Fujikawa PI Method to Anomaly - 2 — Fall 18

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(1)

Ref's

- Anomalies in QFT - Bertlmann pg 251, 265
- Fujikawa, Phys. Rev. D29, 285 (1984)

Defs

$x^0 = -ix^4$, $\gamma^0 = -i\gamma^4$, $\partial_0 = i\partial_4$

$A_0 = iA_4$ $\to d^4x = -id^4x$

$g^{\mu\nu} = -\delta^{\mu\nu}$

$\gamma^{\mu\dagger} = -\gamma^\mu$, $\gamma_5^\dagger = \gamma_5$

> Recall from last time that under a abelian local chiral transform,

$\psi' = e^{i\beta(x)\gamma_5}\psi$, $\bar\psi' = \bar\psi e^{i\beta(x)\gamma_5}$ $\Rightarrow$ $\mathcal{D}\psi\mathcal{D}\bar\psi$ changes too

and eventually we found the singlet anomaly

$$\boxed{\mathcal{A}[A_\mu] = \frac{-i}{16\pi^2}\varepsilon^{\mu\nu\alpha\beta}\, tr\, F_{\mu\nu}F_{\alpha\beta}}$$

Background field (Non-Abelian or Abelian)

> Now let's consider the affect on the generating functional (Suppressing A_μ source)

$$Z[\eta,\bar\eta,A_\mu,\beta] = \frac{1}{N}\int \mathcal{D}\psi'\mathcal{D}\bar\psi' \exp\left[\int dx\,(\mathcal{L}' + \bar\eta\psi' + \bar\psi'\eta)\right]$$

$$= \frac{1}{N}\int d\psi d\bar\psi \exp\left[\int dx\,(\mathcal{L} + \bar\eta\psi + \bar\psi\eta)\right] \times \exp\left[\int dx\,\beta(x)\left(\partial^\mu j_\mu^5 - 2imP - \mathcal{A}[A_\mu] + i\bar\eta\gamma_5\psi + i\bar\psi\gamma_5\eta\right)\right]$$

where
- $j_\mu^5 = \bar\psi\gamma_\mu\gamma_5\psi$, $P = \bar\psi\gamma_5\psi$
- $N = \det(i\not{D} - m)$, $\mathcal{L} = \bar\psi(i\not{D} - m)\psi$

- Expanding in $\beta(x)$

$$Z[\eta,\bar\eta,A_\mu,\beta] = \frac{1}{N}\int \mathcal{D}\psi\mathcal{D}\bar\psi \exp\left[\int dx\,(\mathcal{L} + \bar\eta\psi + \bar\psi\eta)\right]$$

$$\times\left[1 + \int dx\,\beta(x)\left(\partial^\mu j_\mu^5 - 2imP - \mathcal{A}[A_\mu] + i\bar\eta\gamma_5\psi + i\bar\psi\gamma_5\eta\right)\right]$$

$$= Z[\eta,\bar\eta,A_\mu,0] + \int dx\,\beta(x)\frac{\delta}{\delta\beta(x)} Z[\eta,\bar\eta,A_\mu,\beta]\Big|_{\beta=0}$$

$$\frac{\delta f(t_2)}{\delta f(t_1)} = \delta(t_1 - t_2)$$

- Now since we simply shifted $\psi, \bar\psi$ we expect the quantum action to remain invariant ~~[illegible]~~

$$\Rightarrow Z[\eta,\bar\eta,A_\mu,\beta] \equiv Z[\eta,\bar\eta,A_\mu,0] \quad\text{or}\quad \frac{\delta}{\delta\beta(x)}Z[\eta,\bar\eta,A_\mu,\beta]\Big|_{\beta=0} = 0$$

$\therefore$ The anomalous divergence of the axial-current is given by

$$\partial^\mu\langle j_\mu^5\rangle = 2im\langle P\rangle + \mathcal{A}[A_\mu]$$

(2)

- Ward Ids
- Using this generating functional formalism we can explore the Ward Ids (WI's) through functional differentiation of the sources.

- $$\frac{\delta^2 Z[\eta,\bar\eta,A_\mu,\beta]}{\delta\eta(x_2)\delta\beta(x)}\Bigg|_{\beta=0} = \frac{1}{N}\cdot\frac{\delta}{\delta\eta(x_2)}\Bigg(\int \mathcal{D}\psi\mathcal{D}\bar\psi \exp\left[\int dx\,(\mathcal{L}+\bar\eta\psi+\bar\psi\eta)\right]$$

$$\times\left(\partial^\mu\gamma_\mu^5 - 2imP - \mathcal{A}[A_\mu] + i\bar\eta\gamma_5\psi + i\bar\psi\gamma_5\eta\right)\Bigg)\Bigg|_{\beta=0} = \frac{1}{N}\int \mathcal{D}\psi\mathcal{D}\bar\psi \exp\left[\int dx\,(\mathcal{L}+\bar\eta\psi+\bar\psi\eta)\right]$$

$$\times\Big\{-\bar\psi(x_2)\left[\partial^\mu\gamma_\mu^5 - 2imP - \mathcal{A}[A_\mu] + i\bar\eta\gamma_5\psi + i\bar\psi\gamma_5\eta\right](x)$$

$$- i\bar\psi(x)\gamma_5\,\delta(x-x_2)\Big\}$$

- $$\frac{\delta^3 Z[\eta,\bar\eta,A_\mu,\beta]}{\delta\bar\eta(x_1)\delta\eta(x_2)\delta\beta(x)}\Bigg|_{\beta=0} = \frac{1}{N}\int \mathcal{D}\psi\mathcal{D}\bar\psi \exp\left[\int dx\,(\mathcal{L}+\bar\eta\psi+\bar\psi\eta)\right]$$

$$\times\Big\{-\psi(x_1)\bar\psi(x_2)\left[\partial^\mu\gamma_\mu^5 - 2imP - \mathcal{A}[A_\mu] + i\bar\eta\gamma_5\psi + i\bar\psi\gamma_5\eta\right](x)$$

$$- i\psi(x_1)\bar\psi(x_2)\gamma_5\,\delta(x-x_2) + i\bar\psi(x_2)\gamma_5\psi(x)\,\delta(x-x_1)\Big\}$$

- With the invariance condition: $Z' = Z$

$$\frac{\delta^3 Z[\eta,\bar\eta,A_\mu,\beta]}{\delta\bar\eta(x_1)\delta\eta(x_2)\delta\beta(x)}\Bigg|_{\beta=\eta=\bar\eta=0} = 0$$

$$\Rightarrow \partial^\mu_x\langle \gamma^5_{\mu(x)}\psi(x_1)\bar\psi(x_2)\rangle = 2im\langle P(x)\psi(x_1)\bar\psi(x_2)\rangle$$

$$+\langle \mathcal{A}[A_\mu](x)\psi(x_1)\bar\psi(x_2)\rangle - i\langle\gamma_5\psi(x)\bar\psi(x_2)\rangle\delta(x-x_1) - i\langle\psi(x_1)\bar\psi(x)\gamma_5\rangle\delta(x-x_2)$$

WI from Casey's talk before!!

- Non-Abelian Chiral Anomaly

→ $\underset{(m=0)}{\mathcal{L}} = \bar\psi i\not{D}\psi$ where $\not{D} = \not\partial + \not{V} + \not{A}\gamma_5$ and $V_\mu = V_\mu^a T^a$, $A_\mu = A_\mu^a T^a$

- But now $\not{D}$ isn't hermitian: $\not{D}^\dagger = \not\partial + \not{V} + \gamma_5\not{A}$
$= \not\partial + \not{V} - \not{A}\gamma_5 \neq \not{D}$!

→ $\not{D}^\dagger = \not{D}^\dagger(V,A) = \not{D}(V,-A)$

which means that $\not{D}\phi_n = \lambda_n\phi_n$ has $\mathbb{C}$ eigenvalues!

$\boxed{T^{a\dagger} = -T^a \text{ (pg 160)}\ \text{In Bertleman they use } T^a = \frac{\sigma^a}{2i},\ \text{etc.}\ \ \mathrm{Tr}\,T^aT^b = -\frac{1}{2}\delta^{ab}}$

- Not to dispair though, the Laplacian operators definitely have acceptable eigenvalues
  - $\not{D}^{+}\not{D}\, \phi_n = \lambda_n^2 \phi_n$
  - $\not{D}\not{D}^{+} \chi_n = \lambda_n^2 \chi_n$

$$\left.\begin{matrix} \not{D}^{+}\not{D}\, \phi_n = \lambda_n^2 \phi_n \\ \not{D}\not{D}^{+} \chi_n = \lambda_n^2 \chi_n \end{matrix}\right\} \Rightarrow \boxed{\begin{matrix} \not{D}\, \phi_n = \lambda_n \chi_n \\ \not{D}^{+} \chi_n = \lambda_n \phi_n \end{matrix}}$$

- We can preform similar procedure as in Singlet case but now with Laplacian operators $\underline{\not{D}\not{D}^{+}, \not{D}^{+}\not{D}}$
  - $\psi(x) = \sum_n a_n \phi_n(x) = \sum_n a_n \langle x | \phi_n \rangle$
  - $\bar{\psi}(x) = \sum_m \chi_m^{+}(x) \bar{b}_m = \sum_m \langle \chi_m | x \rangle \bar{b}_m$ where $\{\phi_n\}$ $\{\chi_m\}$ are $\underline{\text{complete orthonormal}}$ sets

  $\hookrightarrow$ $D\psi D\bar{\psi} = \left[\det \langle \chi_m | x \rangle \det \langle x | \phi_n \rangle\right]^{-1} \prod_n da_n \prod_m d\bar{b}_m$
  - $\int dx\, \bar{\psi} i \not{D} \psi = \sum_n i \lambda_n \bar{b}_n a_n$
- $N = \det i\not{D} = \int D\psi D\bar{\psi} \exp\left[\int dx\, i\bar{\psi} \not{D} \psi\right]$

$$= \left[\det \langle \chi_n | x \rangle \det \langle x | \phi_n \rangle\right]^{-1} \int \prod_n da_n d\bar{b}_n \exp\left[\sum_n i \lambda_n \bar{b}_n a_n\right]$$

$$= \left[\det \chi^{+} \det \phi\right]^{-1} \prod_n i \lambda_n$$

- Now preform a non-abelian chiral transform

$$\left.\begin{matrix} \psi'_{(x)} = e^{i\beta(x)\gamma_5} \psi(x) \\ \bar{\psi}'_{(x)} = \bar{\psi}(x)\, e^{i\beta(x)\gamma_5} \end{matrix}\right\} \text{euclidean} \qquad \beta(x) = \beta(x)^a T^a$$

Aside

$\psi'_{(x)} = e^{i\beta(x)\gamma_5} \psi_{(x)}$ $\qquad \beta^{+} = \beta^{+a} T^{a+} = (i\beta)(-T^a) = -\beta$ ✓

$\psi'^{+} = \psi^{+}_{(x)} e^{-i\beta^{+}\gamma_5} = \psi^{+}_{(x)} e^{+i\beta\gamma_5} \rightarrow \bar{\psi}'_{(x)} = \psi^{+}_{(x)}\gamma_0 = \bar{\psi}(x)\, e^{i\beta\gamma_5}$

- And the Grassmann expansion coefficients are related by
  - $a'_n = \sum_m C_{nm} a_m$, $\bar{b}'_m = \sum_n D_{nm} \bar{b}_n$

$$\left.\begin{matrix} C_{nm} = \delta_{nm} + i \int dx\, \phi_n^{+}(x) \beta(x) \gamma_5 \phi_m(x) \\ D_{nm} = \delta_{nm} + i \int dx\, \chi_n^{+}(x) \beta(x) \gamma_5 \chi_m(x) \end{matrix}\right\} \text{before only } \underline{C_{nm}}$$

- The measure then transforms as

$$\prod_n da_n' = [\det C]^{-1} \prod_n da_n \quad , \quad \prod_m d\bar{b}_m' = [\det D]^{-1} \prod_m d\bar{b}_m$$

then

- $[\det C]^{-1} = \exp[-\operatorname{tr} \ln C] = \exp\left[-\operatorname{tr} \ln\left(\delta_{nm} + i\int dx\, \phi_n^\dagger(x)\beta(x)\gamma_5\phi_m(x)\right)\right]$

$$\simeq \exp\left[-i\sum_n \int \phi_n^\dagger(x)\beta(x)\gamma_5\phi_n(x)\,dx\right]$$

and

- $[\det D]^{-1} \simeq \exp\left[-i\sum_n \int \chi_n^\dagger(x)\beta(x)\gamma_5\chi_n(x)\,dx\right]$

$$\Rightarrow D\psi' D\bar\psi' = [\det C \cdot \det D]^{-1} D\psi D\bar\psi = J[\beta]\, D\psi D\bar\psi$$

$$J[\beta] = \exp\Big[-i\int dx \sum_n \big(\underbrace{\phi_n^\dagger(x)\beta(x)\gamma_5\phi_n(x)}_{\delta(0)!} + \underbrace{\chi_n^\dagger(x)\beta(x)\gamma_5\chi_n(x)}_{\delta(0)!}\big)\Big]$$

Regularization

- Similar to talk I of mine, we regularize in a particularly clever way. (let me drop explicit x dep)

$$\sum_n (\phi_n^\dagger\beta\gamma_5\phi_n + \chi_n^\dagger\beta\gamma_5\chi_n) = \lim_{M\to\infty}\sum_n\left(\phi_n^\dagger\beta\gamma_5\exp\left[\frac{-\lambda_n^2}{M^2}\right]\phi_n + \chi_n^\dagger\beta\gamma_5\exp\left[\frac{-\lambda_n^2}{M^2}\right]\chi_n\right)$$

$$= \lim_{M\to\infty}\sum_n\left(\phi_n^\dagger\beta\gamma_5\exp\left[\frac{-D\!\!\!/^\dagger D\!\!\!/}{M^2}\right]\phi_n + \chi_n^\dagger\beta\gamma_5\exp\left[\frac{-D\!\!\!/ D\!\!\!/^\dagger}{M^2}\right]\chi_n\right)$$

$$= \lim_{M\to\infty}\int\frac{d^4k}{(2\pi)^4}\operatorname{Tr}\left\{\beta e^{-ikx}\gamma_5\left(\exp\left[\frac{-D\!\!\!/^\dagger D\!\!\!/}{M^2}\right] + \exp\left[\frac{-D\!\!\!/ D\!\!\!/^\dagger}{M^2}\right]\right)e^{ikx}\right\}$$

← From completion $\sum_n \phi_n^\dagger(x)\phi_n(y) = \delta(x-y)$

- To procede further, I'll now utilize properties of the projection operators to split the Laplacian operators into "+" and "−" chiralities
  - $P_\pm = \frac{1}{2}(1\pm\gamma_5)$, $P_\pm^2 = P_\pm$, $P_+P_- = 0$, $P_+ + P_- = \mathbb{1}$, $P_+ - P_- = \gamma_5$
  - $\psi_\pm = P_\pm\psi$, $\gamma_5\psi = \gamma_5\frac{1}{2}(1\pm\gamma_5)\psi = \pm\frac{1}{2}(1\pm\gamma_5)\psi = \pm\psi_\pm$
  - $A_\mu^+ = V_\mu + A_\mu$, $A_\mu^- = V_\mu - A_\mu$, $D\!\!\!/_\pm = \partial\!\!\!/ + A\!\!\!/_\pm$
  - $\mathcal{L} = \bar\psi iD\!\!\!/\psi = i\bar\psi(\partial\!\!\!/ + V\!\!\!/ + A\!\!\!/\gamma_5)\psi = i\bar\psi_+ D\!\!\!/_+\psi_+ + i\bar\psi_- D\!\!\!/_-\psi_-$

Aside

$$i\bar\psi(P_+D\!\!\!/ + P_-D\!\!\!/)\psi = i\bar\psi\big(P_+(\partial\!\!\!/ + V\!\!\!/ - A\!\!\!/)P_- + P_-(\partial\!\!\!/ + V\!\!\!/ + A\!\!\!/)P_+\big)\psi$$

$$= i\bar\psi_- D\!\!\!/_-\psi_- + i\bar\psi_+ D\!\!\!/_+\psi_+$$

$P_+ A\!\!\!/\gamma_5 = -A\!\!\!/ P_-$

$P_- A\!\!\!/\gamma_5 = +A\!\!\!/ P_+$

5

- Now let's decompose

$$\not{D} = \not{\partial} + (P_+ + P_-)(\not{V} + \not{A}\gamma_5) = \not{\partial} + \not{A}_- P_- + \not{A}_+ P_+$$

$$\not{D}^+ = \not{D}(V, -A) = \not{\partial} + (P_+ + P_-)(\not{V} - \not{A}\gamma_5) = \not{\partial} + \not{A}_+ P_- + \not{A}_- P_+$$

- And for the Laplacian

$$\not{D}^+\not{D} = (\not{\partial} + \not{A}_- P_+ + \not{A}_+ P_-)(\not{\partial} + \not{A}_- P_- + \not{A}_+ P_+)$$

$$= \not{\partial}\not{\partial}(P_+ + P_-) + \not{\partial}\not{A}_+ P_+ + \not{A}_+\not{\partial} P_+ + \not{A}_+\not{A}_+ P_+$$
$$+ \not{\partial}\not{A}_- P_- + \not{A}_-\not{\partial} P_- + \not{A}_-\not{A}_- P_-$$

$$= (\not{\partial}^2 + \not{\partial}\not{A}_+ + \not{A}_+\not{\partial} + \not{A}_+\not{A}_+)P_+ + (\not{\partial}^2 + \not{\partial}\not{A}_- + \not{A}_-\not{\partial} + \not{A}_-\not{A}_-)P_-$$

$$= \not{D}_+^2 P_+ + \not{D}_-^2 P_- = \boxed{P_+\not{D}_+^2 + P_-\not{D}_-^2 = \not{D}^+\not{D}} \quad \text{and} \quad \boxed{\not{D}\not{D}^+ = \not{D}_+^2 P_- + \not{D}_-^2 P_+}$$

- Back to the regulator procedure

$$\sum_n (\psi_n^+ \beta\gamma_5 \psi_n + \chi_n^+ \beta\gamma_5 \chi_n) = \lim_{M\to\infty} \int \frac{d^4k}{(2\pi)^4} \, Tr\left\{\beta e^{-ikx}\gamma_5 \left(\exp\left[\frac{-\not{D}^+\not{D}}{M^2}\right] + \exp\left[\frac{-\not{D}\not{D}^+}{M^2}\right]\right) e^{ikx}\right\}$$

→ Focusing on exponential terms

- $$\exp\left[\frac{-\not{D}^+\not{D}}{M^2}\right] + \exp\left[\frac{-\not{D}\not{D}^+}{M^2}\right] = \left(P_+ \exp\left[\frac{-\not{D}_+^2}{M^2}\right] + P_- \exp\left[\frac{-\not{D}_-^2}{M^2}\right]\right) + \left(P_- \exp\left[\frac{-\not{D}_+^2}{M^2}\right] + P_+ \exp\left[\frac{-\not{D}_-^2}{M^2}\right]\right)$$

$$= \exp\left[\frac{-\not{D}_+^2}{M^2}\right] + \exp\left[\frac{-\not{D}_-^2}{M^2}\right]$$

Aside

- How does $\exp(\not{D}^+\not{D}) \to \exp(\not{D}_+) + \exp(\not{D}_-)$?

Say $M=1$:

$$\exp[-\not{D}^+\not{D}] = \exp[-\not{D}_+^2 P_+ - \not{D}_-^2 P_-] = 1 - (\not{D}_+^2 P_+ + \not{D}_-^2 P_-)$$

$$+ \frac{1}{2!}(\not{D}_+^2 P_+ \not{D}_+^2 P_+ + \cancel{\not{D}_+^2 P_+ \not{D}_-^2 P_-} + \cancel{\not{D}_-^2 P_- \not{D}_+^2 P_+} + \not{D}_-^2 P_- \not{D}_-^2 P_-) + \ldots$$

$$= (P_+ + P_-) - (\not{D}_+^2 P_+ + \not{D}_-^2 P_-) + \frac{1}{2!}(\not{D}_+^4 P_+ + \not{D}_-^4 P_-) + \ldots = P_+\left(1 - \not{D}_+^2 - \frac{1}{2!}\not{D}_+^4 - \ldots\right) + P_-\left(1 - \not{D}_-^2 - \frac{1}{2!}\not{D}_-^4 \ldots\right)$$

$$= P_+ \exp\left[\frac{-\not{D}_+^2}{M^2}\right] + P_- \exp\left[\frac{-\not{D}_-^2}{M^2}\right]$$

$$\Rightarrow \sum_n (\psi_n^+ \beta\gamma_5 \psi_n + \chi_n^+ \beta\gamma_5 \chi_n) = \lim_{M\to\infty} \int \frac{d^4k}{(2\pi)^4} \, Tr\left\{\beta e^{-ikx}\gamma_5 \left(\exp\left[\frac{-\not{D}_+^2}{M^2}\right] + \exp\left[\frac{-\not{D}_-^2}{M^2}\right]\right) e^{ikx}\right\}$$

— Using the same procedure from talk 1 yields

$$\sum_n (\psi_n^+(x)\beta(x)\gamma_5\psi_n(x) + \chi_n^+(x)\beta(x)\gamma_5\chi_n(x)) = \boxed{\frac{-\epsilon^{\mu\nu\alpha\beta}}{32\pi^2} Tr\{\beta(F_{\mu\nu}^+ F_{\alpha\beta}^+ + F_{\mu\nu}^- F_{\alpha\beta}^-)\}}$$ where $F^\pm \sim A_\pm$

- Thus far we've only concerned ourselves with $\partial^\mu j^5_\mu$ but what about the affect of chiral gauge on the vector current $\partial^\mu j_\mu$?

- Gauge transformation: $\psi' = e^{i\alpha}\psi$, $\bar{\psi}' = \bar{\psi} e^{-i\alpha}$; $\alpha = \alpha(x)^a T^a$
- Similar to before, the Grassmann coefficients are now given by

$$C_{nm} = \delta_{nm} + i\int dx\, \phi_n^\dagger(x)\,\alpha(x)\,\phi_m(x)$$
$$D_{nm} = \delta_{nm} - i\int dx\, \chi_n^\dagger(x)\,\alpha(x)\,\chi_m(x)$$

$$\Rightarrow \quad J[\alpha] = \left[\det C \cdot \det D\right]^{-1} \simeq \exp\left[i\int dx \sum_n \left(\phi_n^\dagger(x)\alpha(x)\phi_m(x) - \chi_n^\dagger(x)\alpha(x)\chi_m(x)\right)\right]$$

- Next we regulate

$$\sum_n \left(\phi_n^\dagger \alpha \phi_m - \chi_n^\dagger \alpha \chi_n\right) = \lim_{M\to\infty}\int \frac{d^4k}{(2\pi)^4}\, Tr\left\{\alpha\, e^{-ikx}\left(\exp\left[\frac{-D\!\!\!/^\dagger D\!\!\!/}{M^2}\right] - \exp\left[\frac{-D\!\!\!/ D\!\!\!/^\dagger}{M^2}\right]\right)e^{ikx}\right\}$$

↳ where: $\exp\left[\frac{-D\!\!\!/^\dagger D\!\!\!/}{M^2}\right] - \exp\left[\frac{-D\!\!\!/ D\!\!\!/^\dagger}{M^2}\right] = \left(P_+ \exp\left[\frac{-D\!\!\!/_+^2}{M^2}\right] + P_- \exp\left[\frac{-D\!\!\!/_-^2}{M^2}\right]\right)$

$$- \left(P_- \exp\left[\frac{-D\!\!\!/_+^2}{M^2}\right] + P_+ \exp\left[\frac{-D\!\!\!/_-^2}{M^2}\right]\right) = (P_+ - P_-)\left(\exp\left[\frac{-D\!\!\!/_+^2}{M^2}\right] - \exp\left[\frac{-D\!\!\!/_-^2}{M^2}\right]\right)$$

$$= \underline{\underline{\gamma_5}}\left(\exp\left[\frac{-D\!\!\!/_+^2}{M^2}\right] - \exp\left[\frac{-D\!\!\!/_-^2}{M^2}\right]\right)$$

!! appears due to "−"

$\therefore$ $\sum_n \left(\phi_n^\dagger(x)\alpha(x)\phi_n(x) - \chi_n^\dagger(x)\alpha(x)\chi_n(x)\right)$

$$= \lim_{M\to\infty}\int \frac{d^4k}{(2\pi)^4}\, Tr\left\{\alpha\, e^{-ikx}\gamma_5\left(\exp\left[\frac{-D\!\!\!/_+^2}{M^2}\right] - \exp\left[\frac{-D\!\!\!/_-^2}{M^2}\right]\right)e^{ikx}\right\}$$

$$= -\frac{\epsilon^{\mu\nu\alpha\beta}}{32\pi^2}\, Tr\left\{\alpha\left(F^+_{\mu\nu}F^+_{\alpha\beta} - F^-_{\mu\nu}F^-_{\alpha\beta}\right)\right\}$$ analogous to before!

'Moral: even gauge transformations are plagued by chiral presence