University of Alabama Department of Physics & Astronomy
Graduate Qualifying Exam
Part 1: Classical Mechanics

7 January 2020

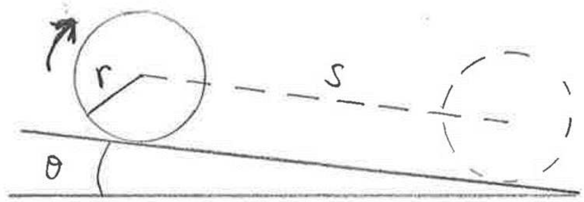
General Instructions

- Do any 5 of the 6 questions. Indicate clearly which 5 questions that you wish to have graded. Each question is worth 20 points.
- 150 minutes are allocated for this exam.
- No reference materials are allowed.
- Do all your work in the corresponding answer booklet (no scratch paper is allowed).
- On the cover of each answer booklet put only your assigned number and the subject - do **not** write your name.
- Turn in this question sheet with your answer booklet.
- **No electronic devices are allowed. In particular, cell phones, handheld computers, and PDAs are explicitly prohibited.**

1. (a) **(4 points)** Show by explicit calculation that the rotational inertia of a solid cylinder of uniform density, mass m, radius r and length h, being rotated around an axis through its center along the length of the cylinder, is given by $\frac{1}{2}mr^2$.

   (b) **(4 points)** Show by explicit calculation that the rotational inertia of a solid sphere of uniform density, mass m and radius r, being rotated around any axis through its center, is given by $\frac{2}{5}mr^2$.

   (c) **(12 points)** If t is the time it takes a solid *cylinder* of mass m and radius r to roll a distance s along an inclined plane without slipping, find the time it takes for a solid *sphere* of the same mass and radius to roll the same distance along the plane without slipping. Assume that both objects are initially at rest, their mass densities are uniform and that the plane is inclined an angle θ from the horizontal, as shown below.



2. Consider a particle of mass m moving in 1 dimension along the x-axis, under the influence of a velocity-dependent drag force that opposes the motion, $F(v) = -cv^{3/2}$, where c is a drag coefficient. The initial velocity of the particle is v_0.

   (a) **(12 points)** Find the velocity of this particle as a function of time, $v(t)$.

   (b) **(8 points)** Find the position of the particle as a function of time, $x(t)$, assuming that it starts at $x = 0$ when $t = 0$. Show that the maximum distance moved by the particle is given by $2m\sqrt{v_0}/c$.

3. Consider the following potential $U(r)$ as a function of the radial distance r from the origin:

$$U(r) = A[(e^{(R-r)/s} - 1)^2 - 1]$$

where the parameters $R, s > 0$ and also $r > 0$.

   (a) **(8 points)** Find the position of any points of equilibrium

   (b) **(7 points)** Determine whether the equilibrium points are stable or unstable. (Explain your answer.)

   (c) **(5 points)** If any equilibrium position is stable, determine the frequency of small oscillations about that equilibrium

4. Consider a particle of mass m moving in two dimensions whose motion is governed by the following Lagrangian:

$$L(x, y, \dot{x}, \dot{y}) = \frac{m}{2}(\dot{x}^2 - \dot{y}^2) - \frac{k}{2}(1 + a\dot{y})x^2 + \frac{k}{2}(1 + a\dot{x})y^2 - kaxy(\dot{x} - \dot{y})$$

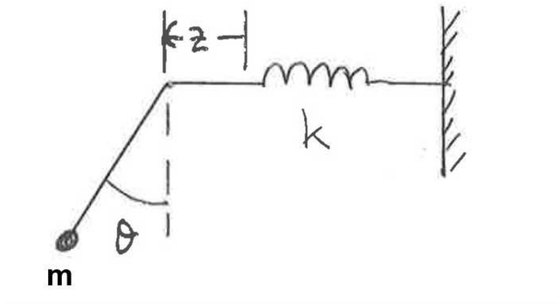
where the parameters k and a are constants.

   (a) **(10 points)** Find and simplify the associated Euler-Lagrange equations of motion

   (b) **(7 points)** Solve these equations and obtain the most general solution for the motion of the particle

   (c) **(3 points)** Do the solutions depend on the parameter a? If not, why not?

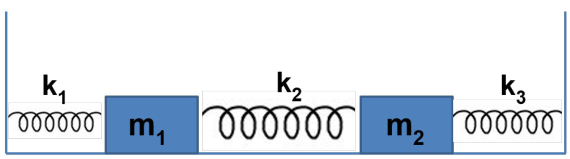
5. As shown in the diagram below, a pendulum is made from a massless rigid rod of length ℓ with a mass m at one end. The other end is attached to the end of a spring of spring constant k that is free to move horizontally. Let z be the displacement from equilibrium of the end of the spring along the horizontal direction, and θ be the angle of the pendulum relative to the vertical.



   (a) **(10 points)** Find the Euler-Lagrange equations of motion for z and θ.

   (b) **(10 points)** Solve the equations of motion in the limit that z and θ (and their time derivatives) are small, and find the common frequency of oscillation of the system.

6. Consider two masses m_1 and m_2 connected to 3 springs with spring constants k_1,k_2 and k_3, as shown in the diagram below. Let x_1 and x_2 be the displacements from equilibrium of m_1 and m_2 respectively.



   (a) **(8 points)** Find the equations of motion for m_1 and m_2 in terms of x_1 and x_2 and the other parameters.

   (b) **(12 points)** Now consider the specific case where the masses are equal ($m_1 = m_2 = m$), and the two outer springs have the same spring constant, but the middle spring is different ($k_1 = k_3 = k \neq k_2$). Find the solutions for this case where both masses oscillate with a common frequency ω, and find the possible values of ω. Describe briefly each obtained oscillation mode.

University of Alabama Department of Physics & Astronomy
Graduate Qualifying Exam
Part 2: Electricity & Magnetism

8 January 2020

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1. Consider a flat circular disk of radius R, carrying a uniform surface charge density σ. The disk sits in the xy-plane, and the z-axis extends out from the center, perpendicular to the disk.

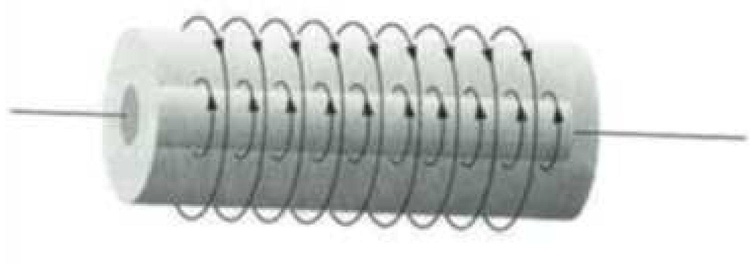
   (a) **(14 points)** Find the electric field $E(z)$ at a distance z above the center of the disk

   (b) **(4 points)** Consider the case when $z >> R$. Find the electric field $E(z)$ in this case by taking the limit of your general expression, and give a brief physical explanation of the result you obtain.

   (c) **(2 points)** Now consider the alternative limit where $R \to \infty$ and determine the electric field $E(z)$ in this case.

2. **(20 points)** Consider two long concentric cylinders. The inner cylinder has radius a and carries charge distributed uniformly throughout its *volume*, with volume charge density ρ. The outer cylinder has radius b and carries a uniform *surface* charge density σ. There is a vacuum between the two cylinders. The volume and surface charges are of opposite sign, and the magnitudes are adjusted so that the combined system is electrically neutral. For this problem, consider r to be the radial distance from the central axis of the cylinders, and find the electric field as a function of r, $E(r)$, in each of the three regions: $r < a$, $a < r < b$, and $r > b$.

3. **(20 points)** Two long coaxial solenoid each carry the same current I, but flowing in opposite winding directions. The inner solenoid has radius a and has n_1 turns per unit length, while the outer solenoid has radius b and n_2 turns per unit length. Find the magnetic field in each of the three regions: inside the inner solenoid; between the two solenoids; and outside the outer solenoid.



4. Two same-sign charged particles q_1 and q_2 are located respectively at $(x, y) = (0, 0)$ and $(r, 0)$ in some inertial reference frame $\mathcal{O}$.

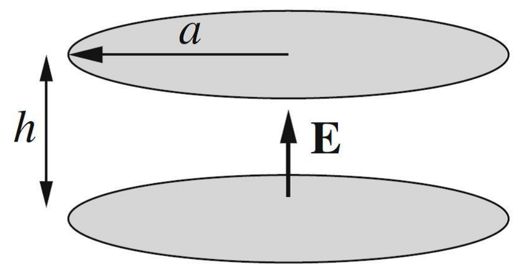
   (a) **(7 points)** What is the total electromagnetic force on q_2 due to q_1 *as seen in a reference frame* $\mathcal{O}'$ *that is moving with constant velocity* $(0, -v)$ *with respect to* $\mathcal{O}$?

   (b) **(7 points)** Say that in frame $\mathcal{O}$, q_1 is held fixed, while q_2, initially at rest, is allowed to move in response to the electromagnetic force from q_1. How long does it take q_2 to get to $x = 2r$ as seen in frame $\mathcal{O}'$?

   (c) **(6 points)** Compare this with the time it take as seen in frame $\mathcal{O}$? Use

$$\int_r^{2r} \frac{dx}{\sqrt{\frac{1}{r} - \frac{1}{x}}} \approx 2.296\, r^{3/2}$$

5. As shown in the figure below, a plane capacitor consists of two parallel circular plates of radius a, a distance $h \ll a$ from each other. The electric field inside the capacitor is slowly varying in time, $\vec{E} = E(t)\hat{\mathbf{z}} = \frac{E_0 t}{\tau}\hat{\mathbf{z}}$. Boundary effects are negligible.



   (a) **(9 points)** Evaluate the magnetic field $\vec{B}$ inside the capacitor.

   (b) **(11 points)** Calculate the Poynting vector $\vec{S}$, and show that the flux of $\vec{S}$ through any surface enclosing the capacitor equals the time variation of the energy associated to the electromagnetic field.

6. Consider an electric field given by:

$$\vec{E} = (\alpha \sin \frac{\omega x_3}{c} \cos \omega t\,,\ \beta \cos \omega t\,, \gamma\, e^{-\kappa x_1} \cos \omega t\ )$$

in some region of space [with Cartesian coordinates (x_1, x_2, x_3)]

   (a) **(8 points)** Find the magnetic field $\vec{B}$. Assume that $\vec{B}$ vanishes everywhere in the region when $t = 0$.

   (b) **(4 points)** Find the charge density in the region.

   (c) **(8 points)** Find the current density in the region.