1999 Qualifier

Part 3. Quantum Mechanics: Answer any 5 of the following questions.

1. a. What is the energy in electron volts of a green photon with a wavelength of 5500 Å?
   b. What is the deBroglie wavelength of a 150 eV electron?
   c. What is the energy in electron volts of a neutron with a 1 Å deBroglie wavelength?
   d. What is the speed in m/sec of a 1 Å neutron?

2. Define the following:
   a. Schrödinger's equation.
   b. Heisenberg uncertainty principle.
   c. Pauli principle.
   d. Degeneracy.
   e. Stationary state.

3. a. Find the wavefunctions for a particle of mass m in an infinite potential well of length L.
   b. What are the energy levels of the particle in the infinite potential well?
   c. What is the energy spacing in ergs between the ground state and the first excited state of a 1 gram particle in a 1 cm long infinite potential well?
   d. What is the energy spacing in electron volts between the ground state and the first excited state of an electron in a 1 Å long infinite potential well?

4. a. Write the expression for the electronic energy levels of a Hydrogenic atom with atomic number Z.
   b. What are the electronic quantum numbers for a Hydrogenic atom?
   c. How many possible electrons can occupy a d shell and what are their quantum numbers?

5. Consider a particle described by a wave packet $\psi(x) = Ae^{i p_0 x}$ for $-L \le x \le L$ and $\psi(x) = 0$ for $|x| > L$.
   a. Calculate the normalization constant A such that the probability is normalized to unity.
   b. Calculate $< x >$.
   c. Calculate $< p_x >$.
   d. Calculate the mean square displacement of the particles position $< (x - < x >)^2 >$.

6. Given the definitions of the spin angular momentum operator:

$$S_z \chi_+ = h/(4\pi)\, \chi_+ \qquad S_z \chi_- = -h/(4\pi)\, \chi_-$$
$$S_x \chi_+ = h/(4\pi)\, \chi_+ \qquad S_x \chi_- = h/(4\pi)\, \chi_-$$
$$S_y \chi_+ = i\, h/(4\pi)\, \chi_+ \qquad S_z \chi_- = -i\, h/(4\pi)\, \chi_-$$

Show:
   a. $S_x^2 = S_y^2 = S_z^2 = [h/(4\pi)]^2$
   b. $\mathbf{S} \times \mathbf{S} = i\, h/(2\pi)\, \mathbf{S}$
   c. The corresponding relations for the raising and lowering operators defined by $S_\pm = S_x \pm i\, S_y$.

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$h = 6.63 \times 10^{-34}$ J•sec ; $e = 1.60 \times 10^{-19}$ C ; $m_e = .911 \times 10^{-30}$ kg; $m_n = 1.63 \times 10^{-27}$ kg; $a_0 = .529 \times 10^{-10}$ m

Qualifying Examination January 1999
Part IV: Mixed Topics - Astrophysics

1. Consider an elliptical orbit around the Sun whose aphelion distance is equal to the Earth's mean orbital radius, 1.0 astronomical units (A. U.), and whose perihelion distance is 0.72 A. U., the radius of Venus' orbit.

(a) What would be the energy per unit mass of such an orbit (in joules/kg), and how does this compare to the energy per unit mass of a perfectly circular orbit of radius 1.0 A. U.?

(b) If a satellite launched along this orbit reached perihelion at the exact same moment as Venus was crossing that point, what would be the orbital velocity of the satellite relative to Venus? (Ignoring any gravitational effect from Venus itself and assuming Venus's orbit is perfectly circular)

(c) What would be the period (in years) of the orbit?

2. The main sequence star β Armadillus A has a radius of $2R_\odot$ and a bolometric absolute magnitude of $M_{bol} = +1.0$. The radius of its companion, β Armadillus B, is $0.6R_\odot$ and $M_{bol} = +6.0$.

(a) What is the ratio of their luminosities?
(b) What is the ratio of their effective temperatures?
(c) If the inclination of their orbits were i=90°, which star would be eclipsed at the primary (deepest) minimum?

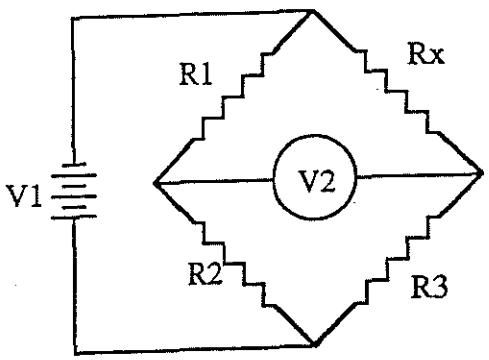
Useful information:
$M_\odot = 1.99 \times 10^{30}$kg
G=6.67×10^{-11} Nm^{2}kg^{-2}

Qualifying Examination January 1999
Part IV: Mixed Topics - Relativity

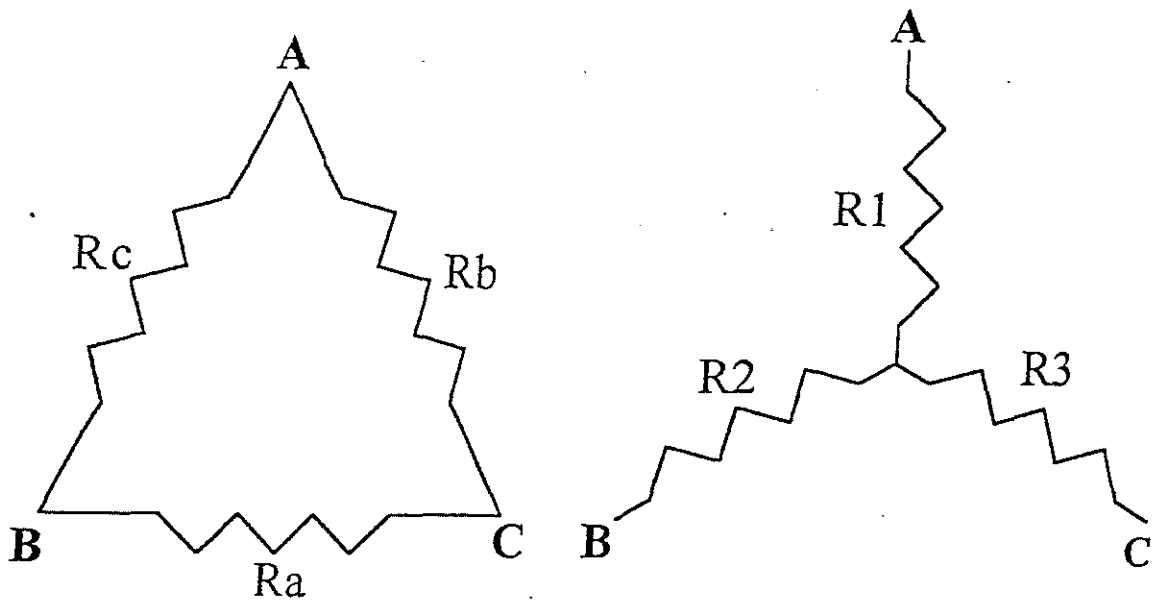
1. An astronaut learns that NASA has chosen him to lead a mission to search for planets orbiting the bright star Sirius, 8.6 light years away. He will leave on his 25th birthday. They assure him that he will get back to Earth before he turns 35, according to the ship's clock. What is the minimum speed the ship will have to achieve to accomplish this goal? Assume the ship will move at a uniform speed for most of the journey, and that the team will spend 6 months exploring the system at speeds much less than that of light.

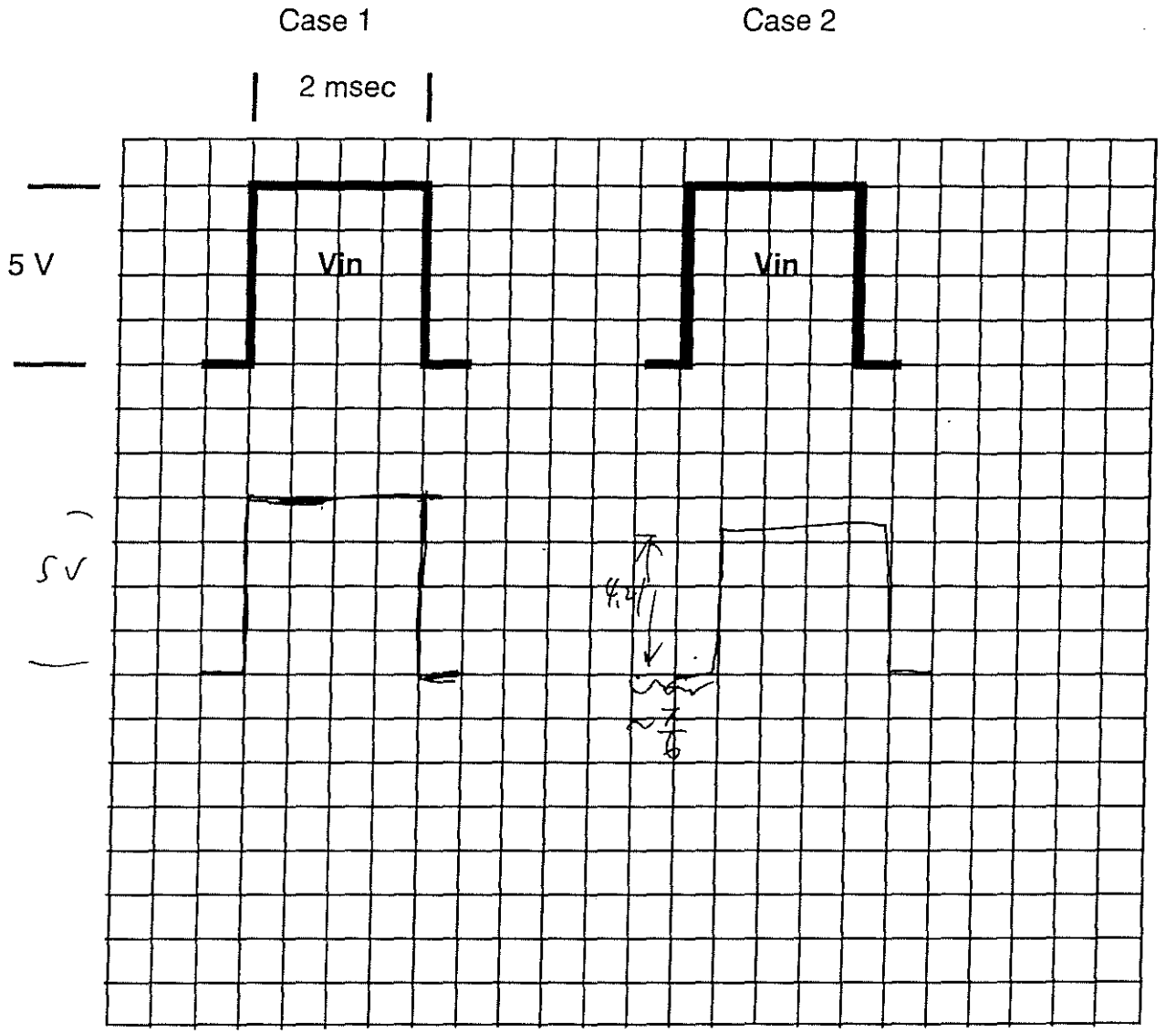
2. A charged pi meson of rest mass 140 MeV/c^2 comes to rest and then disintegrates into a charged muon of rest mass 106 MeV/c^2 and a neutrino of zero rest mass. What is the resulting kinetic energy of the muon?

1. The circuit below is a Wheatstone bridge. This type of circuit is used for measurement of resistance, inductance and capacitance. Derive the expression for the output voltage, V2, in terms of V1, R1, R2, R3 and the unknown resistance Rx.



2. Below is a Delta-star transformation. Find R1, R2, and R3 in terms of Ra, Rb, and Rc such that the resistances between points A, B, and C are the same.

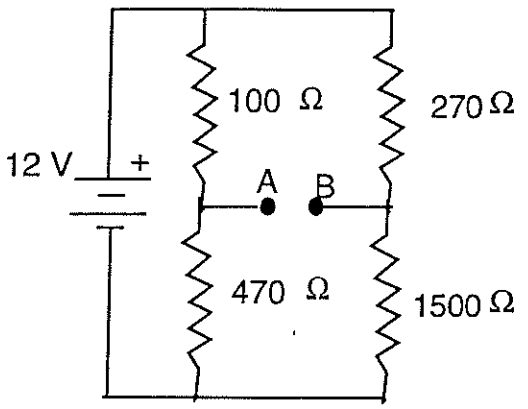


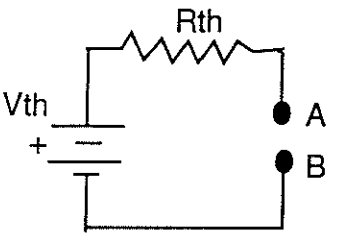


Show calculations for Vout waveforms:

Electronics

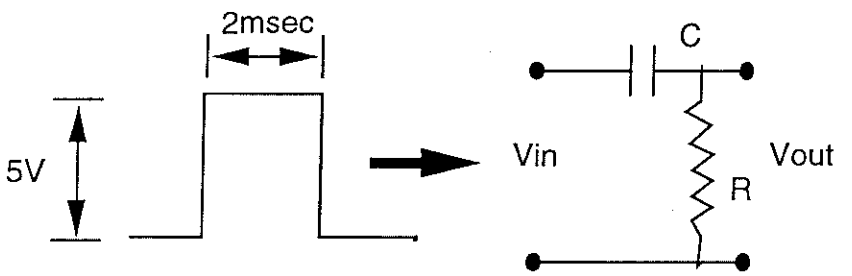
1. A. Convert the circuit shown below to its Thevenin equivalent as shown in the second figure. Show all calculations necessary to get the values of Vth and Rth for the equivalent circuit.





1.B. Use the Thevenin equivalent circuit to find the current in a 100 ohm resistor connected between A and B.

2. Consider the circuit and the input pulse shown below:



Draw the expected shape of the output pulse (Vout) below the input waveform (vin) on the attached graph paper and provide calculations to support your answer for the following two sets of values of C and R:

Case 1
R = 1 megohm
C = 1 microfarad

Case 2
R= 100 Kohms
C= 0.01 microfarad

Thermodynamics

$R = 8.315$ J/mol K
$1 \text{ atm} = 1.013 \times 10^5 \text{ N/m}^2$
$N_A = 6.023 \times 10^{23} \text{ mol}^{-1}$

1. An ideal gas expands from a volume V_1 to V_2. What work is done by the gas

   (a) At constant pressure?

   (b) From pressure P_1 to P_2 at constant temperature?

   (c) From P_1 to P_2 adiabatically?

2. a. How many molecules of He^4 gas are contained in $1 cm^3$ at 27 ^{0}C at a pressure of 10^{-3} mm Hg?

   b. What is their rms speed?

   c. How many atoms strike the walls per cm^2 per sec?

Optics

1. Glass optical components are often coated with thin films of transparent substances like MgF_2 (index of refraction 1.38) in order to reduce reflection from the glass. How thick a coating of MgF_2 is needed on a glass optical component of refractive index 1.50 in order to produce a minimum reflection with a wavelength of 550 nm?

2. If a single slit diffracts 550-nm light so that the diffraction maximum is 3 cm wide on a screen 1.5 m away, what will be the width of the diffraction maximum for light with a wavelength of 400 nm?