi} = \xi_L^\dagger(x)\,\xi_R(x)$$

$\pi(x) \equiv \pi^a(x) \frac{\tau^a}{2}$

parametrized as

$$\xi(x)_{L,R} = e^{i\sigma(x)/f_\sigma}\, e^{\pm i\pi(x)/f_\pi}$$

$\sigma(x) \equiv \sigma^\alpha(x) \frac{\tau^\alpha}{2}$

& transforming as

$$\xi(x)_{L,R} \to \xi'(x)_{L,R} = h(x)\,\xi_{L,R}(x)\,g^\dagger_{L,R}$$

$g_{L,R} \in [SU(2)_L \times SU(2)_R]_g$

$h(x) \in [SU(2)_V]_l$

The covariant derivative is then

$$D_\mu \xi_{L,R}(x) \equiv \partial_\mu \xi_{L,R}(x) - i V_\mu(x)\,\xi_{L,R}(x)$$

$V_\mu(x) = V_\mu^a(x) \frac{\tau^a}{2}$

$\sim \rho$-meson

and the 1-forms are

$$\hat{\alpha}_{\mu\parallel}(x) = (D_\mu \xi_L \cdot \xi_L^\dagger + D_\mu \xi_R \cdot \xi_R^\dagger)/2i$$

$$\hat{\alpha}_{\mu\perp}(x) = (D_\mu \xi_L \cdot \xi_L^\dagger - D_\mu \xi_R \cdot \xi_R^\dagger)/2i$$

$$\hat{\alpha}_{\mu\parallel,\perp}(x) \to \hat{\alpha}'_{\mu\parallel,\perp}(x) = h(x)\,\hat{\alpha}_{\mu\parallel,\perp}\,h(x)$$

There are two resulting $[SU(2)_L \times SU(2)_R]_g \otimes [SU(2)]_{local} \otimes$ parity invariants:

$$\mathcal{L}_V = f_\pi^2\, tr(\hat{\alpha}_{\mu\parallel}(x))^2 = f_\pi^2\, tr(V_\mu(x) - \alpha_{\mu\parallel}(x))^2$$

$$\mathcal{L}_A = f_\pi^2\, tr(\hat{\alpha}_{\mu\perp}(x))^2 = f_\pi^2\, tr(\alpha_{\mu\perp}(x))^2$$

↳ $\mathcal{L} = \mathcal{L}_A + a\,\mathcal{L}_V$ in general

$$\alpha_{\mu\perp,\parallel} = \frac{\partial_\mu \xi_L \cdot \xi_L^\dagger \pm \partial_\mu \xi_R \cdot \xi_R^\dagger}{2i}$$

Unsurprisingly, finding & plugging in the V_μ EOM and fixing the Gauge with $\sigma(x) = 0$ eliminates $\mathcal{L}_V$ and $\xi_L^\dagger(x) = \xi_R(x) = e^{i\pi(x)/f_\pi} = \xi(\pi)$, so

$$\mathcal{L} = \mathcal{L}_A = f_\pi^2 \, tr(\alpha_{\mu\perp}(x))^2 = \mathcal{L}_{CCWZ}$$ as before.

- In this framework, it's also possible to finally gauge $G_{global} = [SU(2)_L \times SU(2)_R]_{global}$ with the elementary Gauge fields

$$\mathcal{L}_\mu = e \, \mathcal{L}^a_\mu(x) \frac{\tau^a}{2} \equiv \mathcal{V}_\mu(x) - \mathcal{A}_\mu(x)$$
$$\mathcal{R}_\mu = e \, \mathcal{R}^a_\mu(x) \frac{\tau^a}{2} \equiv \mathcal{V}_\mu(x) + \mathcal{A}(x)$$

- These can be assoc. with γ, W, Z from Glashow-Salam-Weinberg model.

- So now there are two types of Gauge Bosons, External ($\mathcal{V}_\mu(x)$ & $\mathcal{A}_\mu(x)$) and Hidden ($V_\mu(x)$), which couple independently from each other.

↳ The covariant derivatives must change to reflect this (& will be redefined to only be w.r.t. external gauge fields)

$$D_\mu \xi_L = (\partial_\mu - i V_\mu)\xi_L + i \xi_L \mathcal{L}_\mu \equiv \mathcal{D}_\mu \xi_L(x) - i V_\mu \xi_L$$

- $(L \leftrightarrow R, \mathcal{L} \leftrightarrow \mathcal{R})$

Accordingly, the $\mathcal{L}$ becomes

$$\mathcal{L} = \mathcal{L}_A + a \mathcal{L}_V + \mathcal{L}_{kin}(\mathcal{L}_\mu, \mathcal{R}_\mu)$$
$$\mathcal{L}_V = f_\pi^2 \, tr\left(V_\mu - \frac{(\mathcal{D}_\mu \xi_L \cdot \xi_L^\dagger + \mathcal{D}_\mu \xi_R \cdot \xi_R^\dagger)}{2i}\right)^2$$
$$\mathcal{L}_A = f_\pi^2 \, tr\left(\frac{(\mathcal{D}_\mu \xi_L \cdot \xi_L^\dagger - \mathcal{D}_\mu \xi_R \cdot \xi_R^\dagger)}{2i}\right)^2$$
$$= (f_\pi^2/4) \, tr(D_\mu U D^\mu U^\dagger)$$

- $\mathcal{L}_{kin}$ - ext. boson kinetic term

- Next, assume the kinetic term of $V_\mu(x)$ is dynamically generated via underlying QCD dynamics and add it to the total $\mathcal{L}$, & identify V_μ with the ρ-meson field.

- In the case $\mathcal{L}_\mu - \mathcal{R}_\mu = \mathcal{V}_\mu$ and $\mathcal{V}_\mu = \mathcal{B}_\mu$ (photon),

$$\mathcal{D}_\mu \xi_{L,R} = \partial_\mu \xi_{L,R} + ie\, \xi_{L,R}\, \mathcal{B}_\mu \tau_3/2$$

e – EM coupling constant.

& rescaling $V_\mu \to g V_\mu$,

$$\mathcal{L}_V = f_\pi^2\, tr\left(g V_\mu - i[\pi, \partial_\mu \pi] \frac{1}{2 f_\pi^2} - e\, \mathcal{B}_\mu \frac{\tau_3}{2} + \ldots \right)^2$$

$$\mathcal{L}_A = \frac{1}{4} f_\pi^2\, tr(\partial_\mu U \partial^\mu U^+) + e\, \mathcal{B}^\mu (\pi \times \partial_\mu \pi)_3 + \ldots$$

In full, then,

$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu}^{(V)})^2 - \frac{1}{4}(\partial_\mu \mathcal{B}_\nu - \partial_\nu \mathcal{B}_\mu)^2 + \frac{f_\pi^2}{4} tr(\partial_\mu U \partial^\mu U^+)$$
$$+ \left[\frac{m}{2} m_\rho V_\mu^2 - e g_\rho V_3^\mu \mathcal{B}_\mu + \frac{1}{2} m_{\mathcal{B}}^2 \mathcal{B}_\mu\right]$$
$$+ g_{\rho\pi\pi} V^\mu \cdot (\pi \times \partial_\mu \pi) + g_{\gamma\pi\pi} \mathcal{B}^\mu (\pi \times \partial_\mu \pi)_3 + \ldots$$

⊛ Massive γ?
It turns out that this term is cancelled out by the self-energy when making corrections to the γ propagator.

with

$$m_\rho^2 = a g^2 f_\pi^2 \qquad g_\rho = a g f_\pi^2$$
$$m_{\mathcal{B}}^2 = a e^2 f_\pi^2 = (e g_\rho / m_\rho)^2 \qquad g_{\rho\pi\pi} = \frac{1}{2} a g$$
$$g_{\gamma\pi\pi} = (1 - \tfrac{1}{2} a)\, e$$

One also finds the KSRF(I) relation, which links ρ-meson & charged pion decays:

$$g_\rho = 2 f_\pi^2 g_{\rho\pi\pi}$$

• Experimental results at the time had $\Gamma^{exp}(\rho \to e^+e^-) \Rightarrow g_\rho \approx .12\, GeV^2$ vs. $2 f_\pi^2 g_{\rho\pi\pi} \approx 0.11\, GeV^2$

which is notably independent of the parameters a & g.

- Furthermore, choosing $a = 2$ leads to the $\mathcal{L}$ becoming that of Sakurai's VMD (with ρ-γ mixing & ρ-dominance) plus reproduces 3 phenom. facts:

i) $g_{\rho\pi\pi} = g$ (ρ-coupling universality)

ii) $m_\rho^2 = 2 g_{\rho\pi\pi}^2 f_\pi^2$ (KSRF II)

iii) $g_{\gamma\pi\pi} = 0$ (ρ-dominance of EM form factor of the pion)

· KSRF II requires VMD or an equivalent choice of parameters

→ U(3) extension

Similar to the SU(2) model, we have

$$\mathcal{L} = \frac{1}{4} f_\pi^2 \, tr(\partial_\mu U(x) \partial^\mu U^\dagger(x))$$

$$U(x) = e^{2i\pi(x)/f_\pi} = \xi(\pi)\cdot\xi(\pi), \quad \pi(x) \equiv \pi^a(x) T^a$$

T^a U(3) generators, $tr(T^a T^b) = \frac{1}{2}\delta^{ab}$

· $\pi^a(x)$ are the (pseudo scalar) meson (π, K, $\bar{K}$, η, η′).

· $U(x) \to U'(x) = g_L U(x) g_R^\dagger$

$g_L \in U(3)_L, g_R \in U(3)_R$

The Lagrangian can be written again as the CCWZ $\mathcal{L}$

$$\mathcal{L} = f_\pi^2 \, tr(\alpha_{\mu\perp}(\pi))^2$$

with

$$\alpha_{\mu\perp}(\pi) = \frac{1}{2i}(\partial_\mu \xi(\pi)\cdot\xi^\dagger(\pi) - \partial_\mu \xi^\dagger(\pi)\cdot\xi(\pi))$$

$$= \xi^\dagger(\pi)(\partial_\mu U \cdot U^\dagger)\xi(\pi)/2i$$

note: In this chiral case, $G/H = U(n)_L \times U(n)_R / U(n)_V$, G is not simple. $g \in G$ are pairs of U(n) matrices (ξ_L, ξ_R). $\xi(\pi) \in G/H$, the coset var, has the form $(\xi, \xi^\dagger)$. So the M.C. 1-form is $(\frac{1}{i}\partial_\mu\xi\cdot\xi^\dagger, \frac{1}{i}\partial_\mu\xi^\dagger\cdot\xi) \Rightarrow \alpha_\mu{}^{\parallel}_{\perp} = \frac{1}{2i}(\partial_\mu\xi\cdot\xi^\dagger \pm \partial_\mu\xi^\dagger\cdot\xi)$

This is true for SU(2) case as well.

– Adding ext. gauge fields $\mathcal{L}_\mu$ and $\mathcal{R}_\mu(x)$ to $U_{L,R}$ as usual:

$$D_\mu U(x) = \partial_\mu U(x) - i\mathcal{L}_\mu(x) U(x) + i U(x)\mathcal{R}_\mu(x)$$

$\mathcal{L}_\mu = \mathcal{L}_\mu^a T^a, \mathcal{R}_\mu = \mathcal{R}_\mu^a T^a$

$$\Rightarrow \mathcal{L} = \frac{1}{4} f_\pi^2 (D_\mu U(x) D^\mu U^\dagger(x))$$

$$= f_\pi^2 \, tr(\hat{\alpha}_{\mu\perp}(\pi))^2$$

$$\hat{\alpha}_{\mu\perp}(\pi) = \frac{D_\mu\xi(\pi)\cdot\xi^\dagger(\pi) - D_\mu\xi^\dagger(\pi)\cdot\xi(\pi)}{2i}$$

$$D_\mu\xi(\pi) = \partial_\mu\xi(\pi) + i\xi(\pi)\mathcal{R}_\mu(x)$$

$$D_\mu\xi^\dagger(\pi) = \partial_\mu\xi^\dagger(\pi) + i\xi^\dagger(\pi)\mathcal{L}_\mu(x)$$

Realistically, our external Gauge Bosons are the EW bosons of Glashow-Salam-Weinberg ($\gamma \sim \mathcal{B}_\mu(x)$, $W \sim \mathcal{W}^i_\mu(x)$, $Z^0 \sim \mathcal{Z}(x)$). These are incorporated by letting

$$\mathcal{L}_\mu(x) = eQ[\mathcal{B}_\mu(x) - \tan\theta_W \mathcal{Z}_\mu(x)] + \frac{e}{\sin\theta_W \cos\theta_W} T_3 \cdot \mathcal{Z}_\mu(x) + \frac{e}{\sqrt{2}\sin\theta_W} \mathcal{W}^i_\mu(x)$$

$$\mathcal{R}_\mu(x) = eQ[\mathcal{B}_\mu(x) - \tan\theta_W \mathcal{Z}_\mu(x)]$$

The relevant terms in $\mathcal{L}$ to get masses & couplings are now

$$\mathcal{L}_{m_V} = ag^2 f_\pi^2 \, \mathrm{tr}\, V_\mu^2$$

$$\mathcal{L}_{V\gamma} = (aeg f_\pi^2)\, 2\, \mathrm{tr}(V_\mu Q)\mathcal{B}^\mu$$
$$= (aeg f_\pi^2)(\rho_\mu + \tfrac{1}{3}\omega_\mu - \tfrac{1}{3}\sqrt{2}\phi_\mu)\mathcal{B}^\mu$$

$$\mathcal{L}_{V\pi\pi} = \left(\frac{ag}{2}\right)\frac{2}{f_\pi^2}\, \mathrm{tr}\, V_\mu[\pi, \partial^\mu \pi]$$

$$\mathcal{L}_{\gamma\pi\pi} = (1 - \tfrac{1}{2}a)e\frac{2}{f_\pi^2}\, \mathrm{tr}\, \mathcal{B}_\mu[\pi, \partial^\mu \pi]$$

→ That are could reproduce phenomen. results with specific parameter choices served as evidence (or at least support) for dynamical gauge bosons in low-energy hadronic physics.

An example given is predictions of/constraints for (intrinsic) parity-odd processes like $\omega \to \pi\gamma$ and $\omega \to 3\pi$ via anomalous low-energy theorems for $\pi^0 \to 2\gamma$, $\gamma \to 3\pi$, to which the Wess-Zumino Non-Abelian anomaly contributes.

$$\mathcal{W} = \begin{pmatrix} 0 & \mathcal{W}^+_\mu \cos\theta_C & \mathcal{W}^+_\mu \sin\theta_C \\ \mathcal{W}^-_\mu \cos\theta_C & & \\ \mathcal{W}^-_\mu \sin\theta_C & & 0 \end{pmatrix}$$

$Q = \frac{1}{3}\,\mathrm{diag}(2, -1, -1)$

$T_3 = \frac{1}{2}\,\mathrm{diag}(1, -1, -1)$

$\theta_{W,C}$ ~ Weinberg, Cabibbo θ

e - EM coupling

intrinsic parity-odd γ-emission processes.

General outline of the WZ anomaly example:

$[U(3)_L \times U(3)_R]_{global} \times [U(1)]_{local}$, global fully gauge) with $\mathcal{L}_\mu$ & $\mathcal{R}_\mu$

Let a ~~transformation of the above symmetry~~ be

$$\delta = \delta_L(\varepsilon_L) + \delta_V(\nu) + \delta_L(\varepsilon_R)$$

s.t. $\xi_{L,R} \to e^{i\nu} \xi_{L,R} e^{-i\varepsilon_{L,R}}$

$$\delta V = d\nu + i[\nu, V] ,$$

$$\delta \mathcal{L} = d\varepsilon_L + i[\varepsilon_L, \mathcal{L}] , \quad \delta\mathcal{R} = d\varepsilon_R + i[\varepsilon_R, \mathcal{R}]$$

• Notation of diff. forms like $V = V_\mu dx^\mu$

The WZ action is

$$\Gamma_{WZ}[U, \mathcal{L}, \mathcal{R}] = \frac{N_c}{240\pi^2} \int_{M^5} tr(\alpha^5) + (\text{covariantization})$$

many terms containing $\mathcal{L}$ & $\mathcal{R}$

N_c = # of colors

$$\alpha = \frac{1}{i}(\partial_\mu U) U^{-1} dx^\mu = \frac{1}{i}(dU) U^{-1} , \quad U = \xi_L^\dagger \xi_R$$

Then the WZ action gives a solution of the WZ anomaly eqn. as

$$\delta\Gamma[\xi_L^\dagger \xi_R, V, \mathcal{L}, \mathcal{R}] = -\frac{N_c}{24(?)\pi^2} \int_{M^4} tr[\varepsilon_L((d\mathcal{L})^2 - \tfrac{i}{2} d\mathcal{L}^3] - (L \leftrightarrow R)$$

to which the soln can be written as

$$\Gamma[\xi_L^\dagger \xi_R, V, \mathcal{L}, \mathcal{R}] = \Gamma_{WZ}[\xi_L^\dagger \xi_R, \mathcal{L}, \mathcal{R}] + \int_{M^4} \sum_{i=1}^{4} c_i \mathcal{L}_i$$

$$\mathcal{L}_1 = i\, tr(\hat{\alpha}_L^3 \hat{\alpha}_R - \hat{\alpha}_R^3 \hat{\alpha}_L)$$
$$\mathcal{L}_2 = i\, tr(\hat{\alpha}_L \hat{\alpha}_R \hat{\alpha}_L \hat{\alpha}_R)$$
$$\mathcal{L}_3 = tr\, F_V(\hat{\alpha}_L \hat{\alpha}_R - \hat{\alpha}_R \hat{\alpha}_L)$$
$$\mathcal{L}_4 = tr(\hat{F}_L \hat{\alpha}_L \hat{\alpha}_R - \hat{F}_R \hat{\alpha}_R \hat{\alpha}_L)$$

} CP conserving, but intrinsic parity violating.

$$\hat{\alpha}_{L,R} \equiv \frac{1}{i} D\xi_{L,R} \xi_{L,R}^\dagger = \alpha_{L,R} - gV + e(\mathring{\mathcal{L}} \text{ or } \mathring{\mathcal{R}})$$

$$F_V \equiv dV - igV^2 , \quad \hat{F}_{L,R} = \xi_{L,R} F_{L,R} \xi_{L,R}^\dagger$$

$\alpha_{L,R} = \frac{1}{i} d\xi_{L,R} \cdot \xi_{L,R}^\dagger$

$\mathring{\mathcal{L}} = \xi_L \mathcal{L} \xi_L^\dagger$

$\mathring{\mathcal{R}} = \xi_R \mathcal{R} \xi_R^\dagger$

$F_L = d\mathcal{L} - ie\mathcal{L}^2$

$F_R = d\mathcal{R} - ie\mathcal{R}^2$

other terms will be linear combinations of these 4.

⇒ These $\mathcal{L}_i$ are gauge inv (i.e., anomaly-free) 4-forms ($\delta \mathcal{L}_i = 0$). Then since the amplitudes for $\pi^0 \to 2\gamma$ & $\gamma \to 3\pi$ are determined only by the anomaly (low energy thms), $\mathcal{L}_i$'s should not contribute to the amplitudes.

Ex: $\mathcal{L}_4 = tr(F_L \alpha_L \alpha_R - F_R \alpha_R \alpha_L)$

$$= i\frac{e}{f_\pi}\left[2g\, Tr(V \partial B^Q \partial\pi) + 2g\, Tr(\partial B^Q V \partial\pi) - 4e\, Tr(B^Q \partial B^Q \partial\pi)\right] + i\left(\frac{i}{f_\pi}\right)^3 4e\, Tr(B^Q (\partial\pi)^3) + \ldots$$

• Where $\mathcal{L}_\mu$ and R_μ are set to be the photon field B_μ, and we use $\mathcal{L} = \mathcal{L}_\mu dx^\mu = R = R_\mu dx^\mu = e B_\mu Q\, dx^\mu = eB^Q$

→ $\{V, \partial B^Q\} \partial\pi$ term $\sim \pi \cdot \gamma \cdot V$ (vector meson vertices)

$B^Q \partial B^Q \partial\pi \sim$ contrib. to $\pi^0 \to 2\gamma$

$B^Q (\partial\pi)^3 \sim$ contrib. to $\gamma \to 3\pi$

· If we consider the $\mathcal{L}$ of the $[U(3)_L \times U(3)_R]_{global} \times [U(3)_V]_{local}$ model, we could find Feynman rules for the behavior of the Vector Mesons dictated by the $\mathcal{L}$:

a) Vector Propagator

$$= -\delta^{ab}\left(g_{\mu\nu} - \frac{P_\mu P_\nu}{M_V^2}\right) / (M_V^2 - p^2)$$

b) Vector-photon Vertex

$$= -e g_V g_{\mu\nu} (2\, Tr(T^a Q))$$

• g_V is $V \sim \gamma$ coupling

c) Vector-pseudoscalar-pseudoscalar vertex

$$= (-i g_{V\pi\pi})\, i(k-q)_\mu (2\, Tr(T^a [T^b, T^c]))$$

Consider two processes:

(1) 

$$= \frac{-\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{m_V^2}\right)}{m_V^2 - p^2}(-e g_V)\, 2\,\mathrm{Tr}(T^a Q)\cdot B_\nu(p)$$

$$\xrightarrow[p^2=0]{} \frac{e}{g}\, 2\,\mathrm{Tr}(T^a Q) B_\mu(p)$$

This implies that V_μ^a at a vertex (connected by V-γ vertex) is effectively replaced by the photon B_μ times a factor.

↳ If the V_μ^a's appear in a matrix form like $\mathrm{Tr}(MV_\mu)$, the replacement is

$$g\,\mathrm{Tr}(MV_\mu) = g\sum_a (\mathrm{Tr}(MT^a))V_\mu^a$$

$$\xrightarrow[p^2=0]{} e\sum_a (\mathrm{Tr}(MT^a))(\mathrm{Tr}(T^a Q))\, B_\mu$$

$$= e\,\mathrm{Tr}(MB_\mu Q)$$

(2) 

$$= -\frac{\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{m_V^2}\right)}{m_V^2 - p^2}(-i g_{V\pi\pi})\, i(k-q)^\nu \times 2\,\mathrm{Tr}(T^a[\pi(q), \pi(k)])$$

$$\xrightarrow[p^2=q^2=k^2=0]{} = i\frac{i}{2 g f_\pi^2}\, i(k-q)^\nu\, 2\,\mathrm{Tr}(T^a[\pi(q), \pi(k)])$$

i.e. $g\,\mathrm{Tr}(MV_\mu) \longrightarrow \frac{i}{2 f_\pi^2}\mathrm{Tr}(M[\pi, \partial_\mu \pi])$

Together, this means we can replace

$$g V_\mu \longrightarrow e B_\mu Q + i(2 f_\pi^2)^{-1}[\pi, \partial_\mu \pi]$$

effectively when $V_\mu \to \gamma$ or two pseudoscalars via these two vertices.

• i.e., Vector propagates and turns into an on-shell photon with phys. polariz. $B_\nu(p)$

→ using $\sum_{a=0}^{8} \mathrm{Tr}(AT^a)\, 2\,\mathrm{Tr}(T^a B) = \mathrm{Tr}(AB)$

(next)

Then the first two terms in $\mathcal{L}_4$ contribute as

$$2 g e f_\pi^{-1} \mathrm{Tr}(\{V, \partial B^\mu\} \partial \pi)$$

$$\longrightarrow 4 e^2 \frac{1}{f_\pi} \mathrm{Tr}(\mathcal{B}^\mu \partial B^\mu \partial \pi) + \frac{i e}{f_\pi^3} \mathrm{Tr}(\{[\pi, \partial\pi], \partial\mathcal{B}^\mu\} \partial\pi)$$

Which cancel the last two terms in $\mathcal{L}_4$.

The remaining terms $\mathcal{L}_{1,2,3}$ can similarly be shown not to contribute to the processes due to cancellation b/w direct terms & vector meson mediated terms.

<u>Note:</u> it can also be shown that $\mathcal{L}_i$'s do not contribute to amplitudes of $\mathcal{L}_\mu$ & $\mathcal{R}_\mu$ ext. gauge bosons.

Final words:

• These come from the diagrams