(1)

Explicit Computation of 2D non-abelian Gauge Anomalies

*Review: For a non-abelian gauge theory in 2D, $S[A] = +\int_{S^2} \bar{\Psi}\, i\gamma^\mu(\partial_\mu + A_\mu P_+)\Psi$, a gauge changes our generating functional

$$Z[A] = \int d\bar{\Psi}\, d\Psi\, e^{-S[A]} \rightarrow Z[A^g] = Z[A]\, e^{i\omega(A,g)}.$$

where $g(\theta,\vec{x}) \in SU(2)$ where $\theta \in [0,2\pi]$, $\vec{X} \in S^2 \ni \vec{x}_0$ and $g(0,\vec{x}) = g(2\pi,\vec{x}) = g(\theta,\vec{x}_0) = \mathbb{1}_2 \;\forall \theta,\vec{x}$

We find that for $Z[A^{(\theta)}] = e^{-\Gamma[\theta]}$

$$\Gamma\Big|_0^{2\pi} = -2\pi i N = -i\int \frac{\partial\omega}{\partial\theta}\, d\theta,\ \boxed{N = \frac{1}{2\pi}\int \frac{\partial\omega}{\partial\theta}\, d\theta}$$

Example of $\frac{\partial\omega}{\partial\theta} = \frac{1}{e^{i\theta}+a}$ $a<1$

How to relate to Abelian

(2)

Pre Talk

Comparing 4D-2D anomalies

Theory: $S_A = \int_{S^4} \bar{\Psi}_A i\gamma^\mu(\partial_\mu + A_\mu)\Psi_A$

$$S_{NA} = \int_{S^2} \Psi i\sigma^\mu(\partial_\mu + A_\mu)\Psi$$

Anomaly: Chiral Anomaly $U = e^{i\alpha\gamma_5}$

Non-Abelian Gauge Anomaly $U = e^{i\sum_a \alpha^a T^a}$

Current: $\partial_\mu J^\mu_{A5} \propto tr(F \wedge F)$ (4D!)

$$\frac{1}{2\pi i}(D_\mu J^\mu_3)^a = \left(\frac{i}{2\pi}\right)^2 tr(T^a dA)$$

$$= -\frac{1}{4\pi^2} tr(T^a \partial_\mu(\sum_b (A^\mu)^b T^b)))$$

$$= \frac{1}{8\pi^2}\partial_\mu (A^\mu)^a \quad \text{(2D!)}$$

$$\partial_\mu J^\mu_{A3} = \frac{i}{2\pi} tr(iA) = 0 \quad \text{(2D Abelian)}$$

(3) With Our family of A configuration

$$A^{t,\theta} = tA^{\theta} = t\, g^{-1}(\theta,x)(A+d)g(\theta,x)$$

where $t \in [0,1]$ $g(0,x) = g(2\pi, x) = g(\theta, x_0) = \mathbb{1}$

$A^{t,\theta}$ defines a gauge theory on $D^2 \times S^m$

With two pieces of $A^{t,\theta}$ on $D^+ \times S^m$ for A_N and on $D^- \times S^m$ for A_S. "Sew" together to form A gauge theory on $S^2 \times S^2$

$$A_N = A^{t,\theta} + g^{-1}\left[d\theta \frac{\partial}{\partial\theta}\right] g \quad , (t,\theta) \in U_N$$

$$A_S = A \qquad (s,\phi) \in U_S$$

A with the t, s and θ components of A are set to vanish where "it" components don't and

$A_M = \{A_N, A_S\}$.

∴(4) With AS-index

$$\text{ind}\, i\not{D}_{m+2} = \nu_+ - \nu_- = \frac{1}{2\pi}\int_0^{2\pi} d\theta\, \frac{\partial\omega}{\partial\theta} = \frac{1}{2\pi}\int_{S^1} d_\theta\, \omega$$

$$\parallel$$

$$\int_{S^1\times S^m} (-1)^{m/2}\left(\frac{i}{2\pi}\right)^{\frac{m}{2}+1} \frac{\left(\frac{m}{2}\right)!}{(m+1)!}\, tr\left(g^{-1}(d+d_\theta)g\right)^{m+1}$$

↑ equator

ex the Hopf Map
$g: S^3 \longrightarrow SU(2),\ m=2$

$$(-1)\left(\frac{i}{2\pi}\right)^2 \frac{1}{6}\, tr\left(g^{-1}(d+d_\theta)g\right)^2$$

$$\int_{S^2\times S^2} \partial_\mu J_5^\mu \propto \nu_+ - \nu_- = -\frac{1}{6}\left(\frac{i}{2\pi}\right)^2 tr\left(g^{-1}(d+d_\theta)g\right)^2$$

$$= N \equiv \text{winding } \#$$

∴ Locally $\int_{S^2\times S^2} \partial_\mu J_5^\mu = \int_{D\times S^2} dQ_3(A_N) - dQ_3(A_S)\Big|_{t,s=1}$

Use Stokes Theorem

$$N = \int_{S^1\times S^m} Q_3(A_N) - Q_3(A_S)\Big|_{t,s=1}$$

$$= \frac{1}{2\pi}\int_0^{2\pi} \frac{\partial\omega}{\partial\theta}\, d\theta$$

Expand w.r.t ω. $O(\omega^1)$ term is the anomaly, Q_2^1

$$\int_0^{2\pi} d\theta \int_{S^2} \mathrm{tr}\left(\omega\, D_\mu \frac{\delta\Gamma}{\delta A_\mu}\right) = i\, d_\theta \omega = 2\pi i \int_{S^2} Q_2^1$$

Now ω instead of v

$$\therefore \quad \omega^a D_\mu \langle J^\mu_a \rangle = 2\pi i\, Q_2^1$$

$$= 2\pi i \left(\frac{i}{2\pi}\right)^2 \mathrm{tr}(T^a dA)\,\omega_a$$

$$= \frac{i}{4\pi} \partial_\mu A^\mu_a \quad \blacksquare$$