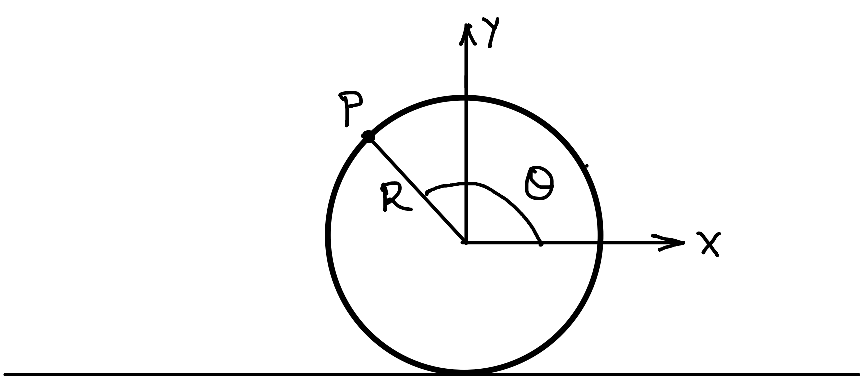
University of Alabama Department of Physics & Astronomy
Graduate Qualifying Examination
Classical Mechanics

8 January 2018

General Instructions

- No reference materials are allowed.
- Do *all* your work in the corresponding answer booklet (no scratch paper is allowed).
- On the cover of each answer booklet put only your *assigned number* and the subject.
- Turn in the questions for each part with the answer booklet.
- 150 minutes are alloted for this part.
- No electronic devices are allowed. In particular, handheld computers, PDAs, and cell phones are explicitly prohibited.
- Answer 5 out of the 6 problems and clearly indicate the problems you wish to be graded.

1. An object with mass m falling vertically under the influence of gravity through a viscous medium is subject to a resistive force that is proportional to the square of its speed, $F_R = cv^2$, where c is a constant. When dropped from rest, the speed of the object is described by the equation $v(t) = v_t tanh(t/\tau)$, where v_t is the terminal speed and τ is the characteristic time.
    a. Find an expression for the characteristic time in terms of only the mass, m, the acceleration due to gravity, g, and the proportionality constant, c.
    b. Verify that your equation is dimensionally correct.
2. A one-dimensional system of mass m is subject to a linear restoring force with force constant k and a frictional force proportional to velocity; denote the proportionality constant by b.
    a. If the system is initially at rest and displaced distance d from equilibrium, find the condition that must be satisfied so that the system does not oscillate when released.
    b. Suppose that the system is now subjected to a harmonic driving force: $F=F_0 \cos\omega t$. Find the amplitude of the velocity. Hint: The solution will have the form $x=A\cos(\omega t+\delta)$
    c. Find the driving frequency ω at which the velocity amplitude is maximum.
3. A hoop of radius R and uniform density is rolling without slipping on a horizontal surface while its center of mass is undergoing constant acceleration a towards the right (see the sketch below). Consider the instant at which the speed of the hoop as measured by a stationary observer is v.
    a. As measured by an observer in whose reference frame the center of mass of the hoop is at rest, find the acceleration of a point on the hoop as a function of θ, v, a, and R and the unit vectors $\hat{x}$ and $\hat{y}$.
    b. As measured by a stationary observer, find the point on the hoop that has the greatest acceleration and give the magnitude and direction of the acceleration relative to the horizontal.



4. The *effective* potential governing the radial motion of a satellite of mass m moving in the neighborhood of a black hole is

$$V_{eff} = \frac{1}{2}mc^2\left[-\frac{R}{r}+\frac{\ell^2}{r^2}-\frac{R\ell^2}{r^3}\right]$$

In the above expression R is the Schwarzschild radius and $\ell = \dfrac{L}{mc}$ where L is the satellite's angular momentum.

   a. Show that for a given $\ell > \sqrt{3}R$ there are possible circular orbits and find their radii in terms of ℓ and R.
   b. Of the orbits found in the previous part, show that only one is stable.
   c. Show that the satellite cannot have a stable circular orbit with radius less than 3R.

5. A speed governor on an old-fashioned steam engine consists of two massive balls that are connected to a rotating rod by light rods. Derive an expression for the angle that the rods make with the vertical as a function of angular velocity ω. Denote the distance from the center of each mass to the opposite end of the connecting rod by L, and denote the mass of each ball by m.

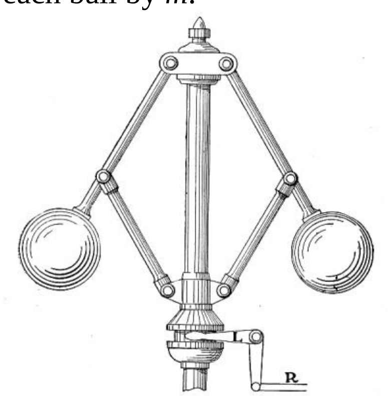
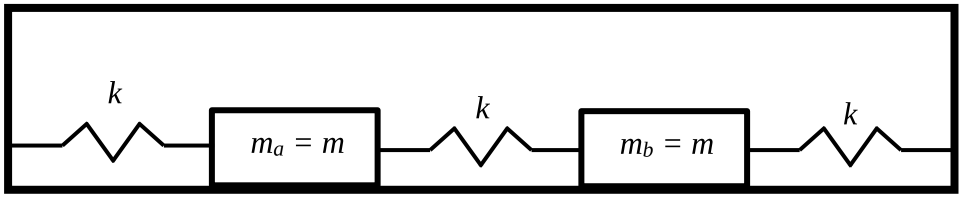


FIG. 15—A FLY-BALL GOVERNOR

6. Consider two identical masses m resting on a frictionless surface and connected by three identical light springs with spring constant k as shown below. Define x_a and x_b as the displacements of the masses from their respective equilibrium positions.
   a) Find the Lagrangian of the system in terms of the variables $u = x_a + x_b$ and $v = x_a - x_b$.
   b) Find the corresponding equation of motion and the resulting frequencies of oscillations.



University of Alabama Department of Physics & Astronomy
Graduate Qualifying Examination
Electricity and Magnetism

9 January 2018

General Instructions

- No reference materials are allowed.
- Do *all* your work in the corresponding answer booklet (no scratch paper is allowed).
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1: Point dipole

A point dipole, $\mathbf{p} = p_0\hat{\mathbf{z}}$, is located at the center of a linear, isotropic, homogeneous sphere with radius R and permittivity $\varepsilon = \kappa\varepsilon_0$. Find the electric potential outside the sphere.

First few Legendre Polynomials

n	$P_n(x)$
0	1
1	x
2	$\frac{1}{2}(3x^2 - 1)$
3	$\frac{1}{2}(5x^3 - 3x)$
4	$\frac{1}{8}(35x^4 - 30x^2 + 3)$
5	$\frac{1}{8}(63x^5 - 70x^3 + 15x)$
6	$\frac{1}{16}(231x^6 - 315x^4 + 105x^2 - 5)$
7	$\frac{1}{16}(429x^7 - 693x^5 + 315x^3 - 35x)$
8	$\frac{1}{128}(6435x^8 - 12012x^6 + 6930x^4 - 1260x^2 + 35)$
9	$\frac{1}{128}(12155x^9 - 25740x^7 + 18018x^5 - 4620x^3 + 315x)$
10	$\frac{1}{256}(46189x^{10} - 109395x^8 + 90090x^6 - 30030x^4 + 3465x^2 - 63)$

Associated Legendre Polynomials

$$P_{\mathrm{n}}^{m}(x) = (-1)^m (1 - x^2)^{m/2} \frac{d^m}{dx^m} \left(P_{\mathrm{n}}(x)\right)$$

2: Shell with fixed charge density

The charge density of a hollow spherical shell of radius R has the form $\sigma(\theta) = \sigma_0 \sin^2\theta$.

(a) Find the potential inside the sphere.

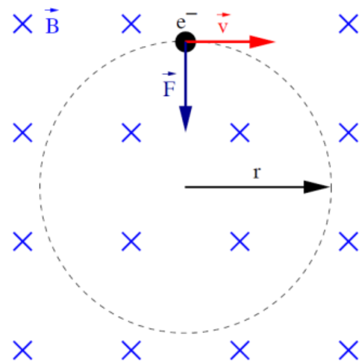
(b) Find the potential outside the sphere.

3: Relativistic Cyclotron

A relativistic electron (rest mass m_0 and charge $-e$) moving in the xy-plane with speed v is subjected to a magnetic field with magnitude B in the z direction. Since the magnetic field does no work, the electron travels in a circular orbit with angular frequency ω_c and radius r_c.

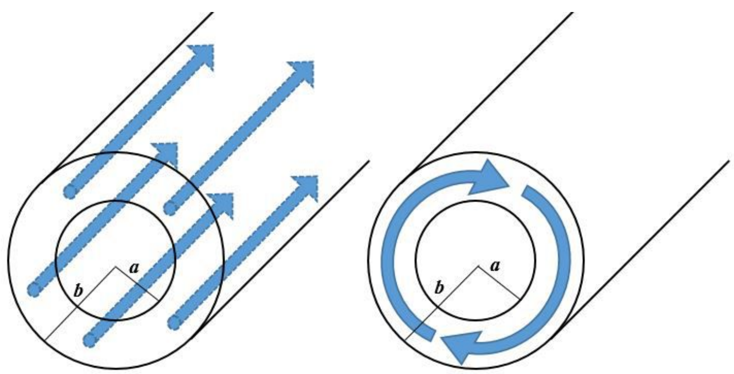
(a) Find the angular frequency in terms of $\gamma = 1/\sqrt{1 - v^2/c^2}$, m_0, e and B.

(b) Find the radius of the orbit in terms of γ, v, m_0, e and B.



4: Cylinders of Current

For each of the geometries shown in figures (i) and (ii) below find the magnetic field ($\vec{B}$) everywhere in space and then sketch the plot of |B| as a function of the distance from the central axis. Assume that the current density $\vec{J}$ is uniform within the shell. Express your answer in terms of the total current I.



(i) $\vec{J} = j_o\hat{z}$ for $a < \rho < b$

(ii) $\vec{J} = j_o\hat{\phi}$ for $a < \rho < b$

5: Concentric Spherical Shells

Three concentric spherical metallic shells have radii c>b>a. They are initially charged with q_c, q_b and q_a, respectively. Finally, the inner shell is grounded. Find the resulting change in potential of the outermost shell.

6: Reflection and Transmission

A plane-polarized EM wave with electric field $\vec{E_I} = \hat{x}E_I e^{i(k_1 z - \omega t)}$ is traveling in a transparent, nonmagnetic medium with index of refraction n_1. It is normally incident on an interface at $z = 0$ with a transparent, nonmagnetic medium with index of refraction n_2. At the interface, the reflected and transmitted electric fields are $\vec{E_R} = -\hat{x}E_R e^{i(-k_1 z - \omega t)}$ and $\vec{E_T} = \hat{x}E_T e^{i(k_2 z - \omega t)}$, respectively with $k_1 = n_1\frac{\omega}{c}$ and $k_2 = n_2\frac{\omega}{c}$.

(a) Find the transmission coefficient, $t_{12} = \frac{E_T}{E_I}$, in terms of the indices of refraction.

(b) Find the reflection coefficient, $r_{12} = \frac{E_R}{E_I}$, in terms of the indices of refraction.