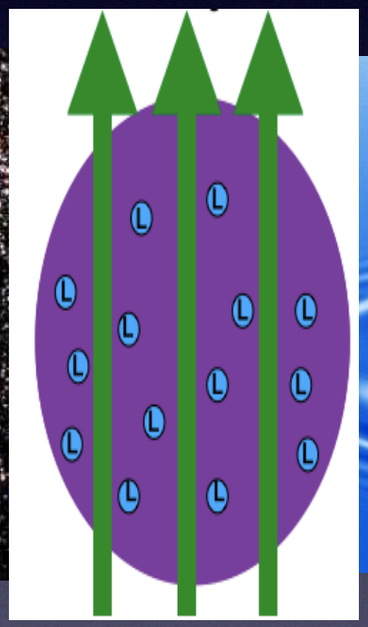
Chiral transport in strong magnetic fields from hydrodynamics & holography

HEP Seminar, University of Alabama

October 04th, 2019



Matthias Kaminski *(University of Alabama)*
in collaboration with
Juan Hernandez *(Perimeter Institute)*
Roshan Koirala, Jackson Wu *(University of Alabama)*
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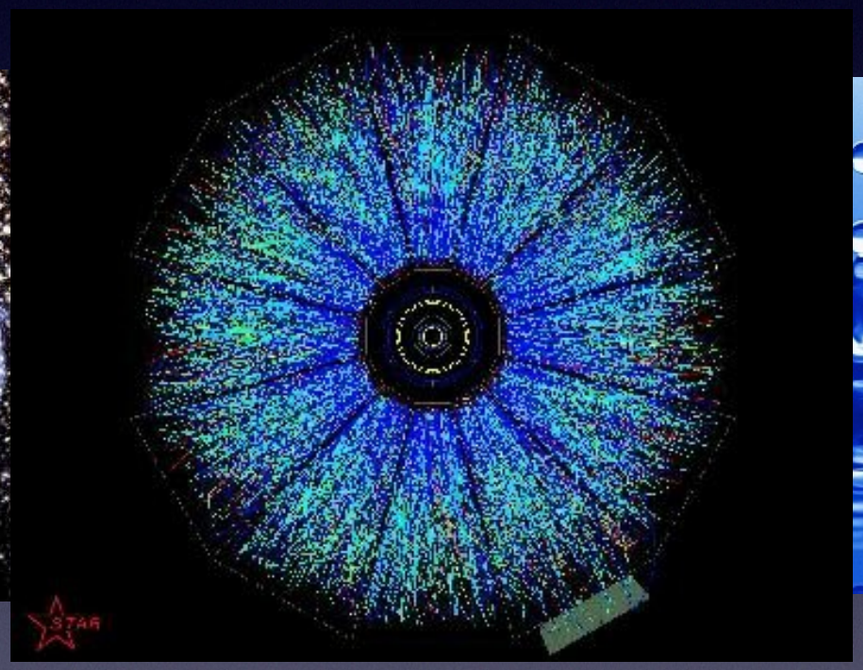

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Odd transport

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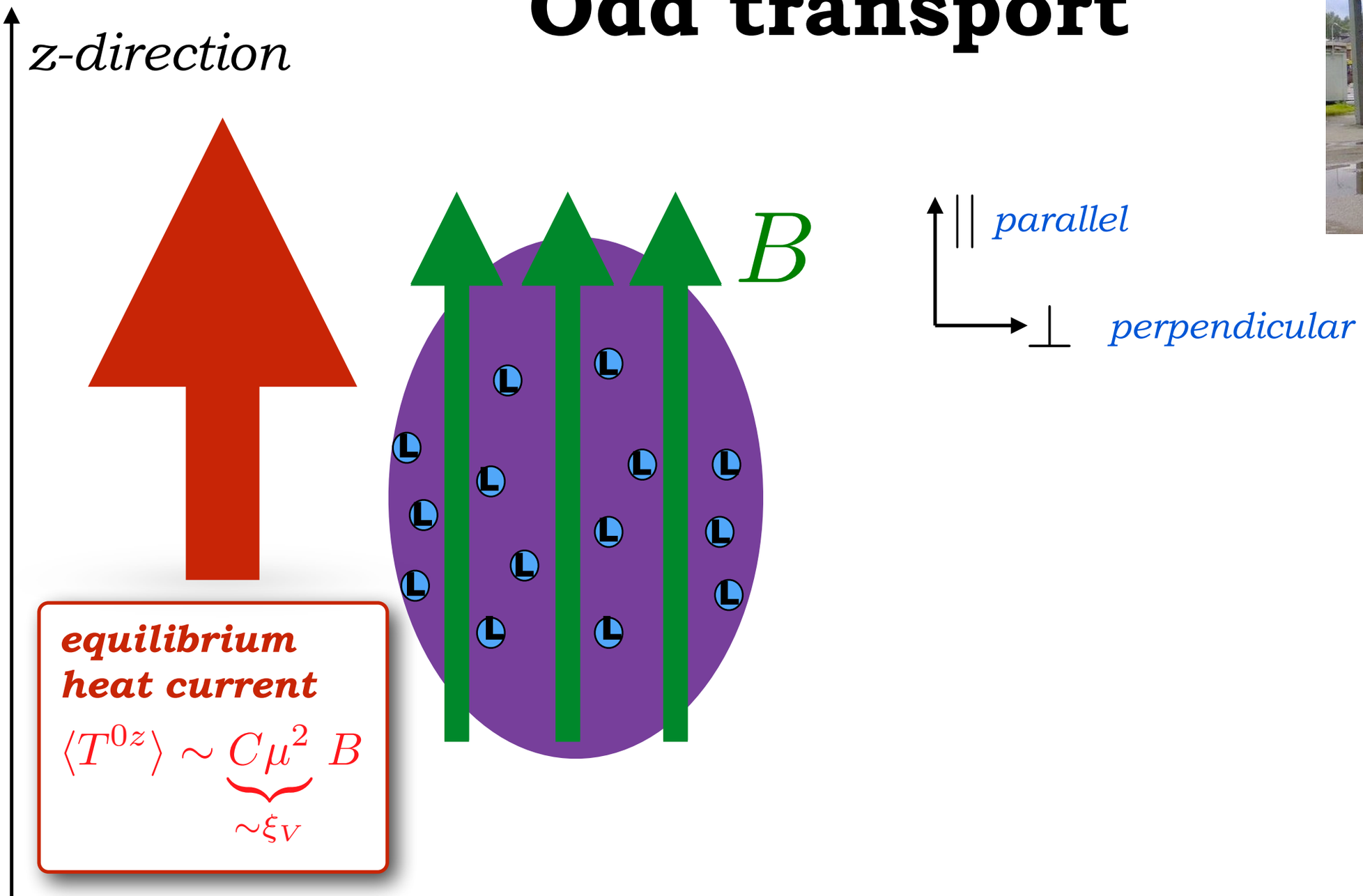

[Ammon, Kaminski et al.; JHEP (2017)]
[Ammon, Leiber, Macedo; JHEP (2016)]

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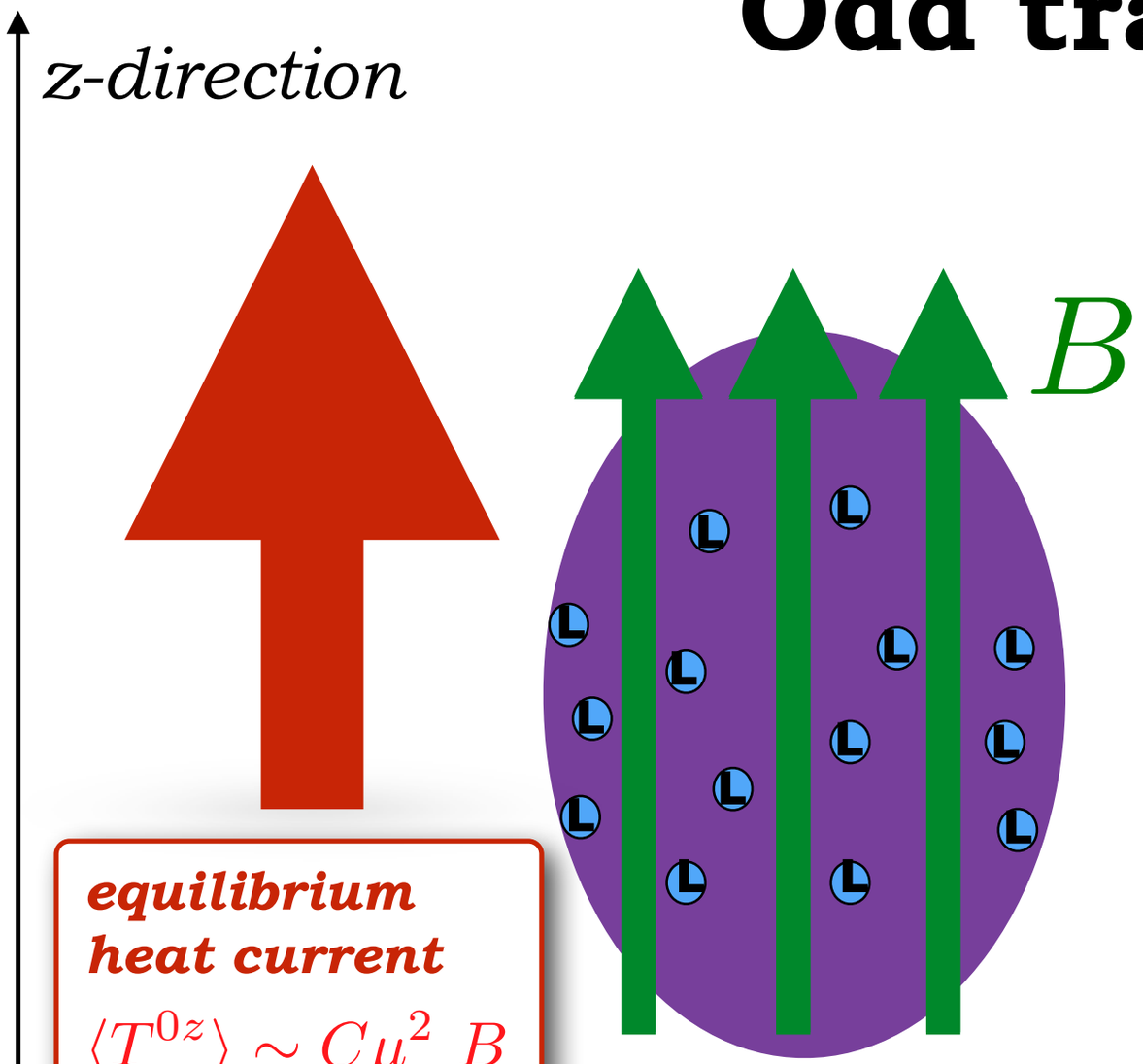
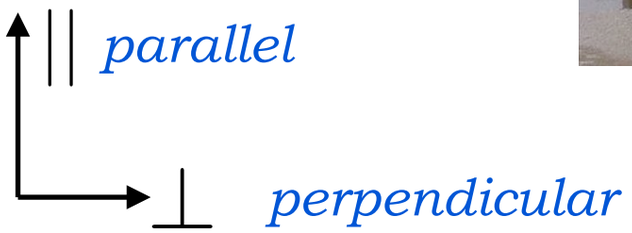




equilibrium heat current

$$\langle T^{0z}\rangle \sim \underbrace{C\mu^2}_{\sim\xi_V} B$$

non-equilibrium parallel conductivity / perpendicular resistivity

$$\langle J^z J^z\rangle(\omega, \mathbf{k}=0) \sim \sigma_{||}$$

$$\langle J^x J^x\rangle(\omega, \mathbf{k}=0) \sim \rho_\perp$$

non-equilibrium parity-odd transport

$$\langle J^x J^y\rangle(\omega, \mathbf{k}=0) \sim \frac{n}{B} - \omega^2 \frac{w^2}{B^4}\tilde{\rho}_\perp + \dots$$

Hall type

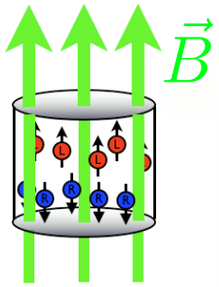
$$\langle J^x J^y\rangle(\omega=0, \mathbf{k}) \sim -ik\, \underbrace{\xi_B}_{C\mu}$$

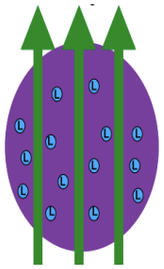
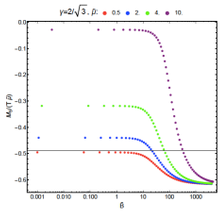
anomaly type

[Ammon, Kaminski et al.; JHEP (2017)]
[Ammon, Leiber, Macedo; JHEP (2016)]

Outline

✓ Invitation: Odd transport

1. Review: hydrodynamics & holography
2. (Chiral magnetic) hydrodynamics 

3. Holographic setup 
4. Results 
5. Discussion

Method: holography & hydrodynamics

Hard problem e.g. QCD at strong coupling

?

gravity dual to QCD or standard model? (not known yet)

model

Hard problem in "similar" model QFT

HOLOGRAPHY

Simple problem in a particular gravitational theory

EFT

Solve problem in effective field theories, e.g.:

- **hydrodynamic approximation of original theory (QCD)**

check of model theory

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➡ Holography good at **qualitative** or **universal** predictions.
➡ **Checks** of model theory.
➡ Understand holography as an **effective description**.

Reminder: Hydrodynamics

universal **effective field theory (EFT)**, expansion in gradients of temperature, chemical potential and velocity

- fields $T(x),\ \mu(x),\ u^\nu(x)$

- conservation equations

$$\nabla_\nu j^\nu = 0$$

- constitutive equations

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Conserved current $\langle j^\mu \rangle = n u^\mu + \nu^\mu$

[Landau, Lifshitz]

Example: hydrodynamic correlators in 2+1

Simple (non-chiral) example in 2+1 dims:

$$j^\mu = n u^\mu + \sigma \left[E^\mu - T \Delta^{\mu\nu} \partial_\nu \left(\frac{\mu}{T} \right) \right]$$

$$\Delta^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$$

$$u^\mu = (1, 0, 0)$$

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sources $\quad A_t,\, A_x \propto e^{-i\omega t + ikx} \qquad u^\mu = (1,0,0)$

fluctuations $\quad n = n(t,x,y) \propto e^{-i\omega t + ikx}$ (fix T and u)

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$$\langle j^t \rangle = n(\omega, k) = \frac{ik\sigma}{\omega + ik^2 \frac{\sigma}{\chi}} (\omega A_x + k A_t)$$

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Einstein relation: $\quad D = \dfrac{\sigma}{\chi}$

$\Longrightarrow$ two point functions $\quad \langle j^x j^x \rangle = \dfrac{\delta \langle j^x \rangle}{\delta A_x} = \dfrac{i\omega^2 \sigma}{\omega + iDk^2}$

$\Longrightarrow$ hydrodynamic poles in spectral function

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$\Longrightarrow$ Kubo formulae $\quad \sigma = \lim\limits_{\omega \to 0} \dfrac{1}{i\omega} \langle j^x j^x \rangle (\omega, k=0)$

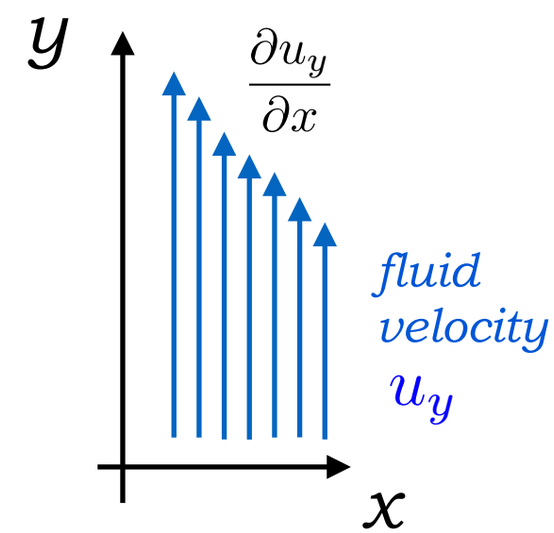
1. Review: hydrodynamics & holography

Famous result: low shear viscosity over entropy density

[Policastro, Son, Starinets; JHEP (2002)]
[RHIC measurement; (2004)]
KSS "bound": [Kovtun, Son, Starinets PRL (2005)]

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

Shear viscosity measures transverse momentum transport:



Kubo formula derived from hydrodynamics:

$$\eta = \lim_{\omega \to 0} \frac{1}{2\omega} \int dt\, d\boldsymbol{x}\, e^{i\omega t} \langle [T_{xy}(x),\, T_{xy}(0)] \rangle$$

from constitutive relation:

$$\langle T_{xy} \rangle \sim \eta\, \sigma_{xy}$$
$$\sim \eta (\nabla_x u_y + \nabla_y u_x)$$

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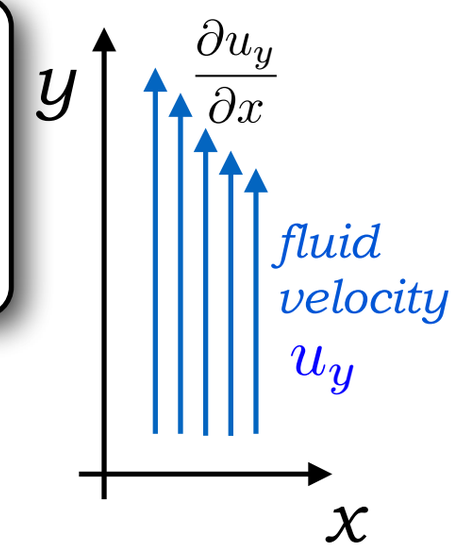

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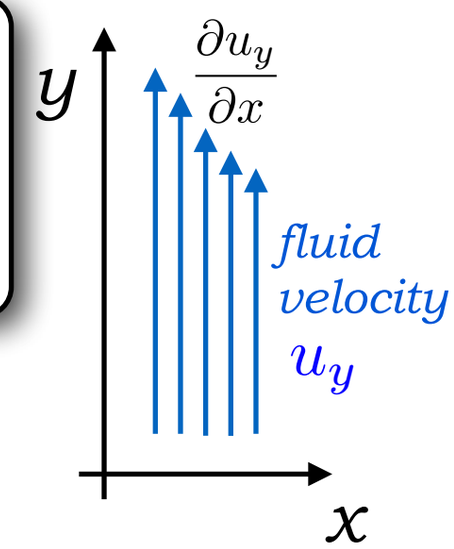

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Holographic calculation:

$$S = \frac{\pi^3 R^5}{2\kappa_{10}^2} \left[\int^1 du \int d^4x \sqrt{-g}\, (\mathcal{R} - 2\Lambda) + 2 \int d^4x \sqrt{-h}\, K \right]$$

$$ds_{10}^2 = \frac{(\pi T R)^2}{u} \left(-f(u) dt^2 + dx^2 + dy^2 + dz^2\right) + \frac{R^2}{4u^2 f(u)} du^2 + R^2 d\Omega_5^2 \qquad \Rightarrow s = \frac{\pi^2}{2} N^2 T^3$$

$f(u) = 1 - u^2$ *black brane metric* *entropy density*

Holographic correlation function: *[Son, Starinets; JHEP (2002)]*

$$G_{xy,xy}(\omega, \boldsymbol{q}) = -\frac{N^2 T^2}{16} \left(i\, 2\pi T \omega + q^2\right) \qquad \Rightarrow \eta = \frac{\pi}{8} N^2 T^3$$

shear viscosity

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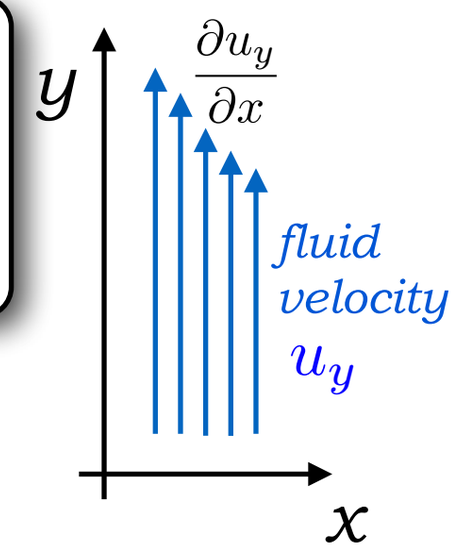

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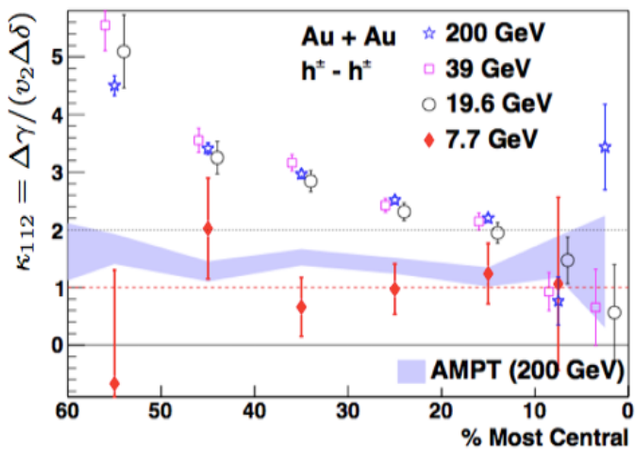
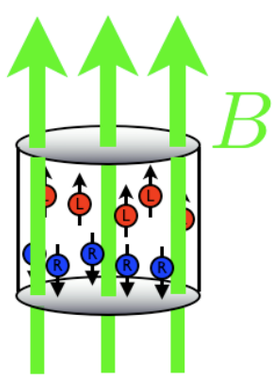
shear viscosity

2. Chiral magnetic hydrodynamics - Motivation

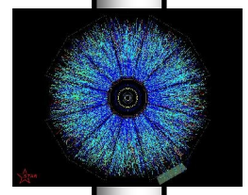
Chiral magnetic effect
- heavy ion collisions (HICs)



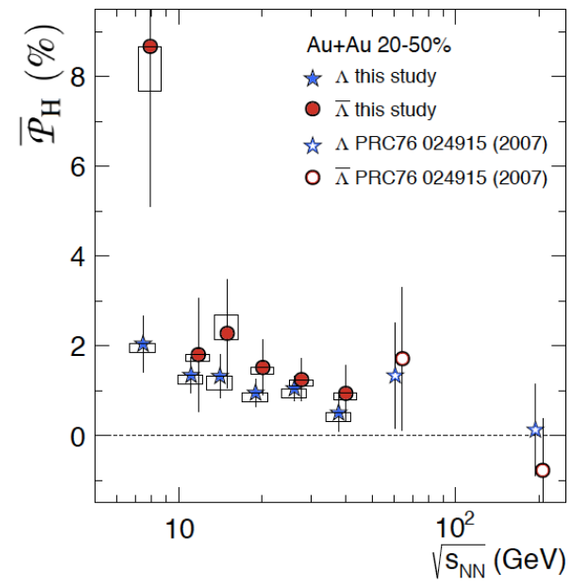
Beam Energy Scan;
Isobaric collisions: Zr / Ru
[RHIC STAR Collaboration; PoS (2018)]



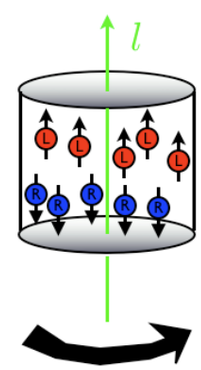
[Fukushima, Kharzeev, Warringa; PRD (2008)]
[Son,Surowka; PRL (2009)] ...
also ***cond-mat*** *and* ***plasma physics***



Most vortical fluid in HICs
- Lambda hyperon polarization



[RHIC STAR Collaboration; Nature (2017)]



vorticity

[Erdmenger, Haack, Kaminski, Yarom; JHEP (2008)]
[Banerjee et al.; JHEP (2011)]
[Landsteiner] *[Son,Surowka; PRL (2009)]* ...

see Koenraad Schalm's talk

Deriving chiral magnetic hydrodynamics

Consider a quantum field theory with a chiral anomaly, in a charged thermal plasma state, subjected to a strong external magnetic field

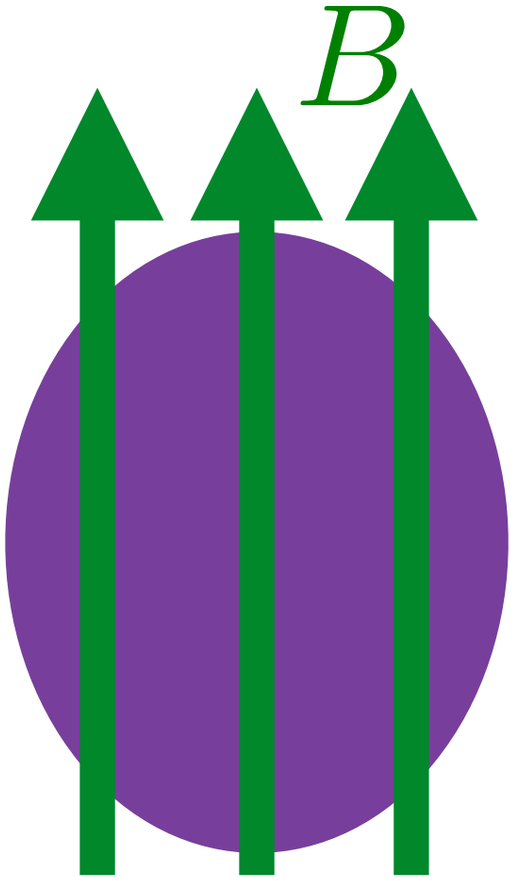
Hydro poles / eigenmodes, and QNMs: [Ammon, Kaminski et al.; JHEP (2017)]

Range of validity $B_0 \sim \mathcal{O}(1)$ $\quad B_0 \ll T_0^2$

$\omega, k \ll T_0$

- equilibrium generating functional
  [Jensen, Kaminski, Kovtun, Meyer, Ritz, Yarom; PRL (2012)]
  [Kovtun; JHEP (2016)]
- equilibrium constitutive equations
  [Kovtun; JHEP (2016)]

$$W_s = \int d^4x\sqrt{-g}\left(p(T,\mu,B^2) + \sum_{n=1}^{5} M_n(T,\mu,B^2)s_n + O(\partial^2)\right)$$



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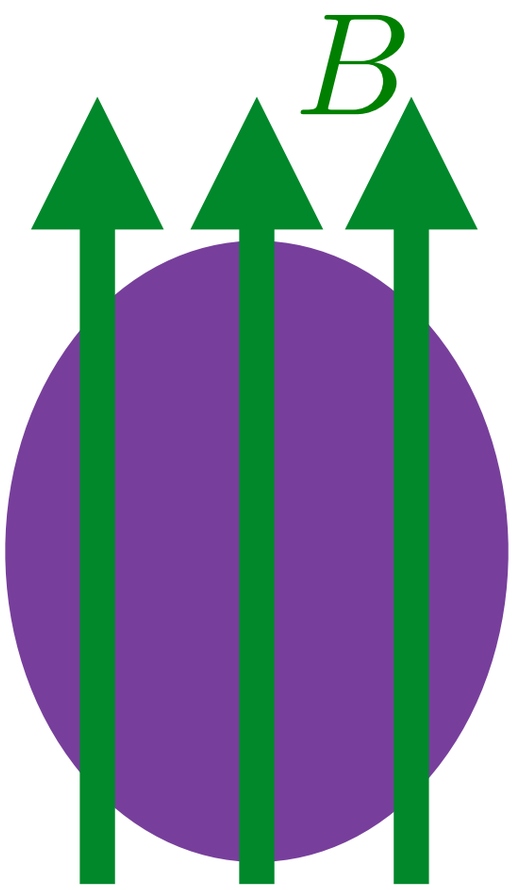
- equilibrium constitutive equations
  [Kovtun; JHEP (2016)]
- add time-dependent hydrodynamic terms
  [Kovtun, Hernandez; JHEP (2017)] $\Rightarrow$ **Kubo formulae**
- constrain through Onsager relations
  and
  entropy current

$$G^R_{\varphi_a\varphi_b}(\omega,\mathbf{k};\chi) = \eta_{\varphi_a}\eta_{\varphi_b}G^R_{\varphi_b^\dagger\varphi_a^\dagger}(\omega,-\mathbf{k};-\chi)$$

$$\nabla_\mu s^\mu \geq 0$$



Example relation for bulk viscosities:

$$3\zeta_2 - 6\eta_1 - 2\eta_2 = 0$$

* thermodynamic frame
* consistent current

Leading order: strong B thermodynamics

[Ammon, Kaminski et al.; JHEP (2017)]
[Ammon, Leiber, Macedo; JHEP (2016)]

Strong B thermodynamics with anomaly in **thermodynamic "frame":**

Energy momentum tensor:

$$\boxed{B \sim \mathcal{O}(1)}$$

equilibrium heat current

$$\langle T^{\mu\nu}_{\text{EFT}} \rangle = \begin{pmatrix} \epsilon_0 & 0 & 0 & \underline{\xi_V^{(0)} B} \\ 0 & P_0 - \underline{\chi_{BB} B^2} & 0 & 0 \\ 0 & 0 & P_0 - \underline{\chi_{BB} B^2} & 0 \\ \underline{\xi_V^{(0)} B} & 0 & 0 & P_0 \end{pmatrix} + \mathcal{O}(\partial)$$

"magnetic pressure shift"

Axial current:

$$\langle J^{\mu}_{\text{EFT}} \rangle = \left(n_0,\, 0,\, 0,\, \underline{\xi_B^{(0)} B} \right) + \mathcal{O}(\partial)$$

equilibrium charge current

➡ **new contributions to thermodynamic equilibrium observables**

based on previous work:

[Kovtun; JHEP (2016)]

[Jensen, Loganayagam, Yarom; JHEP (2014)]

[Israel; Gen.Rel.Grav. (1978)]

One point functions in thermodynamics

Zeroth order CME $\boxed{B \sim \mathcal{O}(1)}$

-thermodynamic chiral currents

[Ammon, Kaminski et al.; JHEP (2017)]
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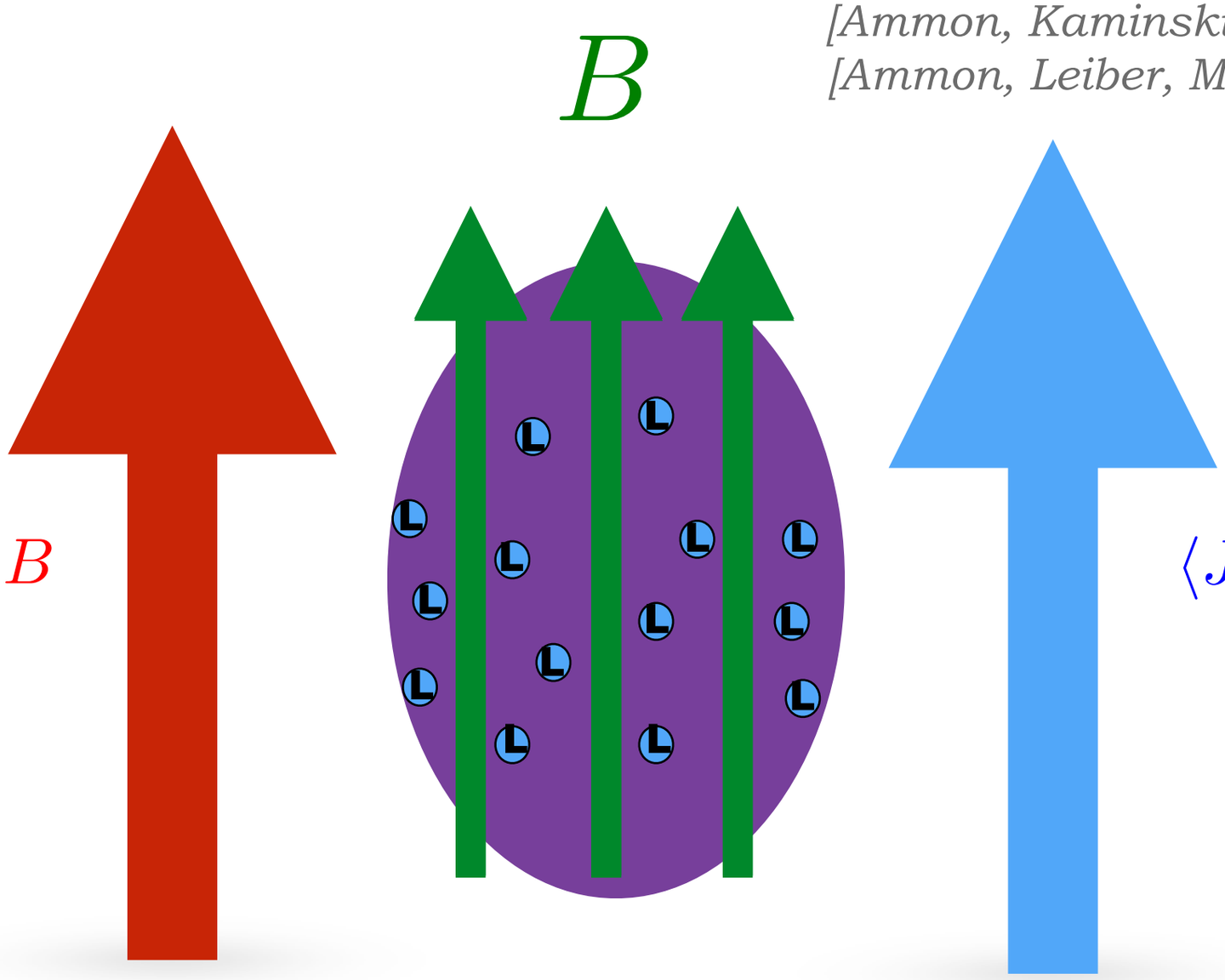

$\langle T^{0z} \rangle = \xi_V^{(0)} B$

heat current

$\langle J^z \rangle = \xi_B^{(0)} B$

axial current

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- constitutive equations

Reminder: Hydrodynamics

universal **effective field theory (EFT)**, expansion in gradients of temperature, chemical potential and velocity

- fields $T(x)$, $\mu(x)$, $u^\nu(x)$

  temperature *chemical potential* *fluid velocity*

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- constitutive equations

Conserved current $\langle j^\mu \rangle = n u^\mu + \nu^\mu$

[Landau, Lifshitz]

Example: hydrodynamic correlators in 2+1

Simple (non-chiral) example in 2+1 dims:

$$j^\mu = nu^\mu + \sigma\left[E^\mu - T\Delta^{\mu\nu}\partial_\nu\left(\frac{\mu}{T}\right)\right]$$

$$\Delta^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$$

$$u^\mu = (1,0,0)$$

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sources $\quad A_t,\, A_x \propto e^{-i\omega t + ikx} \qquad u^\mu = (1,0,0)$

fluctuations $\quad n = n(t,x,y) \propto e^{-i\omega t + ikx}$ (fix T and u)

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one point functions $\quad \nabla_\mu j^\mu = 0$

$$\langle j^t \rangle = n(\omega, k) = \frac{ik\sigma}{\omega + ik^2 \frac{\sigma}{\chi}} (\omega A_x + k A_t)$$

$$\langle j^x \rangle = \frac{i\omega\sigma}{\omega + ik^2 \frac{\sigma}{\chi}} (\omega A_x + k A_t)$$

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Einstein relation: $\quad D = \dfrac{\sigma}{\chi}$

$\Longrightarrow$ two point functions $\quad \langle j^x j^x \rangle = \dfrac{\delta \langle j^x \rangle}{\delta A_x} = \dfrac{i\omega^2 \sigma}{\omega + iDk^2}$

$\Longrightarrow$ hydrodynamic poles in spectral function

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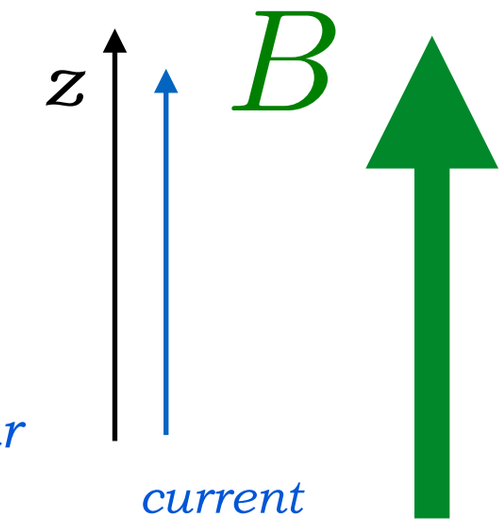
$\Longrightarrow$ hydrodynamic poles in spectral function

$\Longrightarrow$ Kubo formulae $\quad \sigma = \lim\limits_{\omega \to 0} \dfrac{1}{i\omega} \langle j^x j^x \rangle (\omega, k=0)$

Kubo formulae I

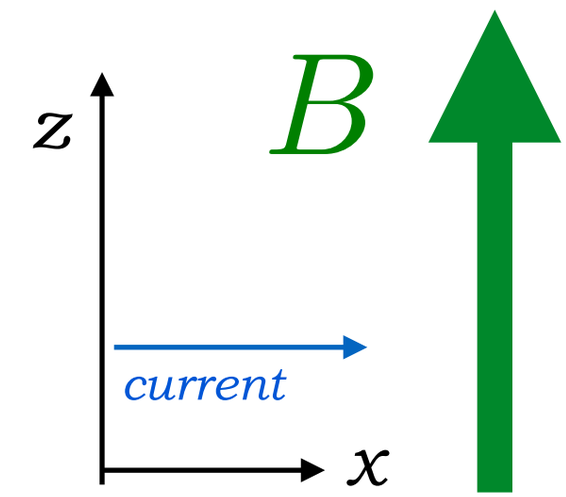
Parallel conductivity

$$\frac{1}{\omega}\mathrm{Im}\, G_{J^z J^z}(\omega, \mathbf{k}{=}0) = \sigma_{||} + \cdots$$



Perpendicular resistivity

$$\frac{1}{\omega}\mathrm{Im}\, G_{J^x J^x}(\omega, \mathbf{k}{=}0) = \omega^2 \rho_\perp \frac{w_0(w_0 - M_{5,\mu} B_0^2)}{B_0^4}$$



Magneto-vortical susceptibility

$$\frac{1}{k_z}\mathrm{Im}\, G_{T^{tx}T^{yz}}(\omega = 0, k_z\hat{k}) = -B_0\, M_5$$

$$W_S \sim M_5\, B \cdot \Omega$$

non-equilibrium parallel conductivity / perpendicular resistivity

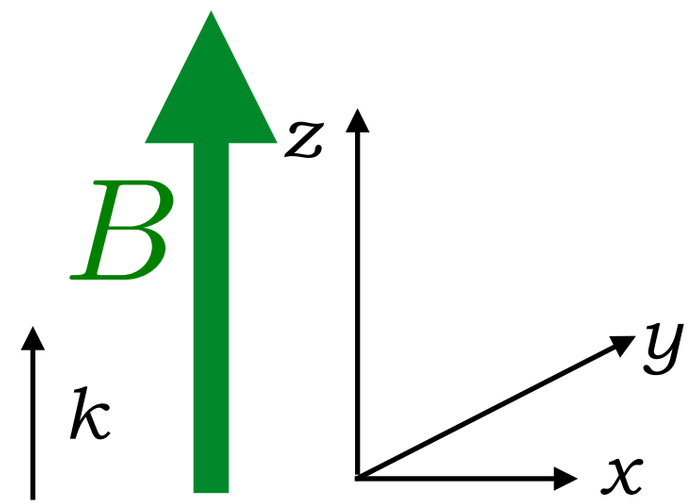
$$\langle J^z J^z \rangle(\omega, \mathbf{k} = 0) \sim \sigma_{||}$$

$$\langle J^x J^x \rangle(\omega, \mathbf{k} = 0) \sim \rho_\perp$$

Kubo formulae II

(Perpendicular) Hall resistivity

$$\frac{1}{\omega}\mathrm{Im}\, G_{J^x J^y}(\omega, \mathbf{k}{=}0) = \frac{n_0}{B_0} - \omega^2 \tilde{\rho}_\perp \frac{w_0(w_0 - M_{5,\mu} B_0^2)}{B_0^4}\, \mathrm{sign}(B_0)$$



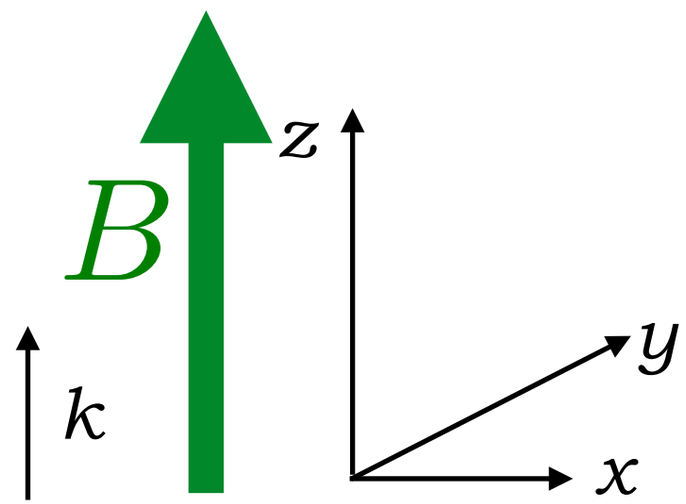
Chiral magnetic conductivity

$$\xi_B = \lim_{k\to 0} \frac{1}{-ik} \langle J^x J^y \rangle (\omega = 0, k) + \frac{1}{3} C\mu$$

Kubo formulae II

(Perpendicular) Hall resistivity

$$\frac{1}{\omega}\mathrm{Im}\, G_{J^x J^y}(\omega, \mathbf{k}{=}0) = \frac{n_0}{B_0} - \omega^2 \tilde{\rho}_\perp \frac{w_0(w_0 - M_{5,\mu} B_0^2)}{B_0^4}\, \mathrm{sign}(B_0)$$



Chiral magnetic conductivity

$$\xi_B = \lim_{k\to 0} \frac{1}{-ik} \langle J^x J^y \rangle(\omega = 0, k) + \frac{1}{3} C\mu$$

non-equilibrium parity-odd transport

$$\langle J^x J^y \rangle(\omega, \mathbf{k} = 0) \sim \frac{n}{B} - \omega^2 \frac{w^2}{B^4} \tilde{\rho}_\perp + \ldots$$

Hall type

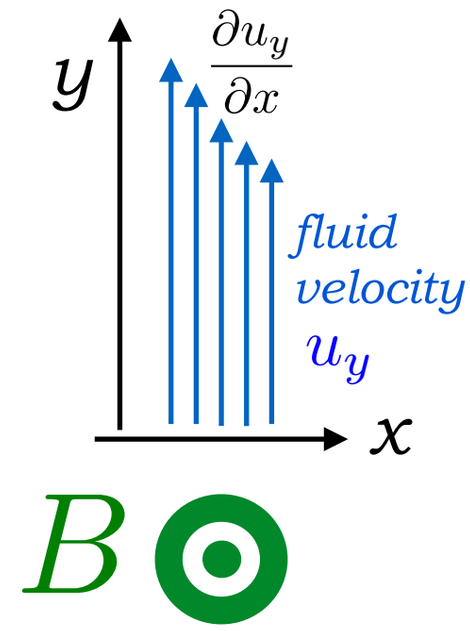
$$\langle J^x J^y \rangle(\omega = 0, \mathbf{k}) \sim -ik\, \underbrace{\xi_B}_{C\mu}$$

anomaly type

Kubo formulae III

Shear viscosity perpendicular

$$\frac{1}{\omega}\mathrm{Im}\, G_{T^{xy}T^{xy}}(\omega, \mathbf{k}{=}0) = \eta_\perp$$

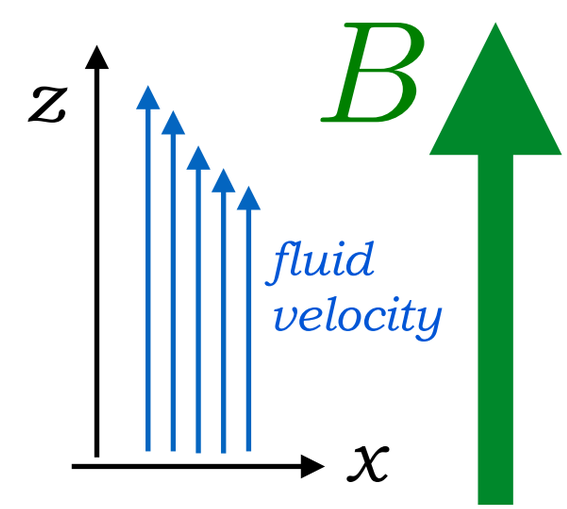


Shear viscosity parallel

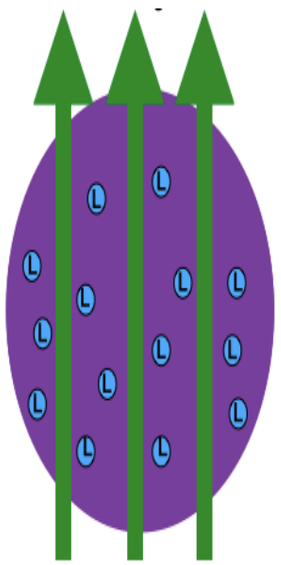
$$\frac{1}{\omega}\mathrm{Im}\, G_{T^{xz}T^{xz}}(\omega, \mathbf{k}{=}0) = \eta_\parallel + (\bar{c}_8 c_{15} - c_{10}\bar{c}_{17})\rho_\perp - (\bar{c}_8\bar{c}_{17} + c_{10}c_{15})\tilde{\rho}_\perp$$

perpendicular resistivity *Hall resistivity*

Holographic model values must satisfy:
- ➡ **constraints**
- ➡ **consistency checks**



3. Holographic setup



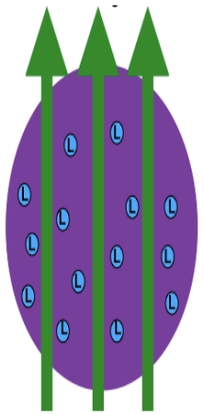
Action and background

Einstein-Maxwell-Chern-Simons action

chiral anomaly

$$S_{grav} = \frac{1}{2\kappa^2}\left[\int_{\mathcal{M}} d^5x\,\sqrt{-g}\left(R + \frac{12}{L^2} - \frac{1}{4}F_{mn}F^{mn}\right) - \frac{\gamma}{6}\int_{\mathcal{M}} A\wedge F\wedge F\right]$$

$$S_{bdy} = \frac{1}{\kappa^2}\int_{\partial\mathcal{M}} d^4x\,\sqrt{-\hat{g}}\left(K - \frac{3}{L} + \frac{L}{4}R(\hat{g}) + \frac{L}{8}\ln\left(\frac{\varrho}{L}\right)F_{\mu\nu}F^{\mu\nu}\right)$$



Magnetic black branes *[D'Hoker, Kraus; JHEP (2009)]*

- charged magnetic analog of RN black brane
- Asymptotically AdS5
- zero entropy density at vanishing temperature

$$\begin{aligned} ds^2 &= \frac{1}{\varrho^2}\left[\left(-u(\varrho) + c(\varrho)^2\,w(\varrho)^2\right)\,dt^2 - 2\,dt\,d\varrho + 2\,c(\varrho)\,w(\varrho)^2\,dz\,dt\right. \\ &\quad \left. + v(\varrho)^2\,\left(dx^2 + dy^2\right) + w(\varrho)^2\,dz^2\right]\,, \\ F &= \underbrace{A_t'(\varrho)}_{\text{charge}}\,d\varrho\wedge dt + \underbrace{B}_{\text{magnetic field}}\,dx\wedge dy + P'(\varrho)\,d\varrho\wedge dz\,, \end{aligned}$$

Correlators from infalling fluctuations

Problem: fluctuation equations are coupled (dual to operator mixing in QFT)

Numerical methods

- matrix method and shooting technique

[Kaminski, Landsteiner, Mas, Shock, Tarrio; JHEP (2010)]

$$G^{(ret)}(\mathbf{k}) = -2 \lim_{\epsilon \to 0} \mathcal{F}(\mathbf{k}, \epsilon)$$

$\Longrightarrow$ frequency and momentum

find independent solutions to coupled systems (pure gauge solutions)

- one-point functions technique and spectral methods

[Ammon, Grieninger, Hernandez, Kaminski, Koirala, Leiber, Wu; to appear]

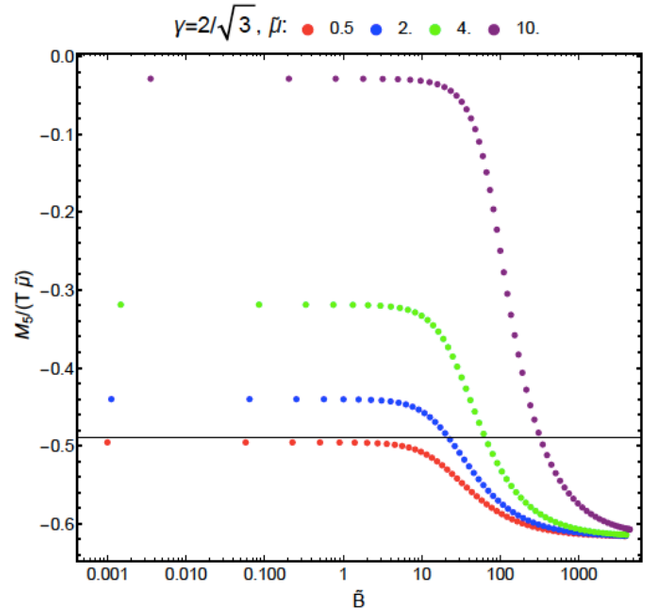
$$\langle \mathcal{O}_A \, \mathcal{O}_B \rangle \sim \frac{\delta \langle \mathcal{O}_B \rangle}{\delta \phi_A}$$

$\Longrightarrow$ analytic relations

find independent solutions to coupled systems (no pure gauge solutions)

preliminary

4. Results



Holographic result: equilibrium

[Ammon, Kaminski et al.; JHEP (2017)]

Background solution: charged magnetic black branes

[D'Hoker, Kraus; JHEP (2009)]
[Ammon, Leiber, Macedo; JHEP (2016)]

- **external magnetic field**
- **charged plasma**
- anisotropic plasma

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- **external magnetic field**
- **charged plasma**
- anisotropic plasma

Thermodynamics

$$\langle T^{\mu\nu}\rangle = \begin{pmatrix} -3\,u_4 & 0 & 0 & -4\,c_4 \\ 0 & -\frac{B^2}{4} - u_4 - 4\,w_4 & 0 & 0 \\ 0 & 0 & -\frac{B^2}{4} - u_4 - 4\,w_4 & 0 \\ -4\,c_4 & 0 & 0 & 8\,w_4 - u_4 \end{pmatrix}$$

$$\langle J^{\mu}\rangle = (\rho,\, 0,\, 0,\, p_1)\ .$$

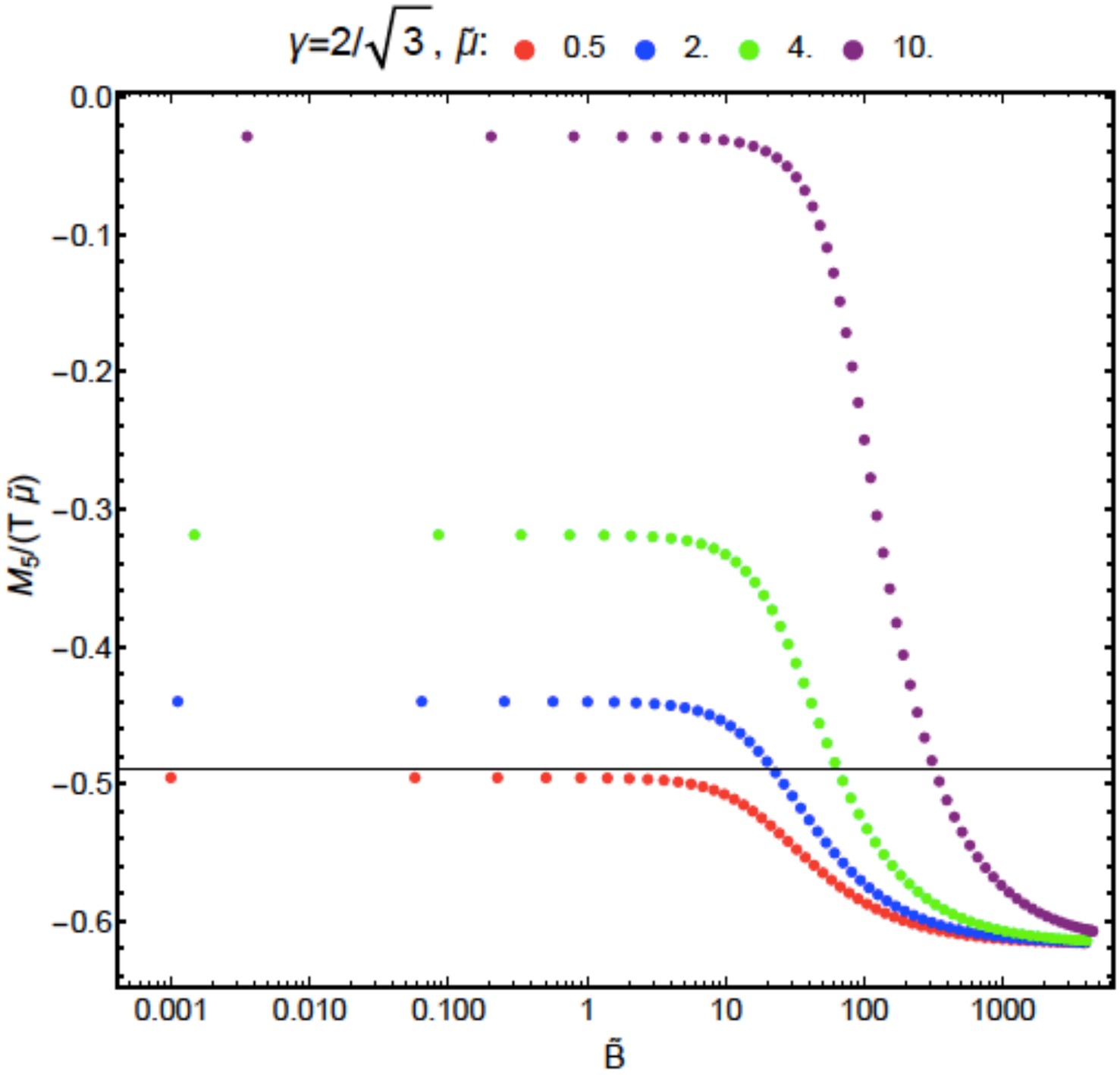
with near boundary expansion coefficients $u_4,\ w_4,\ c_4,\ p_1$

$$\langle T^{\mu\nu}_{\rm EFT}\rangle = \begin{pmatrix} \epsilon_0 & 0 & 0 & \xi_V^{(0)} B \\ 0 & P_0 - \chi_{BB} B^2 & 0 & 0 \\ 0 & 0 & P_0 - \chi_{BB} B^2 & 0 \\ \xi_V^{(0)} B & 0 & 0 & P_0 \end{pmatrix} + \mathcal{O}(\partial)$$

$$\langle J^{\mu}_{\rm EFT}\rangle = \left(n_0,\, 0,\, 0,\, \xi_B^{(0)} B\right) + \mathcal{O}(\partial)$$

➡ **agrees in form with strong *B* thermodynamics from EFT**

Thermodynamic transport



Magneto-vortical susceptibility

$$\frac{1}{k_z}\mathrm{Im}\, G_{T^{tx}T^{yz}}(\omega=0, k_z\hat{k}) = -B_0\, M_5$$

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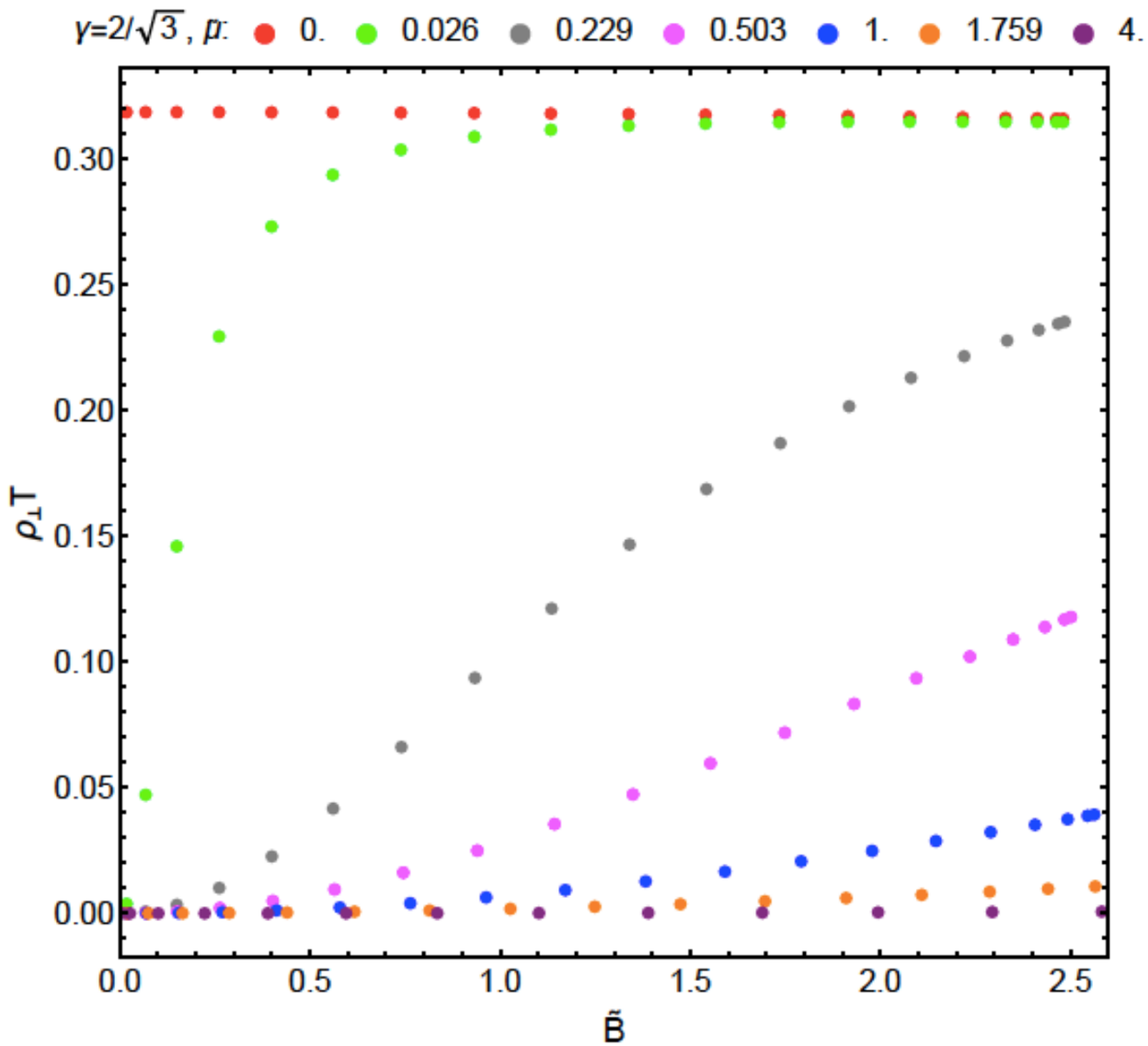
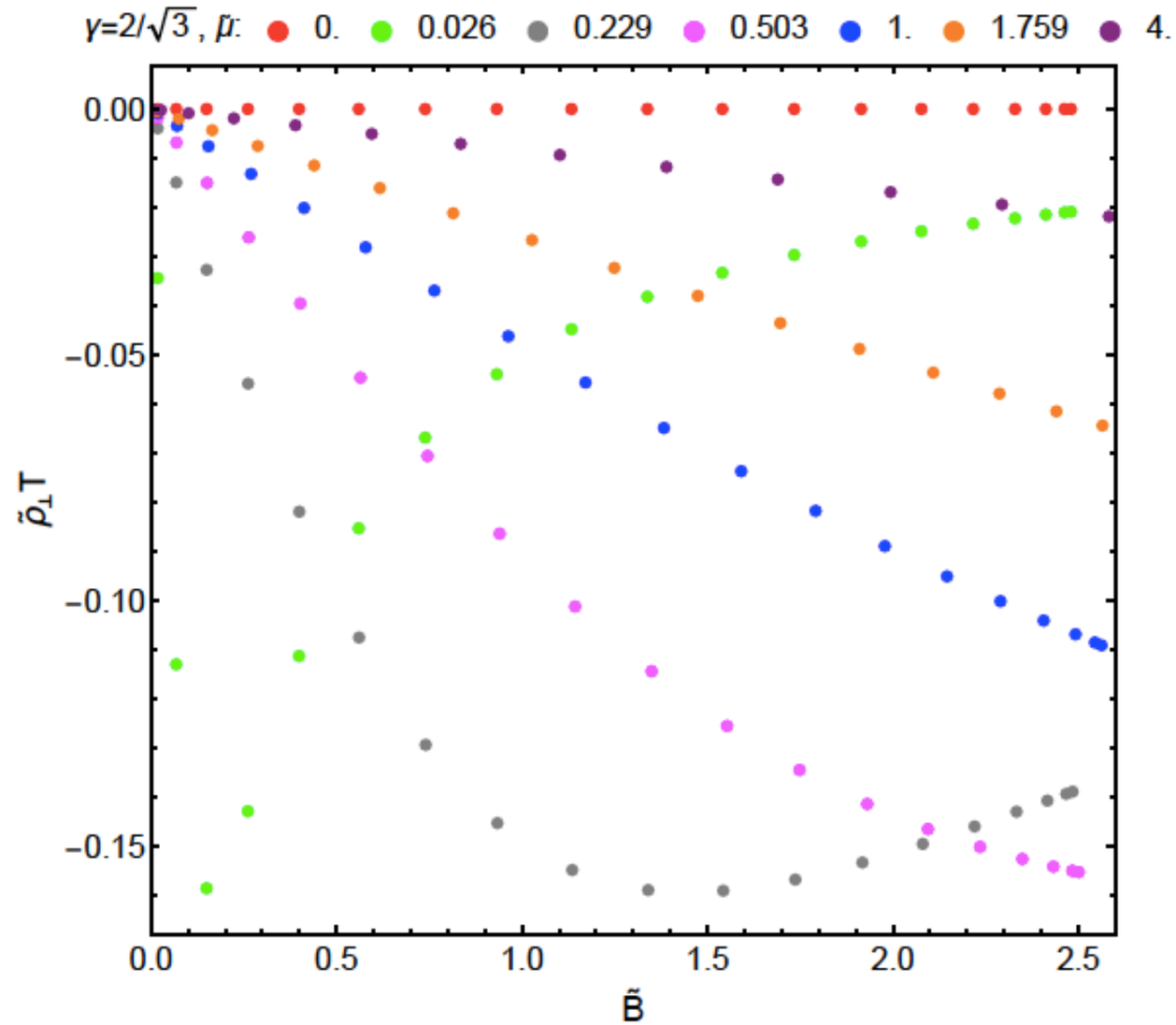
Hydrodynamic transport





$$\frac{1}{\omega}\mathrm{Im}\, G_{J^x J^x}(\omega, \mathbf{k}{=}0) = \omega^2 \rho_\perp \frac{w_0(w_0 - M_{5,\mu} B_0^2)}{B_0^4}$$

$$\frac{1}{\omega}\mathrm{Im}\, G_{J^x J^y}(\omega, \mathbf{k}{=}0) = \frac{n_0}{B_0} - \omega^2 \tilde{\rho}_\perp \frac{w_0(w_0 - M_{5,\mu} B_0^2)}{B_0^4}\, \mathrm{sign}(B_0)$$

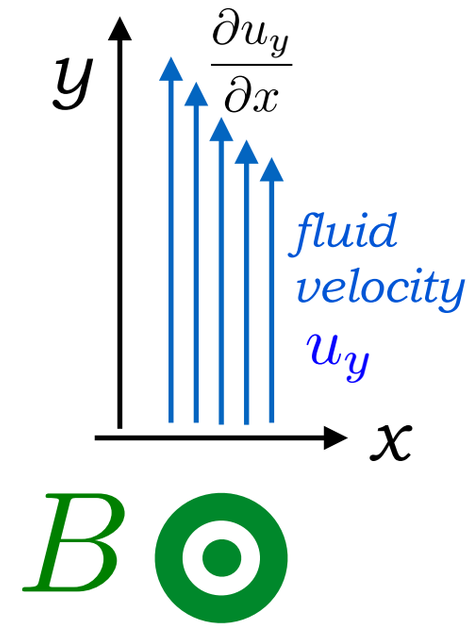
More transport coefficients

$\eta_\perp$	perpendicular shear viscosity
$\eta_{\|\|}$	parallel shear viscosity
$\bar{\eta}_\perp$	perpendicular Hall viscosity
$\bar{\eta}_{\|\|}$	parallel Hall viscosity
ζ_1	bulk viscosity
ζ_2	bulk viscosity
η_1	bulk viscosity
η_2	bulk viscosity
$\sigma_\perp$	perpendicular conductivity
$\sigma_{\|\|}$	parallel conductivity
$\bar{\sigma}$	Hall conductivity

...

Analytic result from one-point function technique

Kubo formula: perpendicular shear viscosity



$$\frac{1}{\omega}\mathrm{Im}\, G_{T^{xy}T^{xy}}(\omega, \mathbf{k}{=}0) = \eta_\perp$$

Analytic result:

$$\eta_\perp = \lim_{\omega\to 0}\frac{1}{\omega}\mathrm{Im}\, G_{T^{xy}T^{xy}}(\omega, \mathbf{k}{=}0) = v(1)^2\, w(1)$$

$$s = 4\pi\, v(1)^2\, w(1)$$

$$\frac{\eta_\perp}{s} = \frac{1}{4\pi}$$

Discussion - Summary

- derived hydrodynamic transport coefficients & Kubo relations for QFT with chiral anomaly, in a charged thermal plasma state, within strong external B
- proof of existence within holographic model (EMCS)
- transport coefficients are nonzero and show non-trivial dependence on B, anomaly coefficient C, and chemical potential
  [Ammon, Grieninger, Hernandez, Kaminski, Koirala, Leiber, Wu; to appear]
- novel transport effects arise (e.g. perpendicular/parallel, unidentified)
- order zero CME (and CVE) *[Ammon, Kaminski et al.; JHEP (2017)]*
  [Ammon, Leiber, Macedo; JHEP (2016)]
- more motivation for strong B model: universal magneto response
  [Endrödi, Kaminski, Schäfer, Wu, Yaffe; arXiv:1806.09632]

Discussion - Outlook

➡ correlations far from equilibrium at high density and magnetic field with chiral anomaly

*[**Cartwright**, Kaminski; JHEP (2019)]*

*[see my **talk at HoloQuark2018***

➡ non-relativistic hydrodynamics & QNMs

*[**Garbiso**, Kaminski; JHEP (2019)]*

[Davison, Grozdanov, Janiszewski, Kaminski; JHEP (2016)]

➡ rotating black holes/branes & QNMs

*[**Garbiso et al** to appear]*

➡ dynamical electromagnetic fields - magnetohydrodynamics *[Kovtun, Hernandez; JHEP (2017)]*

*[**Cartwright, Knipfer, ...** work in progress]*

➡ comparison to experimental data

([Endrödi, Kaminski, Schäfer, Wu, Yaffe; JHEP (2018)])

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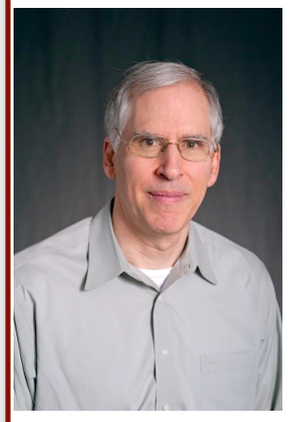
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APPENDIX

Charge, parity, time reversal

quantity	$\mathcal{C}$	$\mathcal{P}$	$\mathcal{T}$
t	+	+	-
x^i	+	-	+
r	+	+	+
T, h_{tt}, T^{tt}	+	+	+
μ_A, A_t, J^t	+	-	+
A_i, J^i	+	+	-
A_r	+	-	-
u^i, h_{ti}, T^{ti}	+	-	-
h_{ij}, T^{ij}	+	+	+
B^i	+	-	-
E^i	+	+	+
$dx^\mu \wedge dx^\nu \wedge dx^\rho \wedge dx^\sigma \wedge dx^\kappa$	+	-	-
$\int_i^f A \wedge F \wedge F$	+	+	+

Constitutive equations

Generic decomposition:

$$T^{\mu\nu} = \mathcal{E} u^\mu u^\nu + \mathcal{P}\Delta^{\mu\nu} + \mathcal{Q}^\mu u^\nu + \mathcal{Q}^\nu u^\mu + \mathcal{T}^{\mu\nu}$$
$$J^\mu = \mathcal{N} u^\mu + \mathcal{J}^\mu$$

$$X = X_{\text{eq.}} + X_{\text{non-eq.}} + X_{\text{anomalous}}$$

Examples:

$$\begin{aligned}
\mathcal{E}_{\text{eq.}} = &-p + T\,p_{,T} + \mu\,p_{,\mu} + \left(TM_{5,T} + \mu M_{5,\mu} - 2M_5\right) B{\cdot}\Omega \\
&+ \left(TM_{1,T} + \mu M_{1,\mu} + 4B^2 M_{1,B^2} + T^4 M_{3,B^2} - M_1\right) s_1 \\
&+ \left(TM_{2,T} + \mu M_{2,\mu} - M_2\right) s_2 \\
&+ \frac{4B^2}{T^4}\left(M_1 - TM_{1,T} - \mu M_{1,\mu} - 4B^2 M_{1,B^2} - T^4 M_{3,B^2}\right) s_3 \\
&+ \left(TM_{4,T} + \mu M_{4,\mu} + \frac{4B^2}{T^4} M_{1,\mu} + M_{3,\mu}\right) s_4\,,
\end{aligned}$$

$$\begin{aligned}
\mathcal{N}_{\text{eq.}} = &\; p_{,\mu} + \nabla{\cdot}p - p{\cdot}a - m{\cdot}\Omega + \left(M_{1,\mu} - T^4 M_{4,B^2}\right) s_1 + M_{2,\mu} s_2 \\
&+ \left(M_{3,\mu} + TM_{4,T} + \mu M_{4,\mu} + 4B^2 M_{4,B^2}\right) s_3 + M_{5,\mu} s_5\,,
\end{aligned}$$

Anomalous parts:

$$\Delta T^{\mu\nu} = u^\mu(\xi_T\,\Omega^\nu + \xi_{TB}\,B^\nu) + u^\nu(\xi_T\,\Omega^\mu + \xi_{TB}\,B^\mu)\,,$$
$$\Delta J^\mu_{cons} = \tfrac{1}{3}CB{\cdot}Au^\mu + \xi\,\Omega^\mu + \left(\xi_B - \tfrac{1}{3}C\mu\right) B^\mu + \tfrac{1}{3}C\epsilon^{\mu\nu\rho\sigma} A_\nu u_\rho E_\sigma\,,$$

$$\xi = \tfrac{1}{2}C\mu^2 + c_1 T^2 + 2c_2 T\mu\,, \quad \xi_B = C\mu + 2c_2 T\,,$$
$$\xi_T = \tfrac{1}{3}C\mu^3 + 2c_1 T^2\mu + 2c_2 T\mu^2\,, \quad \xi_{TB} = \tfrac{1}{2}C\mu^2 + c_1 T^2 + 2c_2 T\mu\,.$$

EFT calculation: chiral hydrodynamics with magnetic field

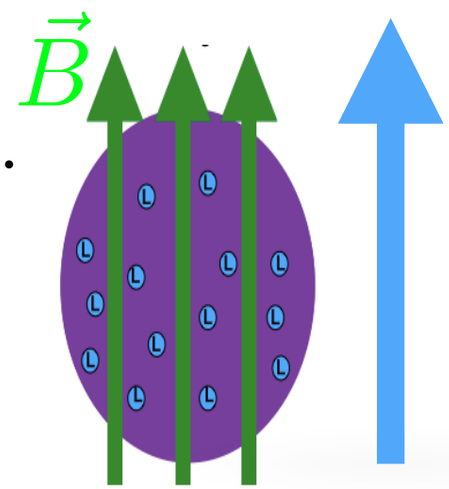
For any theory with chiral anomaly

$$\partial_\mu J_A{}^\mu = C\,\epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}$$

[Son,Surowka; PRL (2009)]
[Erdmenger, Haack, Kaminski, Yarom; JHEP (2008)]
[Banerjee et al.; JHEP (2011)]

Axial current with **weak** external B field:

$$\langle J_A{}^\mu \rangle = n u^\mu + \sigma E^\mu - \sigma T \Delta^{\mu\nu} \nabla_\nu \left(\frac{\mu}{T}\right) + \underline{\xi_B B^\mu} + \xi_V \Omega^\mu + \ldots$$



axial current

Energy momentum tensor with weak external B field:

$$\langle T^{\mu\nu} \rangle = \epsilon u^\mu u^\nu + P \Delta^{\mu\nu} + u^\mu q^\nu + u^\nu q^\mu + \tau^{\mu\nu}$$

EFT calculation: chiral hydrodynamics with magnetic field

For any theory with chiral anomaly

$$\partial_\mu J_A{}^\mu = C\,\epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}$$

[Son,Surowka; PRL (2009)]
[Erdmenger, Haack, Kaminski, Yarom; JHEP (2008)]
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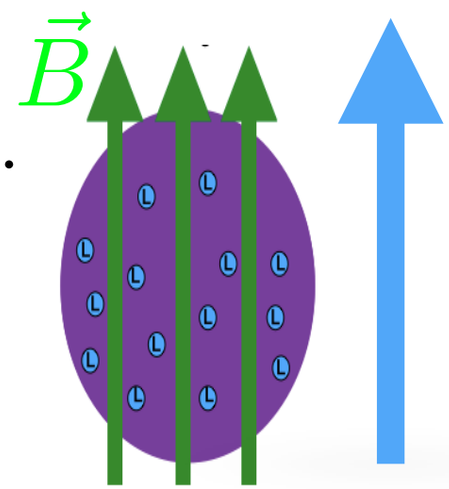
(ideal) charge flow — *conductivity term* — *charge diffusion* — *chiral magnetic conductivity term* — *chiral vortical conductivity term*



axial current

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$$\langle T^{\mu\nu} \rangle = \underbrace{\epsilon u^\mu u^\nu + P \Delta^{\mu\nu}}_{\text{ideal fluid}} + u^\mu q^\nu + u^\nu q^\mu + \tau^{\mu\nu}$$

heat current

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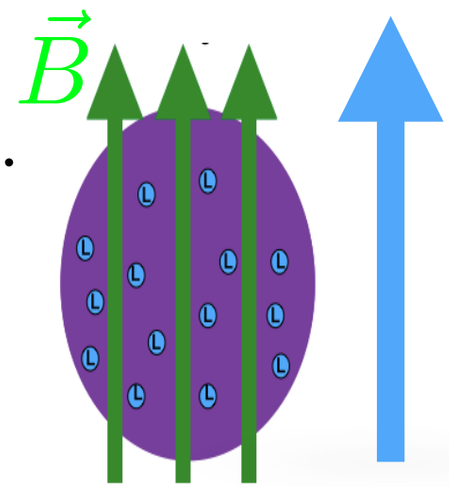

axial current

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$$\langle T^{\mu\nu} \rangle = \underbrace{\epsilon u^\mu u^\nu + P \Delta^{\mu\nu}}_{\text{ideal fluid}} + \underbrace{u^\mu q^\nu + u^\nu q^\mu}_{\text{heat current}} + \tau^{\mu\nu}$$

measured in Weyl semi metals
e.g. [Huang et al; PRX (2015)]

neutron stars?
[Kaminski et al.; PLB (2014)]

Now calculate hydrodynamic 1- and 2-point functions and determine their poles!
[Landau, Lifshitz]
[Kadanoff; Martin]

Dispersion relations: weak B hydrodynamics

[Ammon, Kaminski et al.; JHEP (2017)]
[Abbasi et al.; PLB (2016)]
[Kalaydzhyan, Murchikova; NPB (2016)]

Weak B hydrodynamics - poles of 2-point functions $\langle T^{\mu\nu}\, T^{\alpha\beta}\rangle$, $\langle T^{\mu\nu}\, J^{\alpha}\rangle$, $\langle J^{\mu}\, T^{\alpha\beta}\rangle$, $\langle J^{\mu}\, J^{\alpha}\rangle$:

spin 1 modes under SO(2) rotations around *B*

$$\omega = \qquad\qquad -ik^2 \frac{\eta}{\epsilon_0 + P_0} +$$

former momentum diffusion modes

$$\mathfrak{s}_0 = s_0/n_0$$
$$\tilde{c}_P = T_0(\partial\mathfrak{s}/\partial T)_P$$

Dispersion relations: weak B hydrodynamics

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$$\omega = \mp \underline{\frac{B n_0}{\epsilon_0 + P_0}} - i k^2 \frac{\eta}{\epsilon_0 + P_0} + \underline{k \frac{B n_0 \xi_3}{(\epsilon_0 + P_0)^2} - \frac{i B^2 \sigma}{\epsilon_0 + P_0}}$$

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former momentum diffusion modes

$\mathfrak{s}_0 = s_0/n_0$

$\tilde{c}_P = T_0 (\partial \mathfrak{s}/\partial T)_P$

spin 0 modes under SO(2) rotations around *B*

$$\omega_0 = \underline{v_0\, k} - i D_0\, k^2 + \mathcal{O}(\partial^3)$$

former charge diffusion mode

$$\omega_+ = \underline{v_+\, k - i \Gamma_+\, k^2} + \mathcal{O}(\partial^3)$$

$$\omega_- = \underline{v_-\, k - i \Gamma_-\, k^2} + \mathcal{O}(\partial^3)$$

former sound modes

Dispersion relations: weak B hydrodynamics

[Ammon, Kaminski et al.; JHEP (2017)]
[Abbasi et al.; PLB (2016)]
[Kalaydzhyan, Murchikova; NPB (2016)]

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spin 1 modes under SO(2) rotations around *B*

$$\omega = \mp \frac{B n_0}{\epsilon_0 + P_0} - i k^2 \frac{\eta}{\epsilon_0 + P_0} + k \frac{B n_0 \xi_3}{(\epsilon_0 + P_0)^2} - \frac{i B^2 \sigma}{\epsilon_0 + P_0}$$

former momentum diffusion modes

$$\mathfrak{s}_0 = s_0/n_0$$
$$\tilde{c}_P = T_0 (\partial \mathfrak{s}/\partial T)_P$$

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$$\omega_- = v_-\, k - i \Gamma_-\, k^2 + \mathcal{O}(\partial^3)$$

former sound modes

➡ **a chiral magnetic wave**

[Kharzeev, Yee; PRD (2011)]

$$v_0 = \frac{2 B T_0}{\tilde{c}_P n_0} \left(\tilde{C} - 3 C \mathfrak{s}_0^2 \right)$$

$$D_0 = \frac{w_0^2\, \sigma}{\tilde{c}_P n_0^3 T_0}$$

➡ **dispersion relations of hydrodynamic modes are heavily modified by anomaly and *B***

EFT result III: weak B details

Weak B hydrodynamics - poles of 2-point functions:

[Ammon, Kaminski et al.; JHEP (2017)]
[Abbasi et al.; PLB (2016)]
[Kalaydzhyan, Murchikova; NPB (2016)]

spin 0 modes under SO(2) rotations around *B*

$$\omega_0 = v_0\, k - i D_0\, k^2 + \mathcal{O}(\partial^3)$$ *former charge diffusion mode*

$$\omega_+ = v_+\, k - i\Gamma_+\, k^2 + \mathcal{O}(\partial^3)$$

$$\omega_- = v_-\, k - i\Gamma_-\, k^2 + \mathcal{O}(\partial^3)$$

former sound modes

$$w_0 = \epsilon_0 + P_0$$
$$\mathfrak{s}_0 = s_0/n_0$$
$$\tilde{c}_P = T_0(\partial\mathfrak{s}/\partial T)_P$$
$$c_s^2 = (\partial P/\partial\epsilon)_{\mathfrak{s}}$$

damping coefficients:

$$\Gamma_\pm = \frac{3\zeta + 4\eta}{6w_0} + c_s^2\,\frac{w_0\,\sigma}{2n_0^2}\left(1 - \frac{\alpha_P w_0}{\tilde{c}_P n_0}\right)^2 \qquad D_0 = \frac{w_0^2\,\sigma}{\tilde{c}_P n_0^3 T_0}$$

velocities:

$$v_\pm = \pm c_s - B\,\frac{c_s^2}{n_0}\left(1 - \frac{\alpha_P w_0}{\tilde{c}_P n_0}\right)\left[3CT_0\mathfrak{s}_0 + \frac{\alpha_P T_0^2}{\tilde{c}_P}(\tilde{C} - 3C\mathfrak{s}_0^2) + \frac{1}{2}\xi_B^{(0)} - \frac{n_0}{w_0}\xi_V^{(0)}\right] + B\,\frac{1 - c_s^2}{w_0}\,\xi_V^{(0)}\,,$$

$$v_0 = \frac{2B\,T_0}{\tilde{c}_P n_0}\left(\tilde{C} - 3C\mathfrak{s}_0^2\right)$$

chiral conductivities:

$$\xi_V = -3C\mu^2 + \tilde{C}T^2, \quad \xi_B = -6C\mu, \quad \xi_3 = -2C\mu^3 + 2\tilde{C}\mu T^2$$

known from entropy current argument
[Son,Surowka; PRL (2009)]
[Neiman,Oz; JHEP (2010)]

Caveat: weak B hydrodynamics comparison

Spin-1 modes

No knowledge of anisotropic (B-dependent) transport coefficients *except zero charge: [Finazzo, Critelli, Rougemont, Noronha; PRD (2016)]*
— take B=0 values of this model instead

weak B hydro prediction:

$$\omega = \mp \frac{B n_0}{\epsilon_0 + P_0} - i k^2 \frac{\boxed{\eta}}{\epsilon_0 + P_0} + k \frac{B n_0 \xi_3}{(\epsilon_0 + P_0)^2} - \frac{i B^2 \boxed{\sigma}}{\epsilon_0 + P_0}$$

calculate from holography

We find agreement between hydrodynamic prediction and holographic model for small values of B, increasing deviations for larger B.

Real part of spin-1 modes matches exactly even at large B!

Recent update: strong B hydrodynamics

[Hernandez, Kovtun; JHEP (2017)]

Spin-1 modes

strong B: $$\omega = \pm\frac{B_0 n_0}{w_0} - \frac{iB_0^2}{w_0}\left(\sigma_\perp \pm i\tilde{\sigma}\right) - iD_c k^2$$

weak B: $$\omega = \mp\frac{Bn_0}{\epsilon_0 + P_0} - ik^2\frac{\eta}{\epsilon_0 + P_0} + k\frac{Bn_0\xi_3}{(\epsilon_0 + P_0)^2} - \frac{iB^2\sigma}{\epsilon_0 + P_0}$$

parity-odd

Agreement in form

Exact agreement in real part!

Recent update: strong B hydrodynamics

[Hernandez, Kovtun; JHEP (2017)]

Spin-1 modes

Anisotropic transport coefficients

strong B: $$\omega = \pm\frac{B_0 n_0}{w_0} - \frac{iB_0^2}{w_0}\boxed{(\sigma_\perp \pm i\tilde{\sigma})} - i\boxed{D_c}k^2$$

parity-odd

weak B: $$\omega = \mp\frac{Bn_0}{\epsilon_0 + P_0} - ik^2\frac{\eta}{\epsilon_0 + P_0} + k\frac{Bn_0\xi_3}{(\epsilon_0 + P_0)^2} - \frac{iB^2\sigma}{\epsilon_0 + P_0}$$

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Agreement in form

Exact agreement in real part!

Spin-0 modes

strong B: $$\omega = \pm k v_s - i\frac{\Gamma_{s,\parallel}}{2}k^2\,,$$

$$\omega = -iD_\parallel k^2\,,$$

$$D_\parallel = \frac{\sigma_\parallel\, w_0^2}{n_0^2\chi_{11} + w_0^2\chi_{33} - 2n_0 w_0\chi_{13}}$$

parity-odd

weak B: $$\omega_0 = v_0\, k - iD_0\, k^2 + \mathcal{O}(\partial^3)$$

$$\omega_+ = v_+\, k - i\Gamma_+\, k^2 + \mathcal{O}(\partial^3)$$

$$\omega_- = v_-\, k - i\Gamma_-\, k^2 + \mathcal{O}(\partial^3)$$

$$D_0 = \frac{w_0^2\,\sigma}{\tilde{c}_P n_0^3 T_0} \qquad v_0 = \frac{2B\,T_0}{\tilde{c}_P n_0}\left(\tilde{C} - 3C\mathfrak{s}_0^2\right)$$

Agreement in form

$$\tilde{c}_P = T_0(\partial\mathfrak{s}/\partial T)_P$$

Recent update: strong B hydrodynamics

[Hernandez, Kovtun; JHEP (2017)]

Spin-1 modes

Anisotropic transport coefficients

strong B: $$\omega = \pm\frac{B_0 n_0}{w_0} - \frac{iB_0^2}{w_0}\boxed{(\sigma_\perp \pm i\tilde{\sigma})} - i\boxed{D_c}k^2$$

parity-odd

weak B: $$\omega = \mp\frac{Bn_0}{\epsilon_0 + P_0} - ik^2\frac{\eta}{\epsilon_0 + P_0} + k\frac{Bn_0\xi_3}{(\epsilon_0 + P_0)^2} - \frac{iB^2\sigma}{\epsilon_0 + P_0}$$

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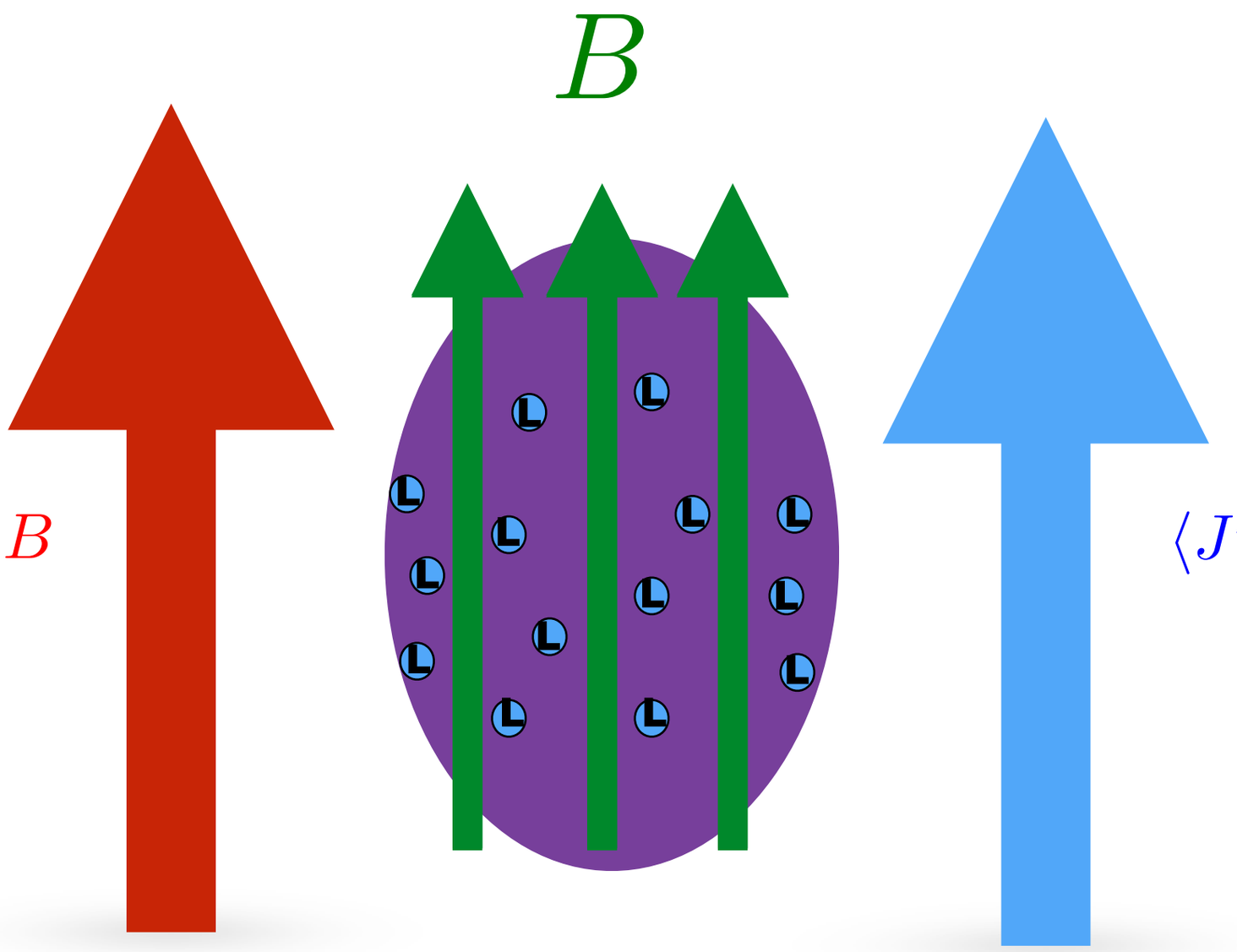
Agreement in form

$$\tilde{c}_P = T_0(\partial\mathfrak{s}/\partial T)_P$$

Zeroth order CME $\boxed{B \sim \mathcal{O}(1)}$
-thermodynamic chiral currents

[Ammon, Kaminski et al.; JHEP (2017)]
[Ammon, Leiber, Macedo; JHEP (2016)]



$\langle T^{0z} \rangle = \xi_V^{(0)} B$

heat current

$\langle J^z \rangle = \xi_B^{(0)} B$

axial current

Previous work: polarized matter at strong B

Generating functionals $W \sim P$ (pressure) for thermodynamics $\boxed{B \sim \mathcal{O}(1)}$

(i) No anomaly: *[Kovtun; JHEP (2016)]*

$$T^{\mu\nu} = Pg^{\mu\nu} + (Ts + \mu\rho)u^\mu u^\nu + T^{\mu\nu}_{\rm EM}$$

$$J^\alpha = \rho u^\alpha - \nabla_\lambda M^{\lambda\alpha}$$

bound current

$$T^{\mu\nu}_{\rm EM} = M^{\mu\alpha} g_{\alpha\beta} F^{\beta\nu} + u^\mu u^\alpha \left(M_{\alpha\beta} F^{\beta\nu} - F_{\alpha\beta} M^{\beta\nu} \right)$$

[Israel; Gen.Rel.Grav. (1978)]

Polarization tensor:

$$M_{\mu\nu} = p_\mu u_\nu - p_\nu u_\mu - \epsilon_{\mu\nu\rho\sigma} u^\rho m^\sigma \qquad M^{\mu\nu} = 2\frac{\partial P}{\partial F_{\mu\nu}}$$

Including vorticity:

$W \sim M_\omega B \cdot \omega$

[Kovtun, Hernandez; JHEP (2017)]

Previous work: polarized matter at strong B

Generating functionals $W \sim P$ (pressure) for thermodynamics $\boxed{B \sim \mathcal{O}(1)}$

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[Kovtun, Hernandez; JHEP (2017)]

(ii) With anomaly: *[Jensen, Loganayagam, Yarom; JHEP (2014)]*

➡ opportunity: single framework allows for polarization, magnetization, external vorticity, *E*, *B*, *and* chiral anomaly

➡ opportunity: dynamical *E* and *B;* magnetohydrodynamics

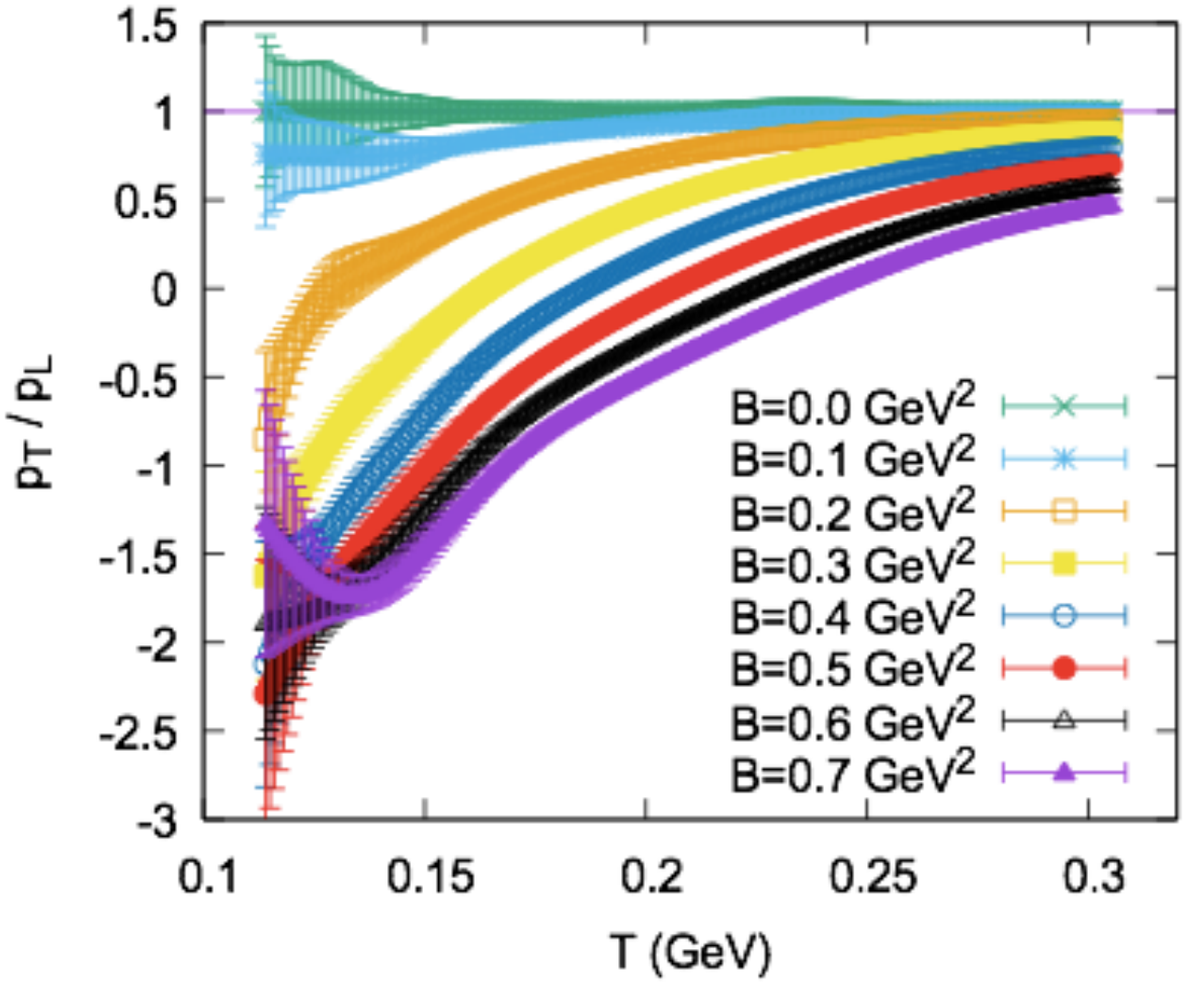
[Kovtun, Hernandez; JHEP (2017)]

➡ opportunity: study equilibrium and near-equilibrium transport

[Ammon, Grieninger, Kaminski, Koirala, Leiber, Wu; to appear]

Universal magnetoresponse in QCD ...

[Endrödi, Kaminski, Schäfer, Wu, Yaffe; arXiv:1806.09632]



Lattice QCD with 2+1 flavors, dynamical quarks, physical masses

transverse pressure: $$p_{\mathrm{T}} = -\frac{L_{\mathrm{T}}}{V}\frac{\partial F_{\mathrm{QCD}}}{\partial L_{\mathrm{T}}}$$

longitudinal pressure: $$p_{\mathrm{L}} = -\frac{L_{\mathrm{L}}}{V}\frac{\partial F_{\mathrm{QCD}}}{\partial L_{\mathrm{L}}}$$

F_{QCD} ... *free energy*

L_{T} ... *transverse system size*

L_{L} ... *longitudinal system size*

V ... *system volume*

Universal magnetoresponse in QCD ...

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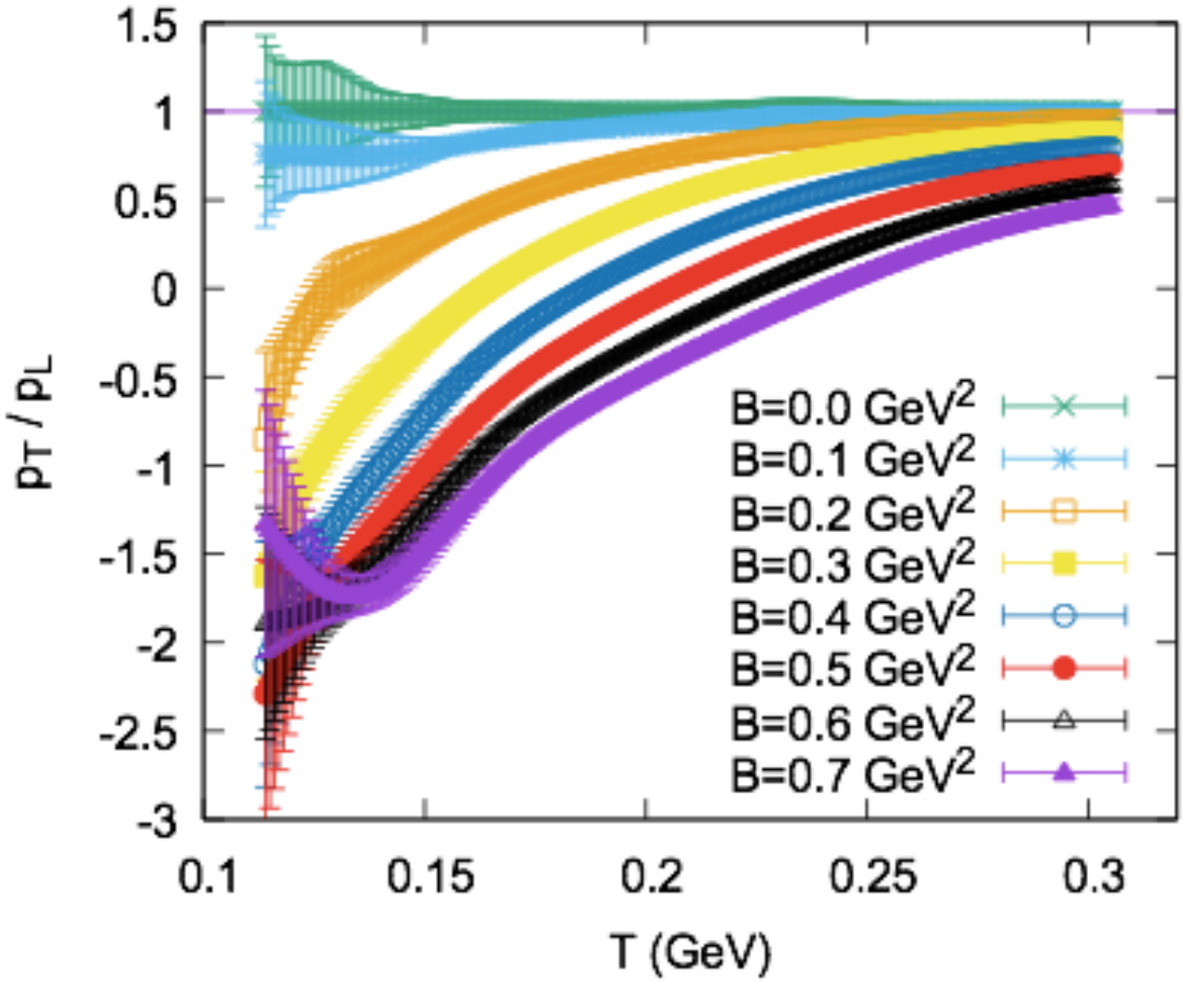

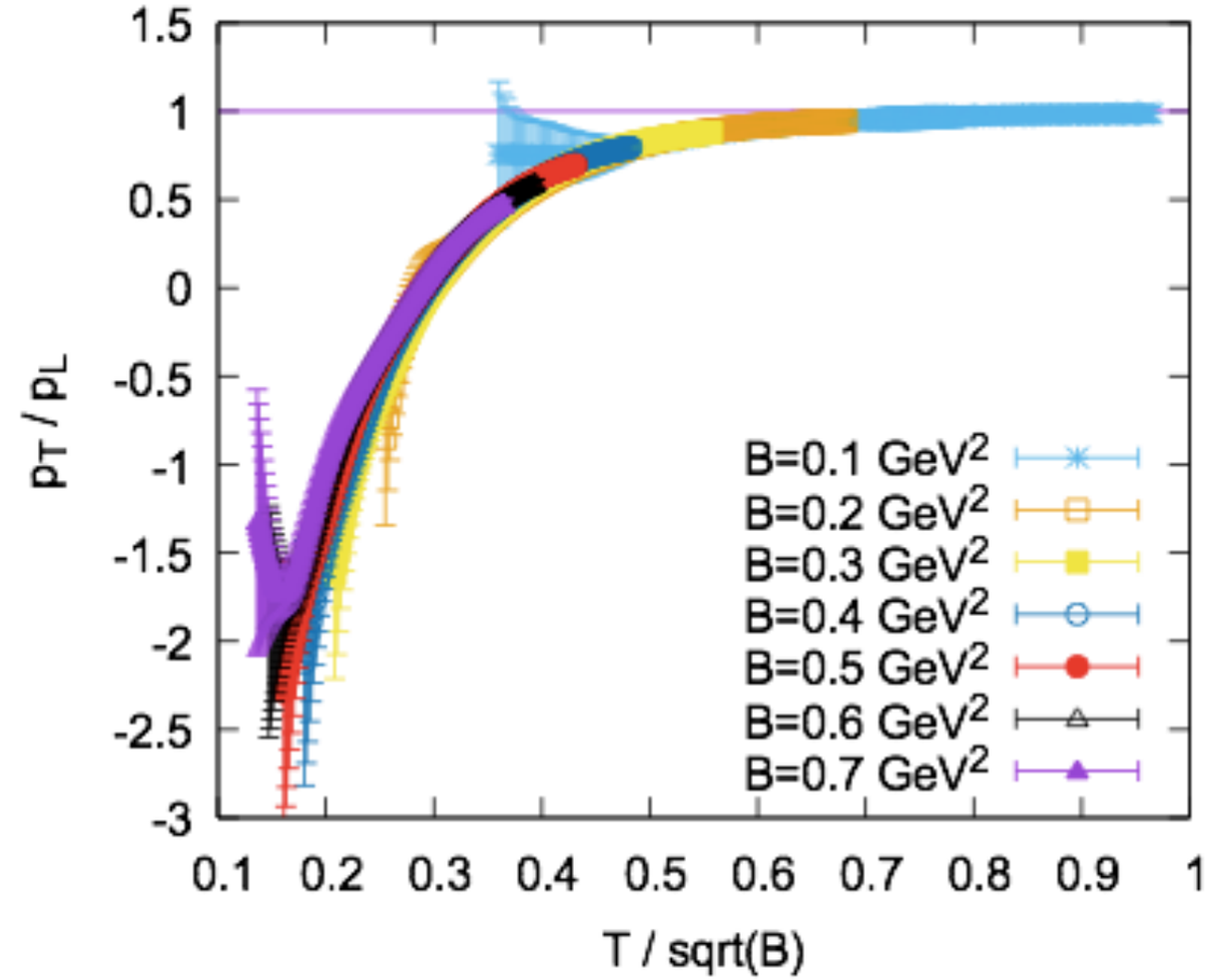

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F_{QCD} ... *free energy*

L_{T} ... *transverse system size*

L_{L} ... *longitudinal system size*

V ... *system volume*

… and *N*=4 Super-Yang-Mills theory

[Endrödi, Kaminski, Schäfer, Wu, Yaffe; arXiv:1806.09632]