Hydrodynamics of Globally Rotating Fluids Via Holography

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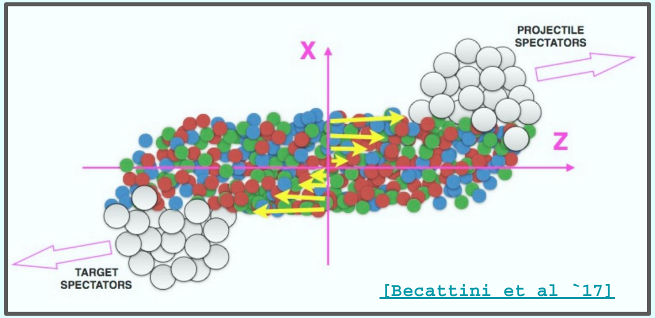
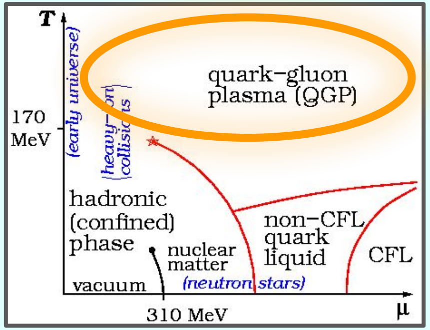


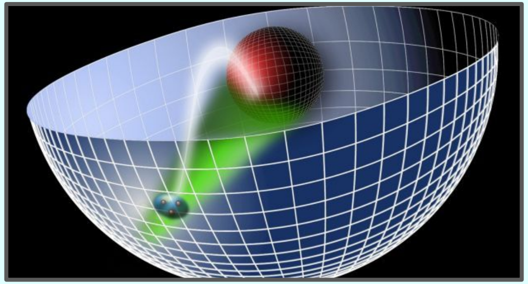
Outline

- Understanding the Field
- Holography and Why?
- Current Research - Globally Rotating Fluid
- Current Research - QNMs
- Current Research - Hydrodynamics
- Current Results - η_{++}
- Current Open Problem and Discussion

Understanding the Field

- Holography (AdS(Anti-de Sitter)/CFT Duality [Maldacena ‘99])
- QGP
    - RHIC [Braun-Munzinger et al. ‘01]
    - Voticity Measured [The STAR Collaboration ‘17]
- Strongly Coupled Systems with **Broken Symmetries**
    - Non-relativistic Systems (Hořava Gravity) [Garbiso et al ‘19]
    - Systems with strong magnetic fields [Cartwright et al ‘19] [Ammon et al ‘17]
    - Rotating fluids from holography? [Hawking ‘99] [Reall et al ‘99]





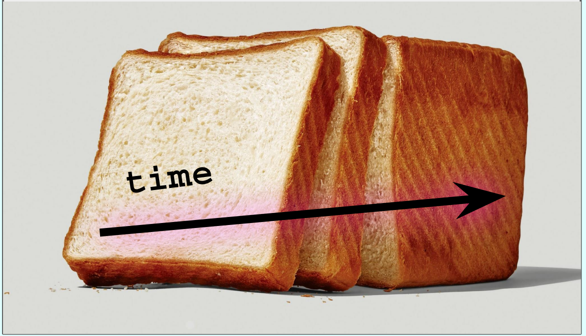
Holography and Why?

- Concrete realization of the holographic principle: AdS/CFT
    - Strongest Statement: Type IIB Superstring theory AdS5×S5 **"Gravity"**/$\mathcal{N}$=4 SYM w/ SU(N) **"CFT"** [Maldacena '99]
    - Weakest Statement (large N limit): *Supergravity* AdS5×S5 **"Gravity"**/*Strongly Coupled QFT* **"CFT"**
    - Gravity/Fluid (large N limit) [Bhattacharyya et al '08] [Bhattacharyya '09]
    - Gravity/QGP [Busza et al '18]
- "Little Bangs" at RHIC
    - Understanding the fluid highly vortical **strongly coupled** fluids (QGP) demands explanation. [Busza et al `18]
- Mysteries of Higher Dimensional Gravity (Final State of Kerr AdS)

Past Research - Hořava

- Using Holography: Non-Relativistic Gravity Theory
- Non-Relativistic Gravity Theory
  - Hořava Gravity [Hořava '09] $t \longrightarrow \tilde{t}(t), \quad x^i \longrightarrow \tilde{x}^i(t, x^i)$
  - Einstein-Aether Gravity [Jacobson et al '01]
  - Equivalent Theories [Bhattacharyya '13]
  - Simultaneity and Causality
- **(3+1)D** black brane: Dual to a **(2+1)D** sheet of non-relativistic material

Bread



Past Research - Einstein- Aether Theory

$$S_{\text{aether}} = \frac{1}{4\pi G_{ae}} \int d^4x \sqrt{-g} \Big(R - 2\Lambda + c_4 (u^\mu \nabla_\mu u^\nu)(u^\sigma \nabla_\sigma u_\nu) - c_3 (\nabla_\mu u^\nu)(\nabla_\nu u^\mu) - c_2 (\nabla_\mu u^\mu)^2 - c_1 (\nabla^\mu u^\nu)(\nabla_\mu u_\nu) \Big)$$

[Janiszewski `15]
[Bhattacharyya '13]

aether vector field and khronon scalar field

$$u_\mu = \frac{\partial_\mu \phi}{\sqrt{-(\partial_\nu \phi)(\partial^\nu \phi)}}$$

coupling constants

$$\alpha = \frac{c_4}{1-c_3} = 0 \qquad 1 + \lambda = \frac{1+c_2}{1-c_3} \qquad \frac{G_H}{G_{ae}} = 1 + \beta = \frac{1}{1-c_3}$$

Previous Research RESULTS

- Is this black brane a linearly stable spacetime?
  - Yes, more **next slide**
- How do you perturb in this theory?
  - ⟶

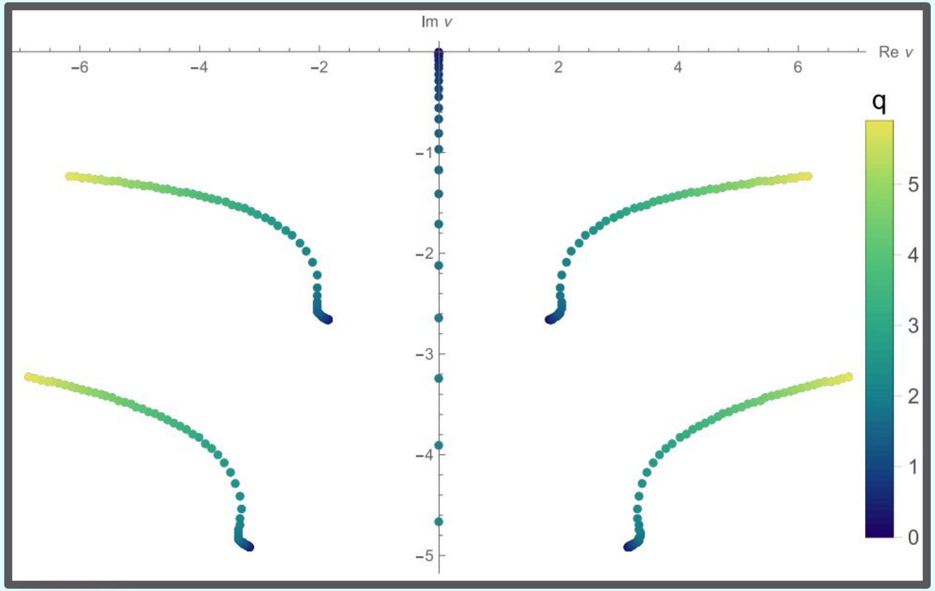
$$g^p_{\mu\nu} = g_{\mu\nu} + \epsilon\, h_{\mu\nu}(x^\sigma)$$
$$\phi^p = \phi + \epsilon\, \chi(x^\sigma)\,,$$

- Are there scale invariances?
  - YES wrt β and r_h
- Any connections to the relativistic version?
  - Yes $\omega^{\text{axial}}_{\text{Horava}} = \sqrt{1+\beta}\,\omega^{\text{axial}}_{\text{Einstein}}$
  - and $\omega^{\text{polar}}_{\text{Horava}} = \sqrt{1+\beta}\,\omega^{\text{polar}}_{\text{Einstein}}$ **MOSTLY**

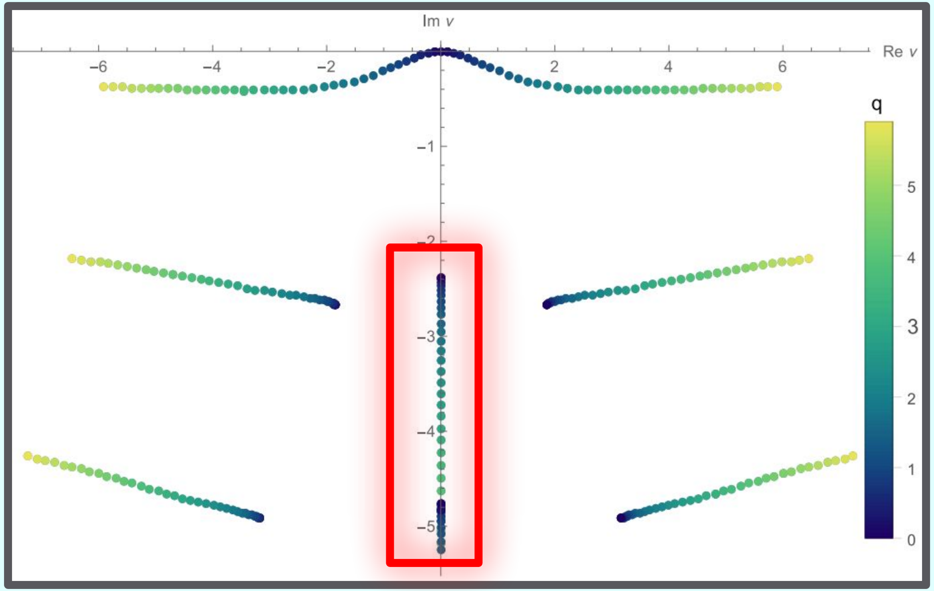
$$\nu = r_h \omega / (2^{1/3}\sqrt{1+\beta})$$

Previous Research RESULTS

Axial QNMs



Polar QNMs



= "Aether Modes"

q = momentum, starting at 0

$$\nu = r_h \omega / (2^{1/3} \sqrt{1+\beta})$$

Current Research - Globally Rotating Fluid

- RHIC & QGP
  - "Non-central collisions have angular momenta of the order of 1,000ħ, and the resulting fluid may have a strong vortical structure 2–4 that must be understood to describe the fluid properly." [The STAR Collaboration '17]
  - Modeling QGP plasma with strong vortical effects where effects are purely rotational
- Gauge/gravity and fluid/gravity
  - A new perspective on rotating strongly coupled fluids
  - How is bulk spacetime rotation encoded on the boundary fluid/field?
  - More QNMs, linear stability analysis?
- Final "AdS-Kerr State"?

Current Research - Theory

$$S = -\frac{1}{16\pi}\int d^5x\sqrt{-g}\left(R + \frac{12}{L^2}\right)$$

5D MP AdS

$$ds^2 = -\left(1 + \frac{r^2}{L^2}\right)dt^2 + \frac{dr^2}{G(r)} + \frac{r^2}{4}\left((\sigma^1)^2 + (\sigma^2)^2 + (\sigma^3)^2\right) + \frac{2\mu}{r^2}\left(dt + \frac{a}{2}\sigma^3\right)^2$$

$$\begin{aligned} G(r) &= 1 + \frac{r^2}{L^2} - \frac{2\mu(1 - a^2/L^2)}{r^2} + \frac{2\mu a^2}{r^4} \\ \sigma^1 &= -\sin\psi d\theta + \cos\psi \sin\theta d\phi \qquad . \\ \sigma^2 &= \cos\psi d\theta + \sin\psi \sin\theta d\phi \\ \sigma^3 &= d\psi + \cos\theta d\phi \end{aligned}$$

Current Research - Theory (TBD)

[Murata `09]

$$ds^2 = -\left(1+r^2\right)dt^2 + \frac{dr^2}{G(r)} + \frac{r^2}{4}\left(4\sigma^+\sigma^- + (\sigma^3)^2\right) + \frac{2\mu}{r^2}\left(dt + \frac{a}{2}\sigma^3\right)^2 .$$

$$\sigma^\pm = \frac{1}{2}(\sigma^1 \mp i\sigma^2)$$

$$e_\pm = \frac{1}{2}(e_1 \pm ie_2)$$

$$\sigma^a e_b = \delta^a_b$$

Current Research - Theory (TBD)

[Murata `09]

$$\xi_x = \cos\phi\,\partial_\theta + \frac{\sin\phi}{\sin\theta}\partial_\psi - \cot\theta\sin\phi\,\partial_\phi$$
$$\xi_y = -\sin\phi\,\partial_\theta + \frac{\cos\phi}{\sin\theta}\partial_\psi - \cot\theta\cos\phi\,\partial_\phi\;.$$
$$\xi_z = \partial_\phi$$

$$\mathcal{L}_{\xi_\alpha}(\sigma^a) = 0$$

$$R_t \times SU(2) \times U(1) \cong R_t \times U(2)$$

$$\mathcal{L}_{\partial_t}(ds^2) = 0$$

$$\mathcal{L}_{e_3}((\sigma^1)^2 + (\sigma^2)^2) = 0$$
$$\mathcal{L}_{e_3}(\sigma^3) = 0$$
$$e_3 = \partial_\psi$$
$$\mathcal{L}_{e_3}(\sigma^\pm) = \pm\sigma^\pm$$

Current Research - Theory (TBD)

[Murata `09]

$$L_\alpha = i\xi_\alpha \ , \ \ W_a = ie_a \ , \ \ \sum_{\alpha=x}^{z} L_\alpha L_\alpha = L^2 = W^2 = \sum_{a=1}^{3} W_a W_a$$

$$[L_\alpha , L_\beta] = i\epsilon_{\alpha\beta\gamma} L_\gamma \ , \ \ [W_a , W_b] = -i\epsilon_{abc} W_c \ , \ \ [W_a , L_\alpha] = 0$$

[Schwinger United States Atomic Energy Commision `52]

Current Research - Linear Analysis

Irreducible Representation of SU(2): **Wigner-D function**

$$L^2 D^{\mathcal{J}}_{\mathcal{K}\mathcal{M}} = \mathcal{J}(\mathcal{J}+1) D^{\mathcal{J}}_{\mathcal{K}\mathcal{M}}\ ,\ L_z D^{\mathcal{J}}_{\mathcal{K}\mathcal{M}} = \mathcal{M} D^{\mathcal{J}}_{\mathcal{K}\mathcal{M}}\ ,\ W_3 D^{\mathcal{J}}_{\mathcal{K}\mathcal{M}} = \mathcal{K} D^{\mathcal{J}}_{\mathcal{K}\mathcal{M}}.$$

$$\mathcal{J} = 0, 1/2, 1, \ldots \quad |\mathcal{K}| \leq \mathcal{J} \qquad |\mathcal{M}| = 0 \leq \mathcal{J}$$

$$g^p_{\mu\nu} dx^\mu dx^\nu = (g_{\mu\nu} + \epsilon\, h_{\mu\nu} + O(\epsilon^2)) dx^\mu dx^\nu$$

$$\dot{R}_{\mu\nu} = \frac{2\Lambda}{D-2} h_{\mu\nu} \quad \dot{R}_{\mu\nu} = -\tfrac{1}{2}\nabla_\mu \nabla_\nu h - \tfrac{1}{2}\nabla^\lambda \nabla_\lambda h_{\mu\nu} + \nabla^\lambda \nabla_{(\mu} h_{\nu)\lambda}$$

[Wald `84]

ex. $\phi = \sum_{\mathcal{K}} \phi^{\mathcal{K}}(r,t) D^{\mathcal{J}}_{\mathcal{K}\mathcal{M}}(\theta, \phi, \psi)$

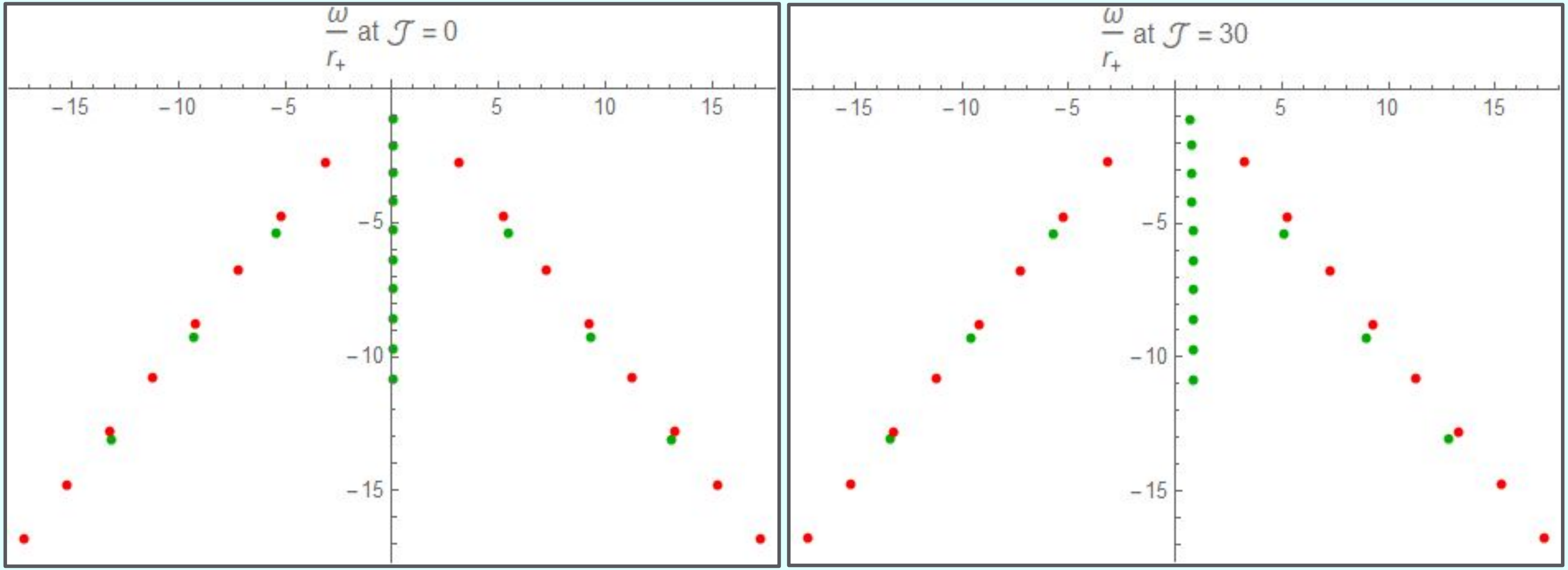
perturbations of different (($\mathcal{J}$,$\mathcal{M}$),$\mathcal{K}$) decouple [Murata et al `08]

Current Research - h++ Modes (TBD)

Current Results - QNMs - “h++” - Real vs Im

At large temperatures ie. $r_+/L \sim 100$

90% Extremality (green) — No Rotation (red)

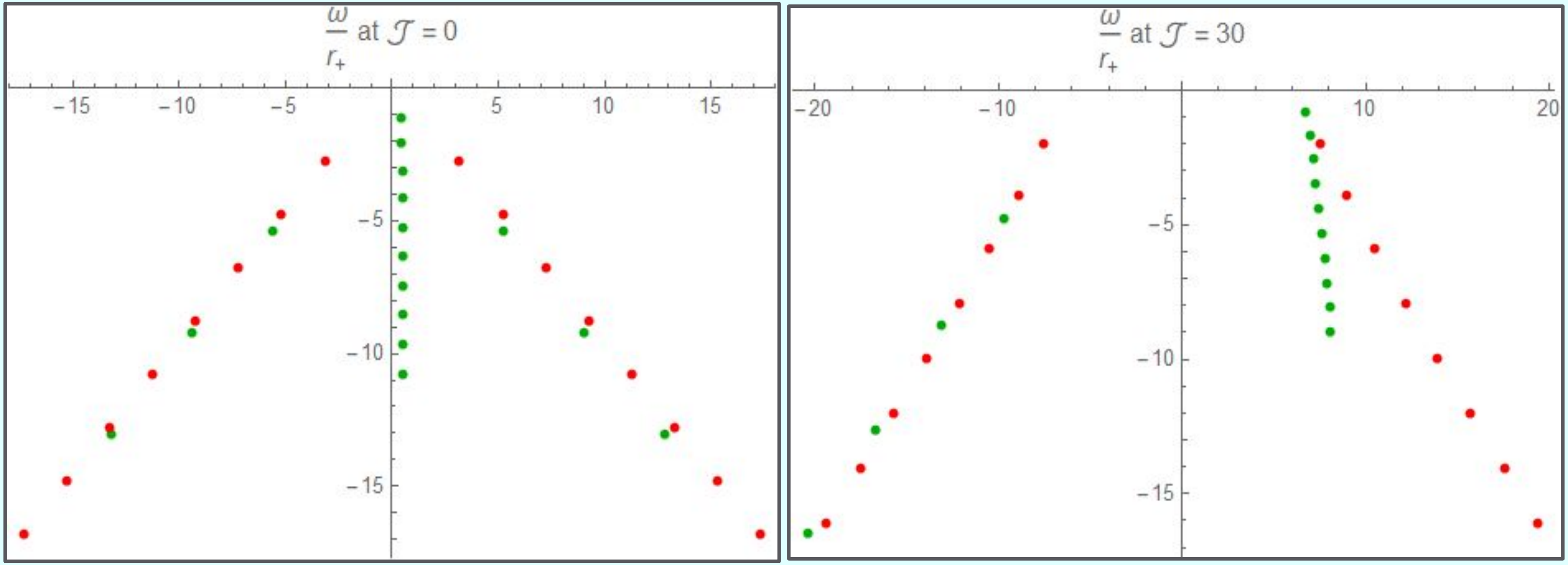


Note new modes close to imaginary axis from larger a.

Current Results - QNMs - “h++” - Real vs Im

At small temperatures ie. $r_+/L \sim 10$

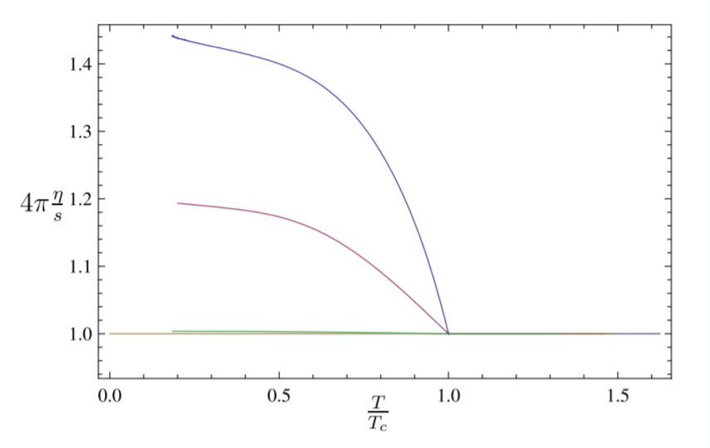
● 90% Extremality ● No Rotation



Note that modes are not symmetric across Imaginary axis.
Note higher J modes are more “unstable”.

Current Research - Hydrodynamics [Cardoso et al. `14]

Rotation seemingly distinguishes a direction so does the **shear viscosity**, μ, change? Shear viscosity doesn't respect the "1/4π" if anisotropy is induced. [Erdmenger et al. `11] [Critelli et al `14]



$$S = \frac{1}{2\kappa_5^2}\int d^5x\left((R - 2\Lambda) - \frac{\alpha^2}{2}\mathrm{Tr}(F^2)\right)$$

[Erdmenger et al. `11]

Current Results - Needed Calculations

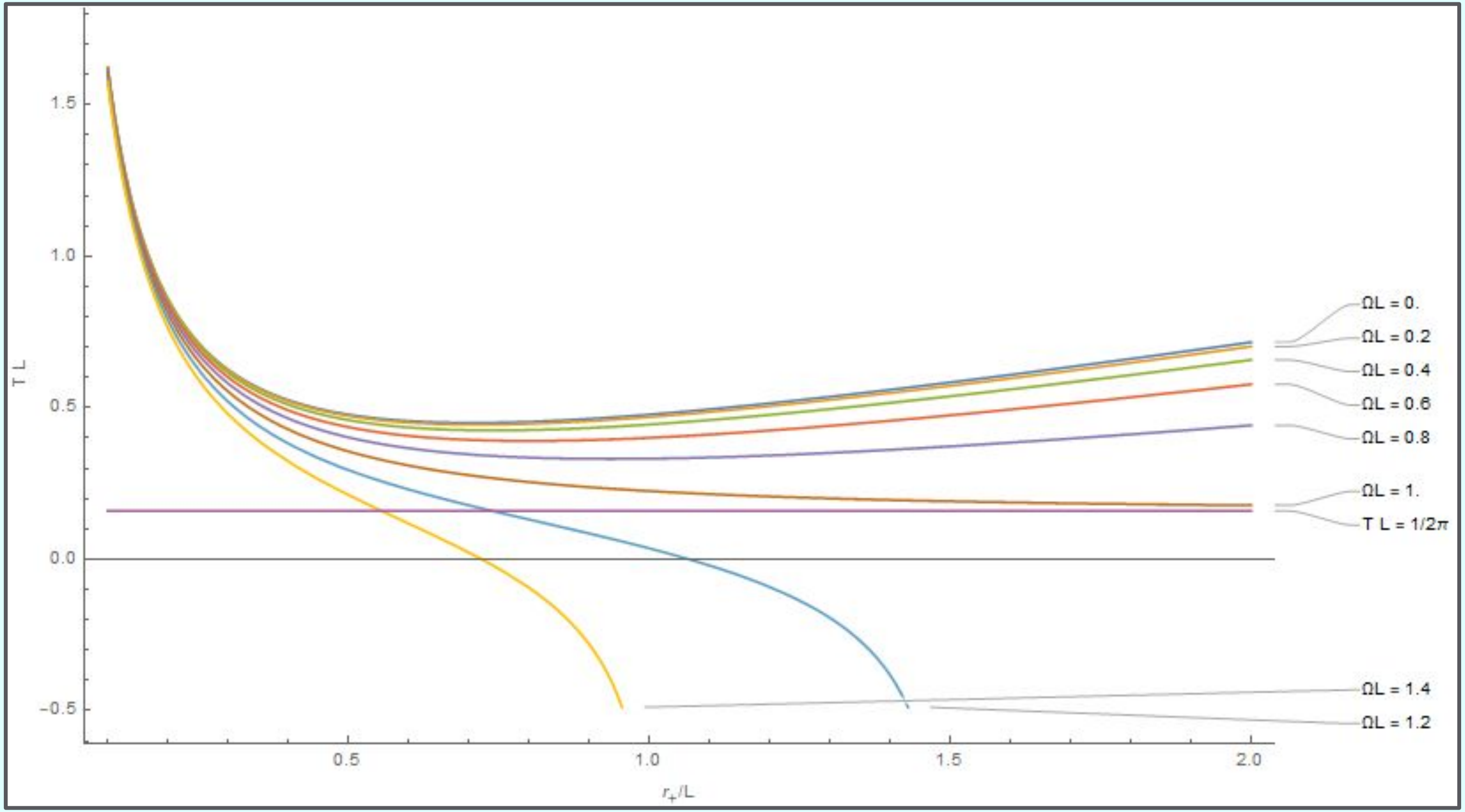
Now, let’s find the temperature of our black hole.

$$T = \kappa/2\pi = \sqrt{\frac{r_+^2+1}{r_+^2\left(1-\Omega^2\right)+1}}\,\frac{2r_+^2\left(1-\Omega^2\right)+1}{2\pi r_+}$$

Hydrodynamics requires large temperature limits and therefore implies ” Ω L<1”.

Furthermore, superradiant instabilities no longer apply [Cardoso et al. `14] [Murata `09], because extremality limit and faster than light limit coincide for .

Hydrodynamics - Large Temperature



Current Results - η++ (in L=1 units)

Kubo Formula for shear viscosity

From papers [Son et al ‘02], we conjectured the expression to the right as a possible expression for η.

$$\eta = -\lim_{\bar{\omega}\to 0^+} \frac{\mathrm{Im} < T_{++} T_{++} >}{\bar{\omega}/T} = -\lim_{\bar{\omega}\to 0^+} \frac{\mathrm{Im}\, h^{(4)}_{++} / h^{(0)}_{++}}{\bar{\omega}/T}$$

$$\sim -\frac{T\,\mathrm{vev}}{\bar{\omega}\ \mathrm{source}}\Bigg|_{\bar{\omega}=\mathrm{small}}$$

Current Results - η++(recap)

$$T = \kappa/2\pi = \sqrt{\frac{r_+^2+1}{r_+^2\left(1-\Omega^2\right)+1}}\,\frac{2r_+^2\left(1-\Omega^2\right)+1}{2\pi r_+}$$

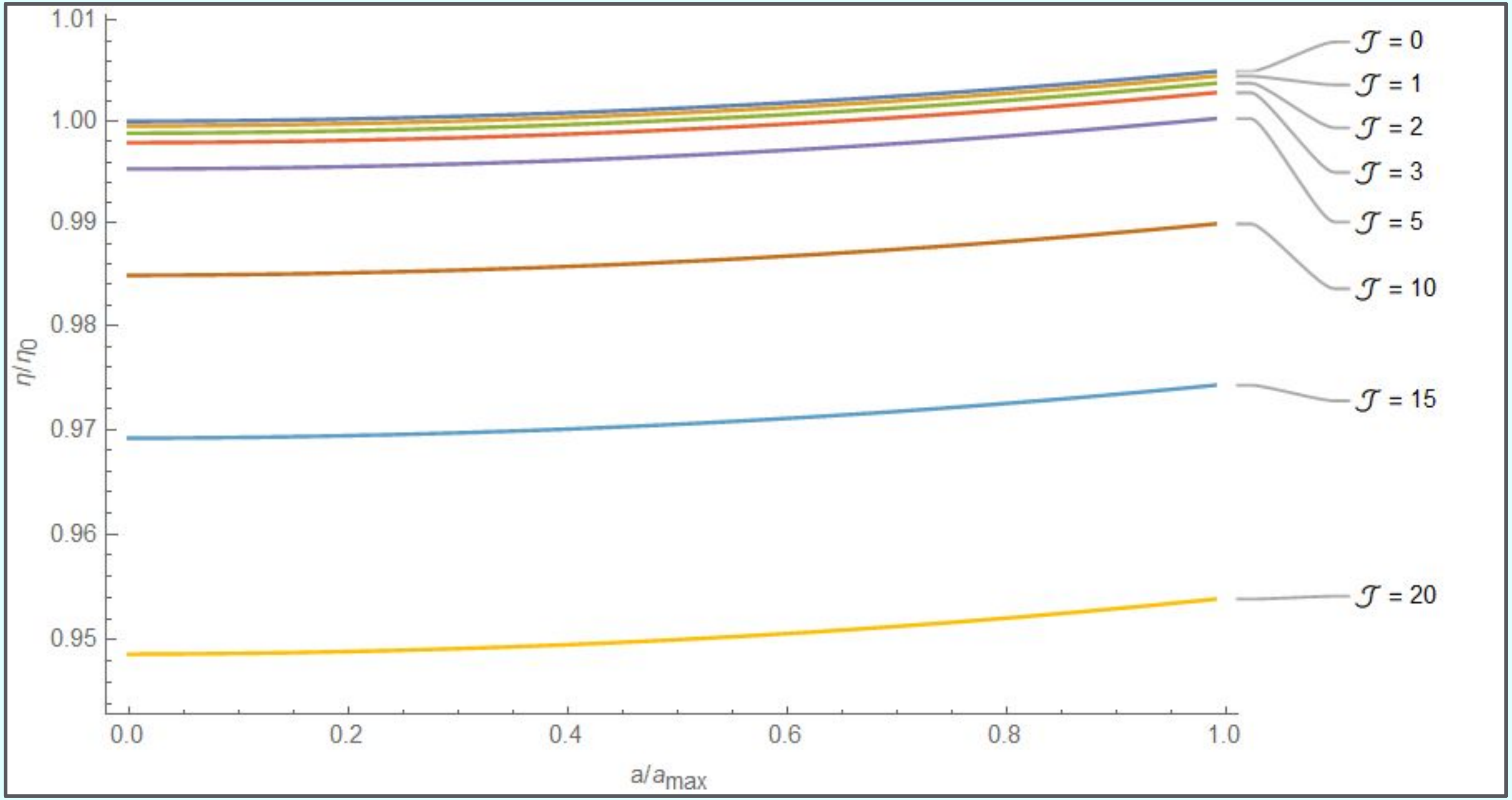
$$\chi^2\Big|_{r=r_+} = 0$$

$$G(r_+) = 0$$

$$\chi = \partial_t - 2\Omega\partial_\psi$$

$$\eta \sim -\frac{T\,\text{vev}}{\bar{\omega}\,\text{source}}\Bigg|_{\bar{\omega}=\text{small}}$$

Current Results - η++ @ (r+ = 100)



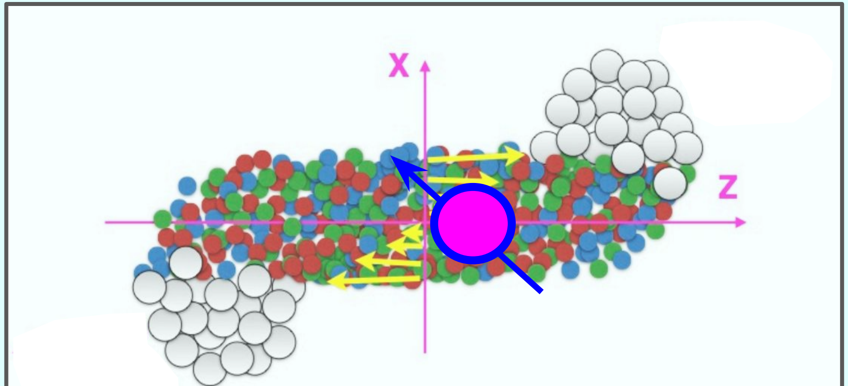
Current Open Problems

How does one model **hyperon**-like fields [Florkowski `19]

QNMs of probe fields

- positive integer spins?
- orbital spin & spin coupling?
- spinors

Finding eta legitimately.



[Becattini et al `17]

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Any Questions?

Useful Refs

https://arxiv.org/pdf/1301.2826.pdf Flow and Viscosity in Relativistic Heavy Ion Collisions
https://arxiv.org/pdf/1905.11309.pdf Quasinormal modes for quarkonium in a plasma with magnetic fields
https://arxiv.org/pdf/1308.2672.pdf Kerr-AdS analogue of triple point and solid/liquid/gas phase transition
https://arxiv.org/pdf/1401.2586.pdf Thermodynamics of rotating black holes and black rings: phase transitions and thermodynamic volume
https://arxiv.org/pdf/1510.04713.pdf Maxwell perturbations on asymptotically anti-de Sitter spacetimes: Generic boundary conditions and a new branch of quasinormal modes
https://arxiv.org/abs/1312.5323 Holographic thermalization, quasinormal modes and superradiance in Kerr-AdS
https://arxiv.org/pdf/1505.04793.pdf Black holes with a single Killing vector field: black resonators
https://arxiv.org/pdf/0803.1371.pdf Stability of Five-dimensional Myers-Perry Black Holes with Equal Angular Momenta
https://arxiv.org/pdf/0901.2574.pdf Warped AdS5 Black Holes and Dual CFTs
https://arxiv.org/pdf/1302.1580.pdf Boundary Conditions for Kerr-AdS Perturbations
https://arxiv.org/pdf/1802.04801.pdf Heavy Ion Collisions: The Big Picture, and the Big Questions
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https://www.sciencedirect.com/science/article/pii/S0370269311003959?via%3Dihub Non-universal shear viscosity from Einstein gravity
https://arxiv.org/pdf/0803.1371.pdf Stability of Five-dimensional Myers-Perry Bla k Holes with Equal Angular Momenta