# HEP Presentation Notes

Markus Garbiso 9/13/2020

- Motivation and Why use symmetry
- Symmetry in General Relativity
    - How to express Symmetry
    - Example (Flat Space)
    - Our Case(Spherical -> then twist it)
        - Parity (Discrete)
- Application (Holographic Fluid)
    - AdS/CFT of Rotating Fluids (MPAdS5D <-> Rotating Strongly Coupled Fluid)
    - Hydrodynamics
        - Decomposition of perturbations
        - Large Temperature Limit
        - Rotating Fluid (Dispersion Relations)
    - Suppression of a direction’s Modes
        - Suppression of momentum at extremality.

# Outline

We consider spinning black holes in five-dimensional spacetime which is asymptotic to Anti-de Sitter space (AdS). We describe in which way this gravitational setup is holographically dual [1] to a spinning quantum fluid with non zero vorticity and angular momentum.

Such a hydrodynamic description is relevant for the understanding of systems such as the quark gluon plasma (QGP) generated in heavy ion collisions. [Janik et al 2005]

We confirmed that our in the large temperature limit our “dual” is a strongly coupled relativistic viscous fluid.

Using symmetry arguments we can reduce the PDE equations to calculate hydrodynamic quantities to ODEs!

## Motivation and Why use symmetry

A space-time  symmetry is a property of the geometry (geodesics, metric, curvature tensor, etc...) that invariant under a transformation (affine transformations, rotations, boasts, etc). For us we will look at transformations that leave the metric invariant ie transformation generated by killing vector fields (KVF).

$$\phi(g) \equiv (\phi^* g)_{\mu\nu} dx^\mu dx^\mu = g_{\mu\nu} (\phi^{*-1} dx)^\mu (\phi^{*-1} dx)^\mu$$

We usual are interested in transformations that be parameterized.

$\phi(g) \to \phi_\tau(g)$, where this can be differentiated

$$\lim_{\tau \to 0} \left( \frac{\phi_\tau(g) - \phi_0(g)}{\tau} \right) =: \mathcal{L}_X g \overset{\text{sym}}{=} 0, \text{ for X a KVF}$$

$$\frac{df(x(\tau))}{dt} = X(f(x(\tau))) \text{ , for some scalar } f$$

## Symmetry in General Relativity: How to express Symmetry

$$g = \eta, \text{ for example, flat space.}$$

$$X, Y \in \mathfrak{iso}(3,1), \text{ the Poincaré Lie algebra}$$

$$\mathcal{L}_X Y = [X, Y], \text{ the Lie Bracket}$$

$$\text{differential translations: } \mathcal{P}_\mu \text{ where} [\mathcal{P}_\mu, \mathcal{P}_\nu] = 0$$

This implies that the metric can be written in terms of a holonomic basis where it's components wrt this basis are constant. Ex) Time and space translations are included (write the expression of lie derivaive)

Symmetry in General Relativity: Example (Flat Space)

Our blackhole's symmetry group $= \mathbb{R}_t \times U(2) \sim \mathbb{R}_t \times SO(2) \times SO(3)$

$$\Rightarrow g \subset f_{\Omega_2} d\Omega_2^2, \text{ from } \mathfrak{so}(3)$$

$$\Rightarrow g \subset f_{\Omega_1} d\psi^2 + g_i dx^i d\psi, \text{ from } \mathfrak{so}(2)$$

$$\Rightarrow g \subset f_{\Omega_1} dt^2 + g_i dx^i dt, \text{ from } \mathcal{P}_t$$

By **Parity** we have the further constraints.

$$(t, \psi) \to (-t, -\psi)$$

$$dt dx^i = 0 \quad d\psi dx^i = 0, \forall dx^i \neq dt \vee dx^i \neq d\psi$$

Summarize part 1 if out of time or to extend if going to fast.

Using the symmetry can help constrain the form of our metric so in our case we only have to solve ODE instead PDEs which further decouple into simpler sets of Diff Eqs.

Emergency (Get Point Across) Let's Summarize

$$S = \kappa \int (R - 2\Lambda), \text{ Einstein-Hilbert in AdS}$$

$$ds^2 = -\left(1 + \frac{r^2}{L^2}\right) dt^2 + \frac{dr^2}{G(r)} + \frac{r^2}{4}\left(4\sigma^+\sigma^- + (\sigma^3)^2\right) + \frac{2\mu}{r^2}\left(dt + \frac{a}{2}\sigma^3\right)^2$$, our rotating metri

[Pope et al 2004]

$\mu =$ Mass Parameter, $a =$ Angular Momentum Parameter [Murata 2009]

$r_+ =$ outer horizon radius, $L =$ AdS Radius

$$G(r) = 1 + \frac{r^2}{L^2} - \frac{2\mu(1 - a^2/L^2)}{r^2} + \frac{2\mu a^2}{r^4}, \qquad \mu = \frac{r_+^4 (L^2 + r_+^2)}{2L^2 r_+^2 - 2a^2 (L^2 + r_+^2)},$$

$G(r_+) = G(r_-) = 0$, extremal when $r_- = r_+$

With this metric that is AAdS, we may plausibly infer that a boundary field theory lives there according to the AdS/CFT correspondence.

$$g^{p}_{\mu\nu}dx^{\mu}dx^{\nu} = (g_{\mu\nu} + \epsilon\, h_{\mu\nu} + O(\epsilon^2))dx^{\mu}dx^{\nu} \,,$$

$$h^{(\mathcal{K})}_{\mu\nu} = \sum_{\mathcal{K}} h^{(\mathcal{K})}_{\mu\nu}(x^{\lambda}) = \sum_{\mathcal{K}=-(\mathcal{J}+2)}^{\mathcal{J}+2} h^{(\mathcal{K})}_{ab}\, \sigma^{a}_{\mu}\sigma^{b}_{\nu} D^{\mathcal{J}}_{\mathcal{K}-Q(\sigma^a)-Q(\sigma^b)} \,,$$

$$Q(\sigma^a) := \begin{cases} 0 & a = r, t, 3 \\ 1 & a = + \\ -1 & a = - \end{cases}$$

Perturbations of different K and J's decouple from each other!!!

We can now solve for omega, the frequency such that the boundary value of the perturbation zero. This defines our QNMs which we will analyze to understand the hydrodynamics.

## Application (Holographic Fluid): Decomposition of perturbations

In the large black hole phase, $T \propto r_+$

Starting with the equations of motion for the perturbations, $\mathcal{J}, \mathcal{K}, r_+/L \to \infty$

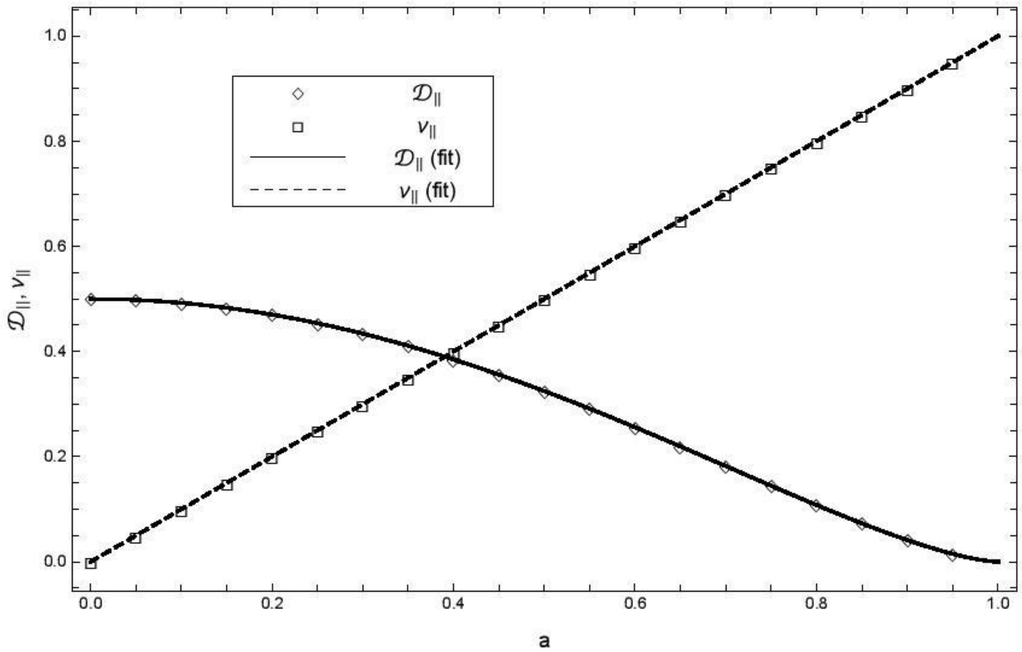
Keep the leading order terms, in terms of a limiting dummy variable. The resulting equations of motion decouple based on the charge of the fluctuation fields' basis.

Now we can easily calculate the quasinormal modes and two-point functions of these modes.

The quasinormal modes correspond to the poles of retarted 2pt green's functions.

$$\langle TT \rangle \propto 1/(\nu - v_s j + i\Gamma j^2)$$

## Application (Holographic Fluid): Large Temperature Limit

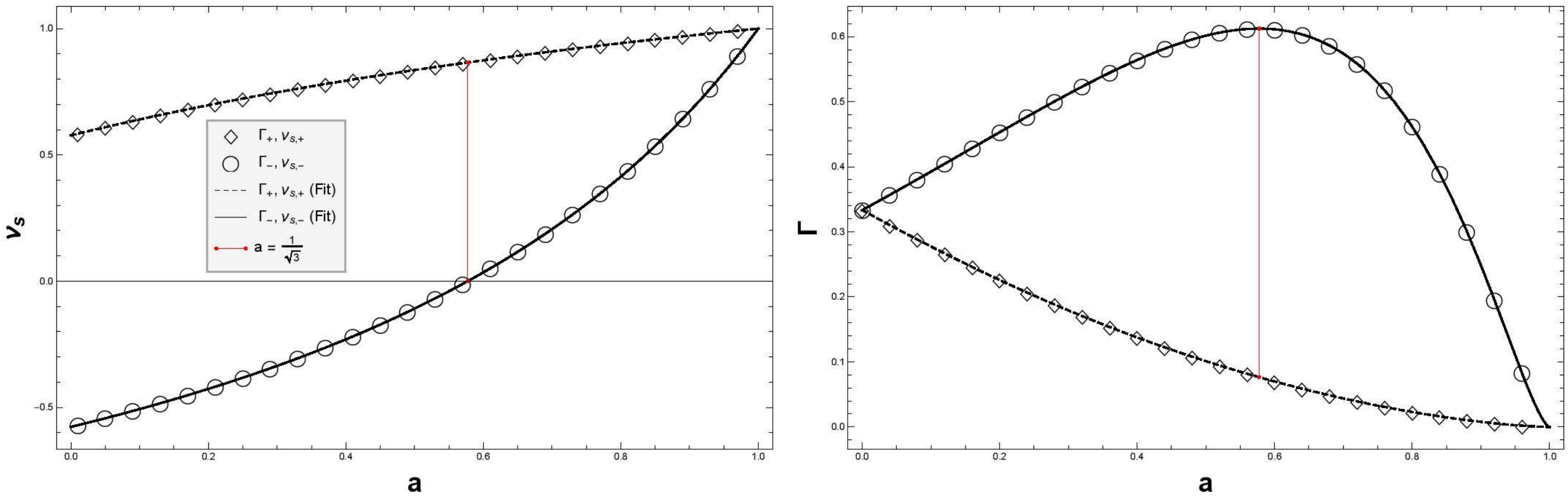



$$\nu = v_{||}(a)j - i\mathcal{D}_{||}(a)j^2$$

$$v_{||} = a$$

$$\mathcal{D}_{||}(a) = \frac{1}{2}(1-a^2)^{3/2}$$

# Application (Holographic Fluid): Rotating Fluid (Dispersion Relations)

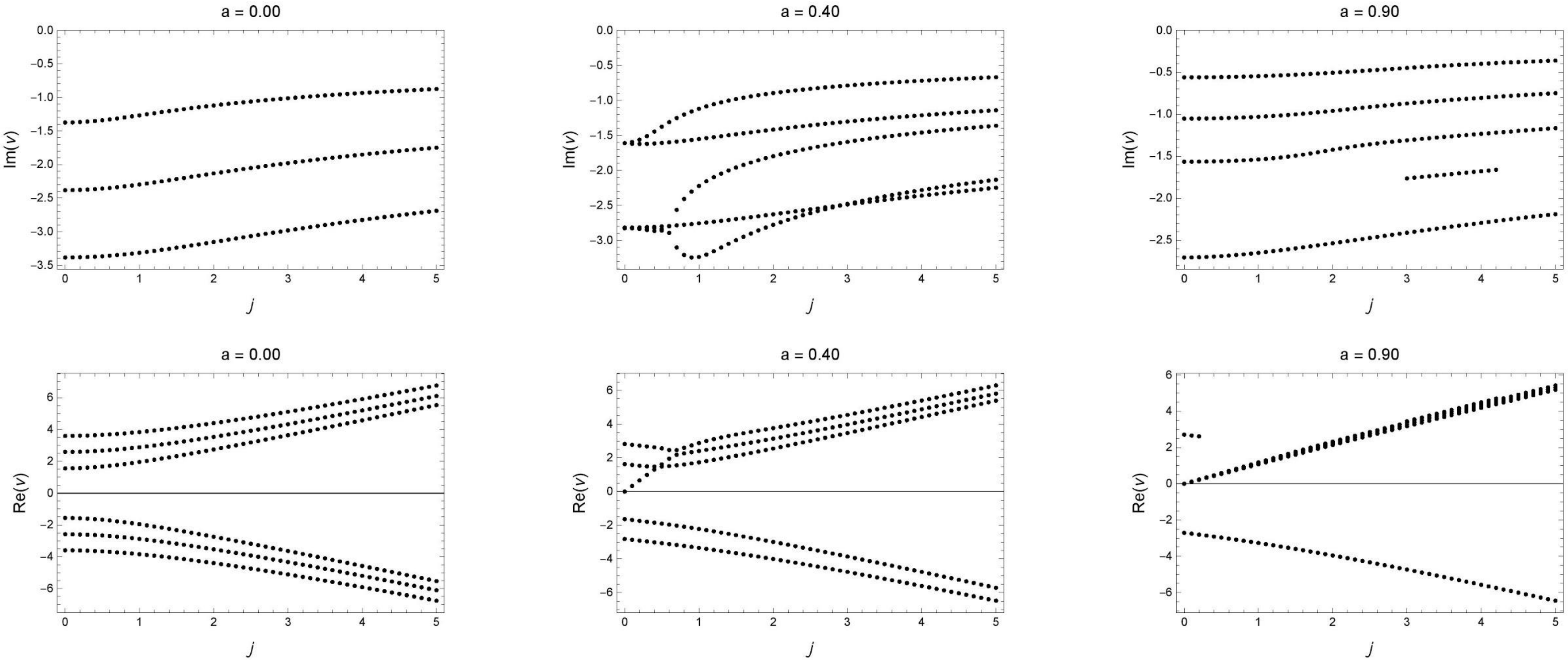



$$\nu = v_{s,\pm} j - i\Gamma_{s,\pm} j^2 \,,$$

$$v_{s,\pm} = \frac{a \pm \frac{1}{\sqrt{3}}}{1 \pm \frac{a}{\sqrt{3}}} \,,$$

$$\Gamma_{s,\pm} = \frac{1}{3} \frac{\left(1 - a^2\right)^{3/2}}{\left(1 \pm \frac{a}{\sqrt{3}}\right)^3} \,,$$

# Application (Holographic Fluid): Rotating Fluid (Dispersion Relations)




# Non-Hydromodes going towards extremality

- Symmetry is useful!
- We can make a 5D black hole in AdS that is dual to a fluid on the AdS’s Boundary.
- There are potentially some interesting mode at extremality for the Tensor equation of motion.

# Conclusion