$SL_q(2)$ Extension of the Standard Model

Intro: - Preons $\longrightarrow$ found this SM extension
- $SL_q(2)$ extension describes particles in terms of knots $\Rightarrow$ Rishon Model Preons
- Particles as knots isn't new, but it's interesting (to me, at least)

- Elementary Knot Theory
  - 3D knots are described in terms of their projections onto a 2D plane.
  - Vertices of a knot ≈ Crossings, with an over/under strand. We consider oriented knots

+1   −1   Crossing Sign

The crossing sign of a vertex is determined by whether the overstrand is aligned with the understrand if rotated ccw (+1) or cw (−1)

- Finkelstein focuses on 3 knot invariants:

Crossing # N ≡ total number of crossings

Writhe ω ≡ Sum of all crossing signs

Rotation r ≡ # of times a tangent of the knot rotates in one traversal.

[Note: These are invariants of regular (2D) isotopy, not ambient (3D) isotopy.]

Ex: Trefoil Knot

$N = 3$

$\omega = 3$

$r = 2$

Ex: Trefoil above is RH trefoil. Via ambient isotopy deformations, can get LH, for which $\omega = -3$.

- Reidemeister Moves & Knot Polynomials

↳ The three reidmeister moves are

I 

II 

III 

oR

↳ We can define the bracket polynomial of a knot K, $\langle K \rangle$, by using the recursive rules

→ Kaufman Polynomials

(A) $\langle \bigcirc \rangle = 1$ (unknot bracket)

(B) $\langle \times \rangle = A \langle)(\rangle + B \langle \asymp \rangle$

(3) $\langle L \cup \bigcirc \rangle = C \langle L \rangle$

↳ L for Link.

- $\langle K \rangle$ must be invariant for any knot or Link, not dependent on A given projection

=> Must be unchanged by R-moves

- Invariance requires $B = A^{-1}$, $C = -A^2 - A^{-2}$
But this is only for moves II & III.

for I, we can define a new polynomial
$$f_K(A) = (-A^3)^{-w(K)} \langle K \rangle$$

$\longleftrightarrow$

→ Signs vary based on convent... Idea is the sam...

Note: Just having inv. under II & I... is enough for regular isotopy inv. $f_K(A)$ for ambient iso.

- The Kauffman rules define a Laurent Polynomial in A.
If we instead consider a new parameter q (with $q^{1/2} \equiv i A^{-1}$), the rules can be written in terms of the matrices
$$\varepsilon_q \equiv \begin{pmatrix} 0 & 1/q^{1/2} \\ -q^{1/2} & 0 \end{pmatrix}, \quad \sigma_\pm \equiv \tfrac{1}{2}(\sigma_x \pm i\sigma_y)$$

→ or $A = i \, \mathrm{Tr}(\varepsilon_q \sigma_-)$

→ ε_q is important for the emergence of $SL_q(2)$

• The Knot Algebra $SL_q(2)$

- The description of the knot by the 3 rules used for constructing our Polynomial is invariant under the transformations
$$\varepsilon_q' = P \varepsilon_q P^T = P^T \varepsilon_q P, \quad \sigma' = \sigma$$
with $P \equiv \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

↳ In particular, requiring $\varepsilon_q' = \varepsilon_q$ and assuming a, b, c, d are not necessarily commuting, we find:

(3)
$$\left.\begin{array}{ll} ab = qba & bc = cb \\ bd = qdb & ad - qbc = 1 \\ ac = qca & da - \frac{1}{q} cb = 1 \\ cd = qdc & \end{array}\right\} SL_q(2)$$

☆ This connection to knots is why $SL_q(2)$ is called the "Knot Algebra"

↳ P is the 2D representation of $SL_q(2)$.

Can also introduce $SU_q(2)$ by setting

$d = \bar{a}$, $c = -\frac{1}{q}\bar{b}$

And the algebra becomes

$ab = qba \qquad a\bar{a} + b\bar{b} = 1$

$a\bar{b} = q\bar{b}a \qquad \bar{a}a + \frac{1}{q^2}b\bar{b} = 1$

$b\bar{b} = \bar{b}b$

- For physical applications, we need higher-dimensional reps. of $SL_q(2)$ (and $SU_q(2)$)

↳ Note that $SL_q(2)$ (and $SU_q(2)$) are Quantum Groups.

↳ $SL_q(2)$ extension isn't as simple as just adding a new Gauge Group to SM

- Want both $SL_q(2)$ & $SU_q(2)$ for physical applications.

→ Quantum groups are actually special cases of Hopf Algebras

- Use q-Binomial theorem to obtain higher-dim. reps. of $SL_q(2)$ & $SO_q(2)$.

$$\Rightarrow (A+B)^n = \sum_s \langle {n \atop s} \rangle_q B^s A^{n-s}$$

$$AB = qBA$$

$$\langle {n \atop s} \rangle_q = \frac{\langle n_q \rangle !}{\langle n-s \rangle_q ! \langle s \rangle !} \; ; \; \langle n \rangle_q \equiv \frac{q^n - 1}{q-1}$$

This is used to compute $SL_q(2)$ transformations on the monomial class

$$\psi^j_m = N^j_m \, x_1^{n_+} x_2^{n_-} \, , \quad -j \leq m \leq j$$

where $[x_1 ; x_2] = 0$

$$n_\pm = j \pm m$$

$$N^j_m = 1 \Big/ \sqrt{\langle n_+ \rangle_{q_1} ! \, \langle n_- \rangle_{q_1} !}$$

↳ Transform $\binom{x_1}{x_2}$ using 2D reps of $SL_q(2)$; P, we get

$$x_1' = a x_1 + b x_2$$
$$x_2' = c x_1 + d x_2 ,$$

$$\psi^{j\,\prime}_m = N^j_m (a x_1 + b x_2)^{n_+} (c x_1 + d x_2)^{n_-}$$

$$= N^j_m \sum_s \langle {n_+ \atop s} \rangle_{q_1} (a x_1)^s (b x_2)^{n_+ - s} \times \sum_t \langle {n_- \atop t} \rangle_{q_1} (c x_1)^t (d x_2)^{n_- - t}$$

$$= N^j_m \sum_{s,t} \langle {n_+ \atop s} \rangle_{q_1} \langle {n_- \atop t} \rangle_{q_1} \times a^s b^{n_+ - s} c^t d^{n_- - t} \times x_1^{s+t} x_2^{n_+ + n_- - s - t}$$

→ This is the q-Analogue of the Binomial thm.

- q-Analogues are generalizations of known expressions that recover familiar forms as $q \rightarrow 1^-$

☆ $q_1 \equiv 1/q$, so just make the substitution

$$\langle n \rangle_{q \rightarrow 1/q} = \frac{(1/q)^n - 1}{(1/q) - 1}$$

$$\Rightarrow \boxed{(A+B)^n = \sum \langle {n \atop s} \rangle_{q_1} A^s B^{n}}$$

→ Assume x_i, a, b commute, so

$(a x_1)(b x_2) = q (b x_2)(a$

- Defining $n_+' = s + t$

$n_-' = n_+ + n_- - s - t$

- note $n_+' + n_-' = n_+ + n_- = 2j$

$$\psi^{j\,\prime}_m = N^j_m \sum_{s,t} \left\langle {n_+ \atop s} \right\rangle_{q_1} \left\langle {n_- \atop t} \right\rangle_{q_1} a^s b^{n_+ - s} c^t d^{n_- - t} \times x_1^{n_+'} x_2^{n_-'}$$

$$= \sum_{s,t} \frac{N^j_m}{N^j_{m'}} \left\langle {n_+ \atop s} \right\rangle_{q_1} \left\langle {n_- \atop t} \right\rangle_{q_1} a^s b^{n_+ - s} c^t d^{n_- - t} \times \delta(s+t, n_+') N^j_{m'} x_1^{n_+'} x_n^{n_-'}$$

Identifying $$D^j_{mm'} = \frac{N^j_m}{N^j_{m'}} \sum_{s,t} \left\langle {n_+ \atop s} \right\rangle_{q_1} \left\langle {n_- \atop t} \right\rangle_{q_1} \ldots \delta(s+t, n_+')$$

$$\underline{\psi^j_m = \sum_{m'} D^j_{mm'} \psi^j_{m'}}$$

- The corresponding $SU_q(2)$ reps are obtained similarly by setting

$d = \bar{a}$, $c = -q_1 \bar{b}$

Then $$D^j_{mm'} = \frac{N^j_m}{N^j_{m'}} \sum_{s,t} \left\langle {n_+ \atop s} \right\rangle_{q_1} \left\langle {n_- \atop t} \right\rangle_{q_1} \delta(s+t, n_+') \times (-q_1)^t a^s b^{n_+ - s} \bar{b}^t \bar{a}^{n_- - t}$$

$\to$ "well-known" procedure for obtaining reps of $SU(2)$

Letting $n_a = s$ $\quad n_+ = n_a + n_b$

$n_c = t$ $\quad n_- = n_c + n_d$

one can write

$$D^j_{mm'}(q|abcd) = \sum_{\substack{\delta(n_a + n_b, n_+) \\ \delta(n_c + n_d, n_-)}} A^j_{mm'}(q|n_a, n_c) \delta(n_a + n_c, n_+') \times a^{n_a} b^{n_b} c^{n_c} d^{n_d}$$

$\to$ $A^j_{mm'}$ contains $\frac{N^j_m}{N^j_{m'}} \left\langle {n_+ \atop s} \right\rangle_{q_1} \left\langle {n_- \atop t} \right\rangle_{q_1}$

Representation of an Oriented Knot

- Previously, we mentioned the knot invariants for 2D projections of knots: N, w, r.
  - ↳ Thinking of these as coordinates (N, w, r) we can make a coordinate transformation to (j, m, m'), the indices labeling the irreducible reps $D^{j}_{mm'}$ of $SL_q(2)$:

$$\boxed{j = \frac{N}{2}, \quad m = \frac{w}{2}, \quad m' = \frac{(r+o)}{2}}$$

- With this, we can introduce the idea of a "Quantum Knot" by interpreting $D^{j}_{mm'}(a, b, c, d)$ as the kinematical description of a quantum state.
  - ↳ For trefoil, let $o = 1$.
    There are 4 choices for (w, r) for a trefoil: $(3, 2)$, $(-3, 2)$, $(3, -2)$, $(-3, -2)$

    Each choice of $D^{3/2}_{\frac{w}{2}, \frac{r+o}{2}}$ will represent distinct quantum states.
- Now we have correspondence b/w states of the quantum knot & topology of classical knot

N - crossing #

w - writhe (sum of all crossing signs)

r - rotation.

→ o is an odd integer.

↳ $\tilde{r} = r + o$ "quantum rotation"
$o \sim$ "zero point" rotation

→ Note that the quantized knot states are restricted by $SL_q(2)$ rep and by knot topology

$$D^{j}_{mm'} = D^{N/2}_{\frac{w}{2}, \frac{r+o}{2}}$$

- Gauge Group of $SL_q(2)$ [& $SU_q(2)$]

- $D^j_{mm'}$ is defined up to the Gauge transformation

$$\left.\begin{array}{ll} a' = e^{i\varphi_a} a & b' = e^{i\varphi_b} b \\ d' = e^{-i\varphi_a} d & c' = e^{-i\varphi_b} c \end{array}\right\} G$$

or $U_a(1) \times U_b(1)$

each term transforms as

$$(a^{n_a} b^{n_b} c^{n_c} d^{n_d})' = e^{i\varphi_a(n_a - n_d)} e^{i\varphi_b(n_b - n_c)} \times (\ldots)$$

which, by the δ-function requirements, can be written in terms of m & m', so that $D^j_{mm'}$ transforms as

$$D^j_{mm'} \to e^{i(m+m')\varphi_a} e^{i(m-m')\varphi_b} D^j_{mm'}$$

$$\text{or } e^{i(\varphi_a + \varphi_b)m} e^{i(\varphi_a - \varphi_b)m'} D^j_{mm'}$$

(Similar for $SU_q(2)$)

→ So we can say

$$D^j_{mm'} \to U_m(1) \times U_{m'}(1) D^j_{mm'}$$

- Now the field operators $\Psi(x)$ are replaced:

$$\Psi^j_{mm'}(x) \longrightarrow \hat{\Psi}^j_{mm'}(x)\, D^j_{mm'}(a,b,c,d)$$

- The Gauge transformation: $U_a(1) \times U_b(1)$

then induces a transformation on $\Psi(x)$:

$$\Psi(x) \longrightarrow U_m(1) \times U_{m'}(1)$$

Under which the new field action must be invariant.

↳ Associate Noether charges with

$U_m(1) \longrightarrow Q_w$, $m = w/2$ "Writhe Charge"

$U_{m'}(2) \longrightarrow Q_r$, $m' = \frac{1}{2}(r+o)$ "Rotation Charge"

- For the quantum trefoil, set $o = 1$ and define

$$Q_w \equiv -k_w m = -k_w \frac{w}{2}$$

$$Q_r \equiv -k_r m' = -k_r \frac{1}{2}(r+1)$$

k_w, k_r are unknown constants with units of electric charge.

→ This knot description is most plausible if simplest particles ⟷ simplest knots

↳ Take most elementary fermions with Isospin $t = \frac{1}{2}$ ~ Trefoils $N = 3, o = 1$.

This is supported by the Empirical observation:

(9) $$\boxed{(t, -t_3, -t_0) = \frac{1}{6}(N, w, r+1)}$$

t, t_3 - isospin

t_0 - $U(1)$ hypercharge

- This empirical observation holds for the left-handed families of the elementary fermions.

  ↳ Each family of fermions (3-fold) is uniquely associated with a specific (w, r) classical trefoil:

$$(e, \mu, \tau)_L \longrightarrow D^{3/2}_{\frac{3}{2}, \frac{3}{2}} \quad (w = 3, r = 2)$$

$$(\nu_e, \nu_\mu, \nu_\tau)_L \longrightarrow D^{3/2}_{-\frac{3}{2}, \frac{3}{2}} \quad (w = -3, r = 2)$$

$$(d, s, b)_L \longrightarrow D^{3/2}_{\frac{3}{2}, -\frac{1}{2}} \quad (w = 3, r = -2)$$

$$(u, c, t)_L \longrightarrow D^{3/2}_{-\frac{3}{2}, -\frac{1}{2}} \quad (w = -3, r = -2)$$

  ↳ Now for $j = \frac{3}{2}$, $t = \frac{1}{2}$, $N = 3$, there is the relation b/w quantum & classical knots as well as the empirical relation b/w fermions & trefoils, leading to

$$(j, m, m') = 3(t, -t_3, -t_0)_L$$

  relating Quantum trefoils to fermions.

- Next, we can compare the electric charges of the elementary fermions to total Noether charges of our trefoils

$$(f_1, f_2, f_3)_L \longleftrightarrow D^{N/2}_{\frac{w}{2}, \frac{r+1}{2}}$$

Reminder

$\tilde{r} = r + 1$

↳ $\tilde{r} = \frac{3}{2} = (2 + 1)\frac{1}{2}$

$\tilde{r} = -\frac{1}{2} = (-2 + 1)\frac{1}{2}$

$\} \rightarrow (j, m, m') = \frac{1}{2}(N, w, r+1)$

$\} \rightarrow (t, t_3, t_0) = \frac{1}{6}(N, w, r+1)$

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↳ Assuming $k_w = k_r = k$ is the same $\forall$ trefoils, we find k by requiring

$$Q_w + Q_r = Q_e$$

$$-k\frac{w}{2} - k\frac{(r+1)}{2} = Q_e$$

Given the values of (w, r) for each lepton family, this holds true for all four families if $k = e/3$

and we see $Q_e = -\frac{e}{6}(w + r + 1)$ for the quantum trefoils

an alternate statement of $Q_e = e(t_3 + t_0)$

$\Rightarrow$ the (t_3, t_0) measures of chg from $SU(2) \times U(1)$ are then related to the (m, m') measures from $SL_q(2)$ at the $j = 3/2$ level via the relation $(j, m, m') = 3(t, t_3, t_0)_L$

$\Rightarrow$ The next step is extending these results beyond $j = 3/2$.

Particularly, for $j = 1/2$.

- $j=\frac{1}{2}$: fundamental Rep & Preon Interpretation

- Already seen fundamental rep of $SL_q(2)$, but Finkelstein attempts to give physical meaning to $D^{1/2}_{mm'}$ by postulating that $(a\,b\,c\,d)$ are creation operators for fermionic preons.

$m \backslash m'$	$\frac{1}{2}$	$-\frac{1}{2}$
$\frac{1}{2}$	a	b
$-\frac{1}{2}$	c	d

Note: These preons aren't "knots" so model as twisted loops with $N=1$, $w=\pm 1$, $r=$

↳ Then the $D^{j}_{mm'}$ from before,

$$D^{j}_{mm} \propto a^{n_a} b^{n_b} c^{n_c} d^{n_d}$$

are clearly polynomials in $(a\,b\,c\,d)$ and can be interpreted as creation ops for a superpos^n of states with n_a, n_b, n_c, n_d preons.

→ # of preons is constrained by fixed (j, m, m') but free (n_a, n_b, n_c, n_d)

↳ In the case of $j=3/2$ we have

- Leptons – $D^{3/2}_{\frac{3}{2},\frac{3}{2}} \sim a^3$ | q_u – $D^{3/2}_{3/2,-1/2} \sim ab^2$
- νs – $D^{3/2}_{\frac{3}{2},\frac{3}{2}} \sim c^3$ | q_d – $D^{3/2}_{3/2,-1/2} \sim cd^2$

Leptons – $3a$'s　　q_u – a, $2b$'s
νs – $3c$'s　　q_d – c, $2d$'s

Interestingly, these agree with Harari-Shupe's Rishon model of quarks

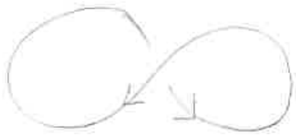
Note: Proposed physical meaning of preon loops as flux tubes, and regarded preons as creation operators for:

particles – if we assume they concentrate energy/momentum at a point.

2D projections of flux tubes – if we assume they concentrate energy/momentum on a curve.

- Graphically, the preons can be drawn as:

a) $D^{1/2}_{1/2,1/2}$ $(w=1, r=0, o=1)$



c) $D^{1/2}_{-1/2,1/2}$ $(w=-1, r=0, o=1)$

b) $D^{1/2}_{1/2,-1/2}$ $(w=1, r=0, o=-1)$

d) $D^{1/2}_{-1/2,-1/2}$ $(w=-1, r=0, o=-1)$

☆ Before drawing fermions, consider the knotted EW vectors and SU(3).

↳ For Vector Bosons, we also impose topo. & empirical restrictions. Letting $t=1$ as in SM, the conditions on J, N, and t imply $J=3$ & $N=6$. Then one can find

$W_\mu^+ : D^3_{-3,0} \sim c^3 d^3$

$W_\mu^- : D^3_{3,0} \sim a^3 b^3$

$W_\mu^3 : D^3_{0,0} \sim f_3(bc)$

Note: W_μ^3 is actuall[y] a superpos^n of 4 states, each with 6 preons, which reduces to a f^n of (bc) by the $SL_q(2)$ Algebra.

- To describe/introduce SU(3), note that the need for SU(3) symmetry appears at the lepton/neutrino level, since they are built from a^3 & c^3. So replace $(a, c) \to (a_i, c_i)$ and

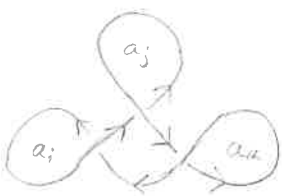
leptons : $a^3 \to \varepsilon^{ijk} a_i a_j a_k$

νs: $c^3 \to \varepsilon^{ijk} c_i c_j c_k$

Now a_i, c_i provide a basis for the fundamental representation of SU(3). If b & d are also color singlets, then the quarks $a_i b^2$ & $c_i d^2$ form the basis. ⟷ in accordance w/SM

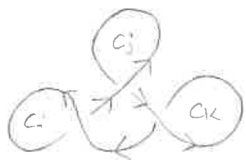
- Now we can draw our fermions like so: (w, r, o)

Leptons: $D^{3/2}_{3/2,3/2} \sim a^3$



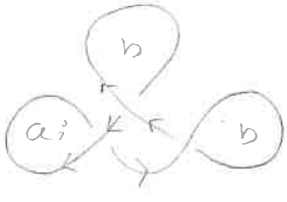
(3, 2, 1)

Neutrinos: $D^{3/2}_{-3/2,3/2} \sim c^3$



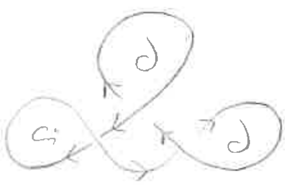
(−3, 2, 1)

q_d: $D^{3/2}_{3/2,-1/2} \sim ab^2$



(3, −2, 1)

q_u: $D^{3/2}_{-3/2,-1/2} \sim cd^2$

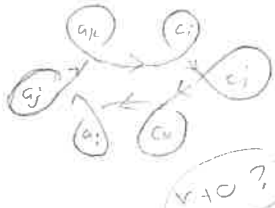


(−3, −2, 1)

As overlapping preon loops

Recall: We are not considering ambient isotopy (3D), so these knots that would unfold into trivial knots do not under regular isotopy (2D).

→ Guess W^-_{N} is



?

r+o?

Representing the model in terms of Preons

- The knot representation of $D^j_{mm'}$ as a function of $(a\,b\,c\,d)$, (n_a, n_b, n_c, n_d) implies constraints on the exponents:

$$n_a + n_b + n_c + n_d = 2j$$
$$n_a + n_b - n_c - n_d = 2m$$
$$n_a - n_b + n_c - n_d = 2m'$$

Then the two relations giving physical meaning to the $D^j_{mm'}$

$$(j, m, m') = \frac{1}{2}(N, w, r+o) \quad \text{field (flux loop) description}$$
$$(j, m, m') = 3(t, -t_3, -t_0)_L \quad \text{particle description}$$

Imply two complementary interpretations of the above relations in terms of preons:

$$\left.\begin{aligned} N &= n_a + n_b + n_c + n_d \\ w &= n_a + n_b - n_c - n_d \\ \tilde{r} &= r + o = n_a - n_b + n_c - n_d \end{aligned}\right\} \text{field / flux interpret.}$$

$$\left.\begin{aligned} t &= \frac{1}{6}(n_a + n_b + n_c + n_d) \\ t_3 &= -\frac{1}{6}(n_a + n_b - n_c - n_d) \\ t_0 &= -\frac{1}{6}(n_a - n_b + n_c - n_d) \end{aligned}\right\} \text{Particle interpret. (only for } j = 3/2)$$

$t \sim$ total preon #

$t_3 \sim$ writhe chg

$t_0 \sim$ rotation chg.

- Consider the field / flux tube interpretation.
This says that the number of crossings
N is equal to the total # of preons.

$\Rightarrow$ fermionic nature of preons then tells us that a knot with N crossings is $\boxed{\text{fermionic if } N \text{ is odd and bosonic if } N \text{ is even.}}$

As a knot, a fermion then becomes a boson when a curl is added/removed.

→ Consistency with view of curl as a twisted preon loop

→ ccw (cw) curl
↕
preon (antipreon) → creation operator

- Since a & d are creation ops for antiparticles (opposite chg & hyperchg) and b & c are neutral (with opposite hyperchg), we can consider preon numbers

$$\nu_a = n_a - n_d$$
$$\nu_b = n_b - n_c$$

$$\Rightarrow \quad \nu_a + \nu_b = w = -6\, t_3$$
$$\nu_a - \nu_b = \tilde{r} = r + 0 = -6\, t_0$$

And we see that conservation of preon #'s / charge & hypercharge is equiv. to writhe & rotation conservation.

↳ quantum conservation laws for preon #s $\longleftrightarrow$ classical conservation for writhe & rotation.

- Charge Measure of $SU(2) \times U(1)$ vs. $SL_q(2)$

- $SU(2) \times U(1) \Rightarrow$ fractional quark charges
- $SL_q(2) \Rightarrow$ Replace the fundamental charge e for chg'd leptons with a <u>new</u> fundamental charge $(e/3)$ for preons

  ↳ quarks no longer have fractional charge.

- $SL_q(2)$ measure (j, m, m') is directly physically interpreted as, $2j$ = total preon #, $2m$ = writhe sources of preonic chg (w), $2m'$ = rotation sources of preonic chg. (r+1)

  ↳ Since these knot descriptors account for the handedness of their source,

  Charge is also measured by source handedness

- The total $SL_q(2)$ charge sums signed CW & CCW turns that an energy-momentum current makes both at crossings & in making a (b/c) circuit of the 2D projected knot.

  ↳ This "knot" charge seems more fundamental than EW isotopic measure.

  Charge → handedness of source.
  Charge becomes a <u>topological concept</u> akin to geometization of Energy-momentum by Curved Spacetime.

(17)

- if Chg <-> handedness
Perhaps $\exists$ a correspondence b/w
(writhe, rotation) ch[illegible]
↕
(Spin, orbital) Ang. moment[illegible]

- Lower Representations

$J = 3 \longrightarrow$ EW vector Bosons

$J = 3/2 \longrightarrow$ Leptons & quarks

$J = 1/2 \longrightarrow$ Preons

- Now consider $J = 1$ & $J = 0$

  - For the $J = 1$ representation, we would obtain $N = 2$ for crossing number.

    $\Rightarrow$ Even; $J = 1$'s are vector bosons. Namely, these are bosons by which the preons interact.

    Ex: For a b, c_i interaction, we might have

    $D^1_{0,m'}$ $(N=2, \omega=0, r+c) = 1+c)$ What should c be here?

These don't play a crucial role in the physics described (at least, as far as SM results go), so Finkelstein does not elaborate on these further

  - For the $J = 0$ representation, then one has $(j, m, m') = \frac{1}{2}(N, \omega, r+c) \Rightarrow (0, 0, c) = \frac{1}{2}(0, 0, c)$ So these correspond to loops with no crossings at all.

> ☆ This means no preonic charge sources, and thus no electroweak interactions!

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[Aside: since $r + \sigma = 0$, $r = \pm 1$, $\sigma = \mp 1$.
Thus, the $J = 0$ quantum states may be realized as loops of field flux with opposite topological rotation that are Quantum Mechanically coupled.]

$\Rightarrow$ Naturally, these non-EW interacting states could be a candidate for Dark Matter in this model.

• Possible interpretation of q Parameter

Recall the matrix

$$\varepsilon_q = \begin{pmatrix} 0 & 1/q^{1/2} \\ -q^{1/2} & 0 \end{pmatrix}$$

which is invariant under

$$P \varepsilon_q P^T = P^T \varepsilon_q P = \varepsilon_q, \quad P = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

- Consider replacing ε_q by

$$\varepsilon = \begin{pmatrix} 0 & \alpha_2 \\ -\alpha_1 & 0 \end{pmatrix}$$

Requiring invariance under

$$P \varepsilon P^T = P^T \varepsilon P = \varepsilon,$$

we can recover the $SL_q(2)$ algebra if $q = \alpha_1 / \alpha_2$. Also imposing $\det(\varepsilon) = 1$, one finds

$$\alpha_1 \alpha_2 = 1 \quad \text{and} \quad \varepsilon_q = \varepsilon$$

Then the knot model can be based on either ε_q or ε.

↳ Since ε is a matrix with 2 parameters rather than 1, it's possible to describe a wider class of theories.

<u>ex:</u> Say the theory we want to describe is characterized by two interacting gauge fields with charges g & g' on the same particle. Physical meaning can be given to q by embedding g & g' into ε:

$$\varepsilon = \begin{pmatrix} 0 & g(E)/\sqrt{\hbar c} \\ -g'(E)/\sqrt{\hbar c} & 0 \end{pmatrix}$$

• g, g' are energy dependent couplings in general.

$\Rightarrow q(E) = g'(E)/g(E)$

↳ $\det(\varepsilon) = 1 \Rightarrow g'(E)g(E) = \hbar c$

Setting (g', g) or $(g, g') = (e, g)$

* ew coupling & gluon coupling

Then $q(E) = e/g$ or g/e

$\det(\varepsilon) = 1 \Rightarrow eg = \hbar c$ ⟷ similarity to Dirac eqn. connecting electric & magn. charge.

↳ If $e(E)$ ↑ $g(E)$ ↓ as E increases, q gets very large or very small @ high energies where interactions/mass terms for preon bound states are relevant.