Motivation

- Lack of DM signals as of yet
- "Naturalness" of DM abundance doesn't necessarily mean naturalness of any other DM properties
- Why not DM at Planck scale, where we may expect new physics?

How to get such heavy DM?

- Only gravitationally coupled
  - Consequently, never in equilibrium with SM/dominant plasma \( \rightarrow \) Freeze-out

  - Such PI_DM can produce DM in mass range \( 10^3 \text{ GeV} < m_X < M_{GUT} \) if \( \text{TeV} \) is high enough

- Charged Hidden charged PI_DM, totally decoupled from SM (time allowing)

Goal: Determine what is required phenomenologically for such PI_DM to be a viable model
- Calculational Setup

\[ J = J_{\text{SM}} + J_{\text{DM}} + J_{\text{EN}} + \frac{1}{2m_p^2} W_{\mu} (T_{\mu}^{\text{SM}} + T_{\mu}^{\text{DM}}) \]

- Some global charge (model dependent) prevents DM-SM sector from coupling directly, forbidding DM decay.
- Non-self-interacting DM (for simplicity).

2-2 amplitude

\[ M = -i 8\pi <p_1|T_{\text{SM}}^{\nu}(p_2 \times k)|T_{\mu}^{\text{DM}}|0> \frac{1}{m_p^2 (k_1 + k_2)^2} \]

\[ w_1 (k_1 + k_2) \cdot T_{\mu}^{\text{DM}} = 0. \]

\[ M = -i \frac{8\pi G}{3} (T_{\mu}^{\text{SM}} T_{\mu}^{\text{DM}} - \frac{1}{2} T_{\mu}^{\text{SM}} T_{\mu}^{\text{DM}}), G \equiv \frac{1}{m_p^2} \]

\[ \text{traces} \]
\textbf{Boltzmann Eqns}

\[ \frac{\partial \rho_b}{\partial t} = -3H (1+w) \rho_b \frac{\Delta S}{\Delta t} \]
\[ \frac{\partial \rho_r}{\partial t} = -4H \rho_r + S + 2\langle \Delta S \rangle \langle \Delta E \rangle \left( n_x^2 - (n_x^S)^2 \right) \]
\[ \frac{\partial n_x}{\partial t} = -3H n_x - 2\langle \Delta S \rangle \left( n_x - (n_x^S)^2 \right) \]

- $\rho_b$ - Inflaton energy density
- $\rho_r$ - radiation
- $n_x$ - DM number density
- $S$ - Describes inflaton decay to SM

- Generally $S(t)$, $W(t)$ have complicated time dependence. Assume $S = \Gamma r \rho$, $\Gamma$ is const.
- $\Gamma_i$ - Hubble rate at end of inflation.
  - $\Gamma_i$ must be real.
  - $T_i$ - Fast reheating.
  - $\Gamma_i \gg H_i$.
- $\Gamma_i$ pert. $\Rightarrow \Gamma_i \ll H_i$.
- Define reheating temperature as $H_i \sim \Gamma_i$ by
  \[ T_R = k_B \left( m_p H_i \right)^{1/2} \]
- Thus, in turn, defines
  \[ Y = \sqrt{\frac{\Gamma_i}{H_i}} \left( \frac{\delta_{\rho}}{\delta_{\rho_0}} \right)^{1/4} e^{-\frac{3}{4} N_R} \frac{(1+\omega)}{} \]

$Y \in (0, 1)$; $Y \ll 1$ perfectly instant. RH.
- For freeze-in scenario to work, need $Y \gg 1$

\[ \sqrt{3 \gamma} = \left( \frac{45}{4 \pi^3} \right)^{1/4} \sim 0.25 \]
- $\gamma_{RH} = 3 \gamma_{SM} \sim 0.75 \text{ at RH.}$
- $N_{RH} \sim$ No. of e-folds

$\rho_{RH}$ = energy density at RH.

- $\Gamma_i \rho_0$ =UED (Inflaton)

\[ \rho_{RH} = \frac{16}{45} \left( \frac{m_p}{\omega} \right)^{3/2} \]

This resembles $\rho_{RH} \approx m_p^2$ approximately large.
For in Planck, assuming it dominates, \( \frac{p}{p} \sim a^{-3(1+w)} \). Thus holds until RH stops & normal plasma dominates.

\[ H = H_0 \left\{ \frac{(a/a_0)^{-3(1+w)/2}}{a/a_0} \right\} \left( \frac{a}{a_{\text{crit}}} \right)^2 \left( a_{\text{crit}} \right) \]

The 2nd eqn., when solved, yields \( T(a) \),

\[ T(a) = K_1 (Y_{\text{H}_0, H_0})^{1/2} \left( \frac{a - \frac{3(1+w)}{2}}{a - 4} \right)^{1/4} \]

needed for Abundance calculations.

Post-RH, \( T(a) = T_{\text{eq}} \frac{c_{\text{eqn}}}{a} \)

3rd eqn requires \( N_x^{eq} \) = \( \frac{g_1}{2\pi^2} \) \( M_x^2 T K_2 \left( \frac{m}{T} \right) \)

As \( T \) bound on \( H_0 \), just \( H_0 \ll H_{\text{crit}} \) is enough, and yields

\[ H_t < 6.6 \times 10^{-6} \quad M_p \left( \frac{r}{0.1} \right)^{1/2} \]

for the DDIM scenario.

**Abundance Calc.**

Rewrite: Define dimensionless abundance \( X = N_x a^3 / T^3 \)

\[ \frac{dX}{da} = \frac{a^2}{T^3 H(a)} \quad <c v> (N_x^{eq})^2 \]

Direct integration possible (if \( \bar{N}_x \) initial abundance remains)

\[ X = \frac{1}{T^3} \int a \quad \frac{a^2}{H(a)} \quad <c v> (N_x^{eq})^2 \]

Yields number density in form of \( M_x, H_0, Y, \& W \).
Resultant abundance is related to present-day abundance:
\[ \mathcal{Q} \propto k^2 = \mathcal{Q} \propto \frac{4}{m} \frac{M_x}{m_p} \frac{M_p}{S_\nu} \mathcal{O}, \quad \mathcal{Q} = \frac{1}{8} \frac{\mathcal{T}^2}{S_\nu} \mathcal{P}_c \]

- For definiteness, consider... [Scalar P IDM]

- Generally, \( \langle \sigma v \rangle = N_0 \langle \sigma v \rangle_0 + N_2 \langle \sigma v \rangle_2 + N_i \langle \sigma v \rangle_i \),

\[ N_0 = 4, \quad N_2 = 45, \quad N_i = 12 \text{ in SM} \]

As usual
\[ \langle \sigma v \rangle = \frac{1}{8 m_x^2 T K_1(m_x/T)^2} \int_0^\infty \frac{d^2 s}{s^2} (s-4m_x^2)^{1/2} \left[ 1 + K_1 \left( \frac{s}{T} \right) \right] \]

\[ \sigma_{TSN} = \frac{1}{16 s (s-4m_x^2)} \int_0^T ds \frac{1}{s} |N|^2 \]

with \( |N|^2 \) depending on P IDM spin.

\[ M = \frac{T_{\text{M}}}{\mathcal{M}} \left( T_{\text{N}} T_{\text{M}} - \frac{1}{2} T_{\text{N}} T_{\text{M}} T_{\text{TM}} \right) \]

\[ G = \frac{T_{\text{N}}}{T_{\text{M}}} - \text{traces} \]

- Skipping explicit lengthy calculations,

\[ \langle \sigma v \rangle_0 = \frac{\pi m_x^2}{8 m_p^4} \left[ \frac{5}{5} K_0^2 + \frac{2}{5} + \frac{4}{5} T \frac{K_1}{K_2} + \frac{8}{5} \frac{T^2}{m_x^2} \right] \]

\[ \langle \sigma v \rangle_2 = \frac{4 \pi T^2}{m_p^4} \left[ \frac{1}{15} \left( \frac{m_x^2}{T^2} \left( \frac{K_2^2}{K_1^2} - 1 \right) + \frac{3m_x^2 K_1}{T} \right) + 3 \right] \]

- Letting \( m_x \gg T \), \( v = 0 \), abundance can be calculated:

\[ X_4 = \frac{N_0 m_x^5}{8 \pi^2 m_p^4 T_\nu^3 H_i} \left[ \frac{1}{T_1} \int_0^{1/a} \frac{da}{a} \left( \frac{2m_x}{a} \frac{a}{T_1} e^{-\frac{a}{T_1}} + \frac{1}{2} \frac{a}{T_1} e^{-\frac{a}{T_1}} \right) \right] \]

\[ \text{am} \gg 1, \text{am} > \text{dark} : \]

\[ X_4 = \frac{N_0 m_x^5}{8 \pi^2 m_p^4 T_\nu^3 H_i} \left[ \frac{1}{T_1} \int_0^{1/a} \frac{da}{a} \left( \frac{2m_x}{a} \frac{a}{T_1} e^{-\frac{a}{T_1}} + \frac{1}{2} \frac{a}{T_1} e^{-\frac{a}{T_1}} \right) \right] \]

\[ g = \left( \frac{32.8}{512} \right)^{1/2} \times 10^{24} T_{\text{max}} = 0.4 \eta \mathcal{T}_\text{max} \]

\[ T_{\text{max}} \text{ is max veloc. T (ca) from before} \]
- For $y < 1$, $T_{\text{max}} > t_{\text{max}}$, so second term can be ignored, and we can solve for $H_i(m_i)$:

$$H_i(m_i) = \frac{4m_i^3}{1.2\gamma M_p^4} \left( -\gamma \right)^{-7/2} \frac{M_p^4}{m_i^2}$$

- If the argument of $W_1(x)$ is less than $-\nu$, no real solution exists. This leads to a restriction on model parameters:

$$\gamma^{7/8} M_X > 2.5 \times 10^{-6} M_p$$

- For small $M_X$, heavy PIDM approx breaks down.

- For large masses, we find a lower bound on RH efficiency $\gamma$ needed for this production mechanism to work.

$$\Rightarrow \text{Scalar vs. Fermion vs. Vector DM}$$

- $<\sigma v>$ for all 3 cases are very similar, only differing in terms of prefactors mainly on a single power of DM mass $M_X$.

- So numerical results don't vary greatly.

\[\text{Nonminimally Coupled PIDM}\]

$$L_{\text{ PIDM}} = \frac{1}{2} \left( g_{\phi} \phi^2 + g_X X^2 \right) R, \quad \phi - \text{SM scalar}\]

- Calculating $<\sigma v>$ in the relativistic limit:

$$<\sigma v> \approx \frac{g_{\phi}^2 m_{\phi}^2}{8} \left( 1 + 4 g_X \right) \left( 1 + 6 g_{\phi} \right)$$

- Effectively $m \rightarrow m \left( 1 + 4 g_X \right) \left( 1 + 6 g_{\phi} \right)$

- For small couplings, previous results still hold.

- Reduced cosmological constraints.
From numerical results (scalar), $M_x$
  can span a large range.
  Very sensitive to RH efficiency $\gamma$.
  If $\gamma \leq 10^{-3}$ ($N_y > 10$
  new e-fold),
  PIDM freeze-in is impossible.

- Higher PIDM masses are more favorable;
  for large $\gamma$, $M_x$ must possible.
- For given bound on $r$, $N_{\text{max}} = 0.023 \sqrt{r/2} r^{1/4} M_p$
  $\leq 25$ for $r \leq 0.07$, $N_{\text{max}} = 0.013 M_p$,
  decreasing for smaller $\gamma$.

If $R$ CMB exponents exclude tensor
  modes to $r \approx 10^{-4}$ or less, PIDM
  viability only significantly below
  the natural cutoff scale.

- Charged PIDM

- In light of similar results by Scalar/fermion/tree,
  we can consider Fermionic DM with only
  gravitational self interactions w/ SM. Also
  with a dark U(1) gauge symmetry; gauge
  born $\gamma_0$.

\[ Z_{\text{dm}} = -\frac{1}{4} V_{\text{dm}} V^{\dagger} + X: \Box X - m_{\text{dm}} X \]

\[ D_{\mu} = \partial_{\mu} - ig_{\text{dm}} V_{\text{dm}} \]

$\Rightarrow q_{\text{dm}} \propto \gamma^2 \gamma_0$ defines dark fine-structure
  constant $\gamma_0$.  

\( \approx 10^{-10} + 10^{-2} M_p \)

Note/reminder:
- used std. reheating
  setup w/ constant $T$ & $\omega$.
  Results could change for more
  general cases.

- Possible SM-DM
  couplings from
  quantum corrections.
  Loops involve gravitons,
  and it turns out
  any contributions from
  these diagrams that
  contribute to leptonic
  mixing go to 0.
For charged PDM, abundance can split into two parts:
1. Around reheating, Dark sector populated by freeze-out, predominantly
2. Subsequent evolution, dependent on whether Dark sector thermalizes. This may affect PDM abundance, depending on x_D.

This calculation is similar to scalar case, just with appropriate changes for fermions, and also accounting for Dark Photon $\gamma_D$ density $x_{\gamma_D}$.

Doing the integrals for $x_x \& x_{\gamma_D}$, we can find for abundances an initial density produced from freeze-out @ around $T_{\text{in}}$:

$$n_{\chi, x} \approx 0.27 \frac{T_{\text{in}}^6}{m_x^3}, \quad n_{\gamma_D, x} \approx 0.65 \frac{T_{\text{in}}^6}{m_{\gamma_D}}$$

Therefore smaller by a factor of $(T_{\text{in}}/m_p)^3$ than would be for an equilibrium dist. at the typical energy scale $T_{\text{in}}$.

Freeze-out gives non-thermal dist $\rho$ produces a Dark sector with momentum dist. $f_x \chi, f_{\gamma_D}(P)$ produced around $T_{\text{in}}$.

But inf. densities are smaller by $(T_{\text{in}}/m_p)^3 \Rightarrow$ Freeze-out produces an underpop. dist.
Dunk Sector won't come to thermal eq. with SM, but might equilibrate with itself.

- Omit detailed analysis. For critical values of \( \alpha_p \) which determines Dunk Sector phase.

\[
\alpha_{\text{crit, med}(a)} \approx 2 \times 10^{-3} \left( \frac{m_x}{100 \text{GeV}} \right)^{2/5} \left( \frac{10^{-4} \text{MeV}}{T_{\text{H}}(a)} \right)^{9/10} \\
\alpha_{\text{crit, med}(b)} \approx 5 \times 10^{-4} \left( \frac{m_x}{100 \text{GeV}} \right)^{1/2} \left( \frac{10^{-4} \text{MeV}}{T_{\text{H}}(b)} \right)^{9/8}
\]

- If \( \alpha_p \gg \alpha_{\text{crit}} \), max (\( \alpha_{\text{crit, med}(a)}, \alpha_{\text{crit, med}(b)} \))

\[\Rightarrow \text{Thermal} \rightarrow \text{Dunk Sector equil.}\]

- Even if \( \alpha_p \ll \alpha_{\text{crit}}, \) no Thermalization.

PIDM self-scattering processes ("Dunk Coulomb Scattering") can bring PIDM alone into kinetic eq.

\[\Rightarrow \text{Overall, Dunk Sector can evolve to an eq. dist. in both weak & strong exp.}\]

\( \alpha_p \ll \alpha_{\text{crit}} \) (weak):

\[
2^{-1/2} \geq 0.12 \left( \frac{m_x}{300 \text{GeV}} \right) \left( \frac{T_{\text{H}}}{6 \times 10^{-4} \text{MeV}} \right)^3
\]

\( \sim 0.064 \) bound \( \Rightarrow T_{\text{H}}/m_p \lesssim 6 \times 10^{-4} \).

Thus, plus \( \Re \rho^2 \approx 0.120 \Rightarrow m_x \gtrsim 400 \text{GeV} \)

\[\rho \text{ due to } \rho \text{ must (nearly) tend to zero, dominated by residual entropy production, } \Re \rho^2 \Rightarrow 8 \times 10^{-6} \text{ in very weak}\]

\( \alpha_p \approx \alpha_{\text{crit}} \) (strong):

- A bit more complicated, but essentially normal freeze-out except with \( T_0 \) (Dunk Sector temp.)

Freeze-out happens before kinetic decoupling for allowed \( T_{\text{H}} \).

Relic density leads to a bound on \( X_f \) (at freeze-out):

\[
X_f \lesssim 3 \times 10^{-5} \left( \frac{m_x}{10^{16} \text{GeV}} \right)^{1/2} \left( \frac{T_{\text{H}}}{10^{-4} \text{MeV}} \right)^{3/4}
\]

\[\Rightarrow X_f \lesssim 15 \]
- as an example, for $\Sigma = 0.5$, $52 \times 10^3$ is obtained.

- More specifically, $\Sigma \propto \left( \frac{\tau_{\nu \nu}}{m_{\nu}} \right)^{3/4} \ll 10^{-2}$

- $\Sigma \approx \Delta m_{\nu}$ is achievable only for $m_x \gg 10^4$ GeV.

- For cleaner/more in-depth treatment, see reference.

**What about Chosen GUT scale PIDM?**

- Assume self-annihilating DM not necessary to explain smallness.

- PIDM @ this mass are already non-viable when produced in SM processes.

$$ \langle \sigma v \rangle_{\text{ann}} \approx \frac{3 \alpha_f}{m_{\nu}} \frac{m_{\nu}}{m^3} \left( \frac{m_x}{m} \right)^2$$

- However, only if $M_\nu \ll \delta_{\text{ann}} / m_x$.

- Solving the usual way yields

$$ n_{\nu \nu} \approx 0.18 \frac{m_x^3 T_{\text{ann}}}{m_{\nu}^3} \exp\left[ -2m_x / T_{\text{ann}} \right]$$

- exp. suppressed compared to $\gamma_\nu$ density.

- No freeze out, but possible $\gamma_{\nu \nu} \rightarrow \gamma \gamma$ production. Company $\nu$ rates

$$ \Gamma_{\gamma \gamma \rightarrow \nu \nu} \approx \frac{1}{N_x} \left( \frac{n_{\gamma \nu}}{n_{\nu}} \right)^2 \left( \frac{n_{\gamma \nu}}{m_{\gamma}} \right)^2 \frac{\pi m_{\gamma}}{m_x} S_{\text{ann}} \left( \frac{m_x}{T} \right)$$

$$ \Gamma_{\text{ann}} \approx \frac{1}{N_x} \left( \frac{n_{\nu}}{m_{\nu}} \right)^2 \frac{2 \pi m_{\nu}}{m_x} S_{\text{ann}} \left( \frac{m_x}{T} \right)$$

$$ \Rightarrow \frac{\Gamma_{\gamma \gamma \rightarrow \nu \nu}}{\Gamma_{\text{ann}}} \sim \left( \frac{n_{\gamma \nu}}{n_{\nu}} \right)^2 \frac{m_{\gamma}}{2 \pi m_{\gamma}} \frac{m_{\gamma}}{m_x} \frac{m_x}{T_{\text{ann}}} \ll 1$$

- $\Gamma_{\gamma \gamma \rightarrow \nu \nu}$ excluded vs. gravitational, but

$$ \left( \frac{n_{\gamma \nu}}{n_{\nu}} \right)^2 \left( \frac{T_{\text{ann}}}{m_{\nu}} \right)^4 \ll 1.$$
- In Situ PIDM

- A few scenarios/models in which PIDM can easily or naturally be incorporated

1. Monodromy Inflation + PIDM (effective description)

- 4D monodromy potential obtained from compactification of 11D SUGRA via mixing of an axion-like particle w/ a 4-form from effective action:

\[ S_{\text{eff}} = \int d^4x \sqrt{-g} \left[ \frac{1}{16\pi^2} \mathcal{R} - \frac{1}{2} \mathcal{F}_{\mu
u} \mathcal{F}^{\mu
u} - \frac{1}{2} (\partial \phi \partial \phi') \phi' + \frac{1}{4} \phi \phi' \mathcal{F}_{\mu
u} \mathcal{F}^{\mu
u} \right] \]

- PIDM incorporated by adding scalar field with a mass \( M \sim O(M_{\mu}) \)

\[ S_{\text{PIDM}} = -\frac{1}{2} \int d^4x \sqrt{-g} \left[ \partial_{\mu} \phi \partial^{\mu} \phi + M^2 \phi^2 \right] \]

and RH mechanism via

\[ S_{\text{RH}} = \int d^4x \sqrt{-g} \frac{\mu^2}{\phi} G_{\mu\nu} G^{\mu\nu} \]

\[ S_{\text{eff}} = S_{\text{SM}} + S_{\text{PIDM}} + S_{\text{RH}} \]

- In the model, the 4-Run describes a membrane moving in 11D SUGRA. 4-form background breaks shift symmetry, provoking quadratically

\[ V_{\phi} = \frac{1}{2} (\dot{\phi} + \mu \phi^2) \]

valid for \( \frac{\mu}{\mathcal{M}} \ll \mathcal{M} / m \)

\( (\mu \sim O(10^3 \mathcal{M}_{\mu})^2 \mathcal{M}_{\mu} \ll \mathcal{M}_{\mu}) \)

\( \Rightarrow \) amplitude of density fluct. \( \delta \rho / \rho \sim 10^{-5} \)

\( \Rightarrow \frac{\mu}{m} \sim 10^{13} \Rightarrow M_{\mu} \sim 10^{16} \text{ GeV} \)

So \( M_{\mu} \sim M_{\mu} \) correctly naturally.
- Effective PIDM from grav. scattering requires high $T_{\text{IR}}$.

- Inflation ends when inflaton B6 value,

\[ \frac{\mu}{\kappa} + \phi < M_{\text{Pl}} \] 4 inflaton begins coalescing.

- Inflation - SM empty with $\mu \leq \phi \lesssim M_{1/2}$ needed to determine decay of inflaton to SM gauge sector:

\[ P = \frac{\mu^3}{8\pi f_\phi^2} \Rightarrow T_{\text{IR}} = \frac{16\pi^3}{(8\pi)^4} \frac{\mu^2}{f_\phi} \]

\[ \Rightarrow \text{Thus, an efficiency parameter} \]

\[ \gamma = \frac{1}{16\pi} \frac{f_\phi}{\mu} \]

- From prev. parts, $m_X \sim M_{1/2} \sim M_{\text{GUT}}$ means

\[ \gamma \lesssim 0.1 \Rightarrow f_\phi \mu \sim 10^{13} \text{ GeV.} \]

(alternatively, can lower $m_X$ to $M_X \sim \mu$)

\[ \text{in turn requires } f_\phi \sim M_{1/2} \sim M_{\text{GUT}} \]

2. Higgs Inflation PIDM

- Higgs (SM) as the inflaton.

\[ L = \left( \frac{1}{16\pi^2} m_\phi^2 + \frac{1}{2} \partial^a H \partial^a H \right) + g^a \left( D^a H \right)^2 + \lambda \left( \frac{v^2}{2} \right)^2 \]

\( \Rightarrow \) also add the asthmatic SPIDM from noninflation.

- Critical vs. noncritical Higgs inf?!

NonCrit. inf. occurs on plateaus of STStab. inf. form

\[ V_{\text{IR}} = \frac{\lambda m_\phi^4}{256\pi^2 v^2} \left( 1 + e^{-\frac{v^2}{\tilde{m}_\phi^2}} \right) \]

\( \Rightarrow \) $\lambda \sim O(1)$, it turns out $\tilde{m}_\phi \sim 10^4$, and

\[ r = 16 \times 10^{-5}, \quad e = 3/4 \nu^2_0 \]

\( \Rightarrow \) Pert. RH \( \Rightarrow T_{\text{IR}} \sim 10^{14} \)

\( \Rightarrow \) with $v \sim 3 \times 10^{-5} \Rightarrow \gamma \sim \frac{\mu v^2}{10^{-5} \mu^2}$
- Acting scale if \( m_x \sim 10^{-5} \mu_p \) warrants a
  connection/compatibility to leptogenesis, as
  \( m_x \sim RHN \) mass scale

Crit.: \( m_\nu \& m_\tau \) are finely tuned s.t.
  second vacuum vanishes & being
  an inflection point of potential
  that can be used for inflation.
- No inf. @ plateau
- \( \phi \sim V_{N^2} \) broken
- \( \nu \) large is possible for
  \( g \sim 10 \) and \( T_{\nu} \ll \text{GUT scale} \)
  \( \Rightarrow m_x \sim M_{\text{GUT}} \), PIDM possible.