A function of soft W6 boson processes is uniquely determined by the low-energy behavior. This is an example of how symmetry helps to simplify the problem. A popular approach is the Standard Model, in which the fermions are constructed in terms of spinorial bosons. The spinors transform under a group $G$. 

- G$\mathbb{H}$ angle compact groups. 
- G non-abelian extensions to $G$. 
- let $T^a$ be the generators of $G$. 
- Split into two parts: $T_{ij} = x^i x^j$. 
- $T_{ij}$ is the algebra of $G$. 
- The Lie algebra of $G$. 

- Let $G/N$ be representing of the left and right: $G/N$: 

  $gN = g'N$. 

- Right multiplication by $g'$ is a left group, which can be uniquely decomposed into a right part and a left null part $L^2$: 
  $T_{ij} = L^2 g g' N$.

- Then, the right mass is equal to the mass of the left.
Amplitudes of soft NG boson processes are uniquely determined by the low-energy theorems.

2) Since low-energy theorems apply pretty universally to systems with G symmetry broken to H, a low energy effective Lagrangian for the symmetry can make it easier to read off soft NG boson amplitudes.

A popular approach is the nonlinear Sigma Model, in which $\phi$ is constructed in terms of nonlinearly dressing NG bosons alone.

- $G \times H$ simple compact groups.
- $G$ spontaneously broken to $H$ subgroup $H$.

Let $T^a$ be the generators of $G$, split into two parts:

$$2\{T^a, e^{i\vphi} x^a(y)\}, x^a \in \Omega$$

Solving $T_a(\sigma_{Im}) = 0$, $T.a(\sigma_{Re}) = 0$.

- All NG bosons with number equal to $\dim(G/H) = \dim G - \dim H$.

NG bosons transform linearly under $H$, but not under $G$.

- Let $\phi^a$ be "representatives" of the left coset space $G/H$:

$$\phi^a = e^{i\phi^a x_a(y)} = \sum_{a \in \Omega} T^a \phi^a$$

Right multiplication by $g^a \in G$ yields $g^a \phi^a g^{-a}$, which can be uniquely decomposed into a constant part $\phi^a$ and an unbroken part $L^a e^{iH}$:

$$g^a = L^a (\phi^a) g^a e^{iH}$$

Thus, the nonlinear twist of $\phi^a$ (or $\phi^a$) under $g^a \in G$ is

$$\phi^a \to \phi^a = k^a \phi^a g^a e^{iH} \quad g^a \in G$$

- Model based on nonlinear $G/H$.

First cut, introduce the nonlinear $G/H$ model in a partonic way from CCWZ (chekhlov el al).

- Script letter denotes the algebra of $G$ groups.

- Unitary Nonlinear rep of $G$, renormalized instanton of $G$, $\phi^a$ decay constant.
A fundamental obj. to be used in the Nauer-Center (form constructs) is
\( \alpha (n) = \alpha (n) \cdot \xi (n) \).

It is expandable in terms of \( \xi (n) \), so we can define the II and I components of \( \alpha (n) \) for \( \xi (n) \):

\[
\begin{align*}
\alpha_{II} (n) &= (2 + \xi (n) \cdot \xi (n)) \cdot \xi (n) \\
\alpha_{I} (n) &= (2 + \xi (n) \cdot \xi (n)) \cdot \xi (n) - \xi (n)
\end{align*}
\]

The transform law of \( \alpha (n) \) can be found from third of \( \xi (n) \):

\[
\begin{align*}
\alpha_{II} (n) - \alpha_{I} (n) &= (2 + \xi (n) \cdot \xi (n)) \cdot \xi (n) \\
\alpha_{I} (n) &= (2 + \xi (n) \cdot \xi (n)) \cdot \xi (n) - \xi (n)
\end{align*}
\]

As \( \xi (n) \cdot \xi (n) \), we get

\[
\begin{align*}
\alpha_{II} (n) \rightarrow \alpha_{II} (n) &= h(n) \cdot \alpha_{II} (n) \cdot \xi (n) \\
\alpha_{I} (n) &= (2 + \xi (n) \cdot \xi (n)) \cdot \xi (n)
\end{align*}
\]

Since only \( \alpha (n) \) transforms homogeneously, we can construct Gr-invariants from \( \alpha (n) \) above: \( E(\alpha (n)) \).

Thus, the most general \( Z \) made up of \( \alpha (n) \) within the smallest \# of dependent is

\[
Z_{\text{core}} = \sum_{n} \tau_{n}^{2} \cdot (\alpha (n))^{2}
\]

Save \( Z \) as formed by CCMZ.
\( \tau \) since to normalize too kinetic forms.
CCMZ call \( \alpha (n) \) the "Constant Masses."
Goal: Write \( \mathcal{F} \) in terms of U(G) fields.

Author uses the formula:

\[
\Delta_{m}(e^{\frac{\Delta m}{2} \mathcal{H}} e^{-\frac{\Delta m}{2} \mathcal{H}}) = \left[ e^{N_{m} H} - 1 \right] \mathcal{H}_{m} \mathcal{H}.
\]

If \( G/H \) is symmetric space, then for the terms in the expansion:

* even \( \rightarrow \) belonging to \( H \)
* odd \( \rightarrow \) belonging to \( G - H \)

and the proper part becomes:

\[
\Delta_{m}(e^{\frac{\Delta m}{2} \mathcal{H}} e^{-\frac{\Delta m}{2} \mathcal{H}}) = \frac{1}{2} \left[ \mathcal{H}_{m} + \frac{1}{2} \left( \mathcal{H}_{m}^{2} \right) \right] + \mathcal{H}_{m} \mathcal{H}.
\]

\[
\Rightarrow \text{trace} \left| \Delta_{m}(e^{\frac{\Delta m}{2} \mathcal{H}} e^{-\frac{\Delta m}{2} \mathcal{H}}) \right| = \text{trace} \left| \frac{1}{2} \left( \mathcal{H}_{m} + \frac{1}{2} \left( \mathcal{H}_{m}^{2} \right) \right) \right| + \mathcal{H}_{m} \mathcal{H}.
\]

To incorporate matter fields:

\( X(\alpha) \) introduced as a linear rep., \( \Omega \) of \( H \):

\[
X \rightarrow X' = \rho_{\alpha}(h)X \quad \text{under } h \in H.
\]

Define the transformation of \( X(\alpha) \) under \( \rho_{\alpha} \):

\[
X(\alpha) \rightarrow X'(\alpha) = \rho_{\alpha}(h_{(\alpha,\alpha)})X(\alpha).
\]

Now we can convert \( X(\alpha) \) into a linear representation field \( Y_{\alpha} \) by using any representation \( \rho_{\alpha} \) of \( G \) whose restriction to \( H \) contains \( \rho_{\alpha} \).

\[
Y_{\alpha}(\alpha) = \rho(\alpha) X(\alpha).
\]

Note limits of \( \Omega \) transformations:

\[
Y \rightarrow Y' = \rho(\alpha) X = \rho(\alpha \xi \xi^{-1} h_{(\alpha,\alpha)}) \rho(h_{(\alpha,\alpha)}) X = \rho(\alpha) \rho(\xi) X = \rho(\alpha) Y.
\]

\[
\triangledown \mathcal{F} \rho(\alpha)
\]
- From this, we can construct commutators for another fields. From the Anti-Permutation Law of $\alpha_{\mu}(x)$ and $\alpha_{\nu}(x)$, one finds the following commutator between the $G = \text{SU}(2)$:

$$\bar{x} \times \bar{y} = \bar{y} \times \bar{x}$$

$$\bar{x} \times \alpha_{\mu}(x) \times (\bar{x} \times \alpha_{\nu}(x)) = \bar{y} \times \alpha_{\mu}(x) \times (\bar{y} \times \alpha_{\nu}(x))$$

$$\bar{x} \times \bar{y} \times [\bar{\alpha}_{\mu}(x) \times \bar{\alpha}_{\nu}(x)] = \bar{y} \times \bar{x} \times [\bar{\alpha}_{\mu}(x) \times \bar{\alpha}_{\nu}(x)]$$

The commutator $\mathcal{Z}$ is given then by:

$$\mathcal{Z} = \bar{x} \times \bar{y} \times [\bar{\alpha}_{\mu}(x) \times \bar{\alpha}_{\nu}(x)] = \bar{y} \times \bar{x} \times \bar{\alpha}_{\mu}(x) \times \bar{\alpha}_{\nu}(x)$$

- External Gauge Fields:

Ex. Gauge field $A_{\mu}(x)$, gauge group $\text{SU}(2)$. According to this requires replacing $\bar{x} \times \bar{y}$ in the Maurer-Cartan forms with the commutator $\mathcal{Z}$:

$$\bar{x} \times \bar{y} = \bar{y} \times \bar{x}$$

$$A_{\mu}(x) = \alpha_{\mu}(x) + \frac{1}{2} \mathcal{Z}$$

$$\frac{d}{dt} \bar{u} = \frac{d}{dt} [\bar{x}, \bar{y}] = \frac{1}{2} \mathcal{Z} \left[ \frac{d}{dt}, \bar{x}, \bar{y} \right] = \frac{1}{8} \mathcal{Z}^{\bar{\n}} \left[ \frac{d}{dt}, \bar{x}, \bar{y} \right]$$

we then replace the $\mathcal{Z}$ terms from before with:

$$\bar{Z}_{\mu}(x) = \alpha_{\mu}(x) \times \alpha_{\nu}(x)$$

$$\bar{Z}_{\mu} = \bar{Z} \left[ \bar{x}, \bar{y} \right]$$

$$\bar{Z}_{\mu} = \bar{Z} \left[ \bar{x}, \bar{y} \right]$$

If $Z/H$ is symmetric, the components of $\bar{Z}_{\mu}$ can be expanded & split into even & odd parts. Non-Abelian vectors:

$$\bar{x} \bar{y} = \frac{1}{2} \left[ \bar{x}, \bar{y} \right] = \frac{1}{2} \left[ \bar{x}, \bar{y} \right]$$

- Assume $\bar{x} \bar{y} \bar{z}$ leading to the fields:

$$\bar{x} \times \bar{y} = \bar{y} \times \bar{x}$$

$$\bar{x} \times \bar{y} \times [\bar{\alpha}_{\mu}(x) \times \bar{\alpha}_{\nu}(x)] = \bar{y} \times \bar{x} \times [\bar{\alpha}_{\mu}(x) \times \bar{\alpha}_{\nu}(x)]$$

$$\mathcal{Z} = \bar{x} \times \bar{y} \times [\bar{\alpha}_{\mu}(x) \times \bar{\alpha}_{\nu}(x)] = \bar{y} \times \bar{x} \times [\bar{\alpha}_{\mu}(x) \times \bar{\alpha}_{\nu}(x)]$$

If $Z/H$ is symmetric, the components of $\bar{Z}_{\mu}$ can be expanded & split into even & odd parts. Non-Abelian vectors:

$$\bar{x} \bar{y} = \frac{1}{2} \left[ \bar{x}, \bar{y} \right] = \frac{1}{2} \left[ \bar{x}, \bar{y} \right]$$

- $Z_{\mu} = \bar{x} \bar{y} \bar{z}$

$$\bar{x} \bar{y} = \frac{1}{2} \left[ \bar{x}, \bar{y} \right] = \frac{1}{2} \left[ \bar{x}, \bar{y} \right]$$

- $Z_{\mu} = \bar{x} \bar{y} \bar{z}$

Note: gauge components become massive.
A Hidden Local Symmetry

Any Palomin model based on G/H is gauge equivalent to a "linear" model with symmetry $G/H \times \text{H}^2/\text{H}^1$.

Let $\phi(x)$ take the value of a unitary matrix rep. of $G$ which transform under the group $G/H^2/\text{H}^1$ as

$\phi(x) \rightarrow \phi'(x) = h(x) \phi(x) h(x)^{-1}$, \hspace{1cm} $\phi(x) \in \text{G/H}$, \hspace{1cm} $h(x) \in \text{H}^2/\text{H}^1$

$\phi(x) = \phi(h(x)\phi(x)) = e^{h(x)\phi(x)h(x)^{-1}}$

\[ \prod_{\text{cos}^2 \pi c \phi(x)} X^n \cdot \sigma_0 \phi(x) \sigma_0 \phi(x)^{-1} \]

Define a Mann-Carden definition of the transformation

$\delta_\mu(x) = \frac{1}{2} \partial_{\nu} \phi(x) \phi(x)$

$\delta_\mu(x) \rightarrow \delta_\mu^\nu(x) = h(x) \phi(x) \phi(x) \frac{1}{2} \partial_{\nu} h(x)^{-1} + \frac{1}{2} \partial_{\nu} h(x)$

With projections $\mu = \nu$:

$\delta_{\mu \mu}(x) \rightarrow \delta_{\mu \mu}(x) = h(x) \phi(x) \phi(x) \frac{1}{2} \partial_{\nu} h(x)^{-1} + \frac{1}{2} \partial_{\nu} h(x)$

$\delta_{\mu \nu}(x) \rightarrow \delta_{\mu \nu}(x) = h(x) \phi(x) \phi(x) \frac{1}{2} \partial_{\nu} h(x)^{-1} + \frac{1}{2} \partial_{\nu} h(x)$

Define a compact derivation

$\delta_\mu(x) = \partial_\mu \phi(x) - \phi(x) \partial_\mu \phi(x)$

and $\delta_{\mu \nu}(x) = \partial_{\mu \nu} \phi(x) - \phi(x) \partial_{\mu \nu} \phi(x)$

So the covariant divergence of $\phi(x)$ is now

$\delta_{\mu}(x) \equiv \partial_\mu \phi(x) - \phi(x) \partial_\mu \phi(x)$

Taking $\phi = 0$

$\delta_{\mu}(x) \rightarrow \delta_{\mu}(x) = h(x) \phi(x) \phi(x) \frac{1}{2} \partial_{\nu} h(x)^{-1} + \frac{1}{2} \partial_{\nu} h(x)$

With these, two invariants can be written:

$\mathcal{L}_A = \frac{1}{2} \partial_\mu \phi(x)(\partial_\nu \phi(x))^2 = \frac{1}{2} \partial_\mu \phi(x)(\partial_\nu \phi(x) - \phi(x) \partial_\nu \phi(x))^2$

$\mathcal{L}_A = \frac{1}{2} \partial_\mu \phi(x)(\partial_\nu \phi(x))^2 = \frac{1}{2} \partial_\mu \phi(x)(\partial_\nu \phi(x))^2$

And the most general $\mathcal{L}$ made of $\phi$ and $\phi_\mu$ with lowest divergence is

$\mathcal{L} = \mathcal{L}_A + a \mathcal{L}_A$
How is the \( Z \) with \( \alpha \) equivalent to the nonlinear sigma model one?

1. Using the EOM, solution for \( V_{\mu} \):

\[
V_{\mu} = Z \text{tr} \left( S^{\sigma}_{\mu} \rho_{\sigma} \right) = Z \text{tr} \left( S^{\rho} \rho_{\rho} \right)
\]

\( \Rightarrow Z \rightarrow 0 \)

Can also be done by setting \( \alpha \rightarrow 0 \):

\[
\rho(\omega) = e^{i0/\hbar} \text{tr} \left( S^{\rho} \rho_{\rho} \right) = \rho(\omega)
\]

Then:

\[
Z = Z_{0} = \text{tr} \left( \rho_{\rho}(\omega) \right) \frac{2}{Z}
\]

\[
= \frac{2}{Z_{0}} \sum_{x, \omega} \text{tr} \left( \rho_{\rho}(\omega) \rho_{\omega} \right)
\]

\( \Rightarrow Z_{0} \omega \)

2. The gauge fixing eliminates \( H_{\mu} \) and \( \tau_{\mu} \).

\( \rho(\omega) \) is not generally preserved under global transformations.

\[
G_{\mu} = \tau_{\mu} \rightarrow G_{\mu} = \tau_{\mu} \rightarrow \pi(0, \omega) = \rho(0, \omega) = \pi(0, \omega)
\]

\( \rho(\omega) \) is not generally preserved under global transformations.

\[
G_{\mu} = \tau_{\mu} \rightarrow G_{\mu} = \tau_{\mu} \rightarrow \rho(0, \omega) = \rho(0, \omega)
\]

Now \( \rho \) and \( \tau_{\mu} \) are the same as the nonlinear sigma model one. The gauge equivalence holds with the addition of \( \rho_{\rho} \) and \( \tau_{\mu} \). The gauge fields

\[
\rho(\omega), \rho_{\rho}(\omega), \tau_{\mu}
\]

note: other gauge fields are treated similarly to \( \rho \).

\( \Rightarrow V_{\mu} \rightarrow 0 \) if \( \rho_{\rho} \)
In some cases, more bosons are needed than
provided by Hoekel or Gılmab × Hoekel
(this is true for QCD). Maybe we can
get a Gılmab × Gholm model.

Process: Start with Gılmab × Hoekel
linear model, then gauge the model
and use EOM to eliminate the
corresponding bosons, leaving us with
Gholm × Gholm.

\[ Z_{\text{1,2}(x)} \] dynamical was \( G_{\text{G} \times H} \times G_{\text{G} \times H} \)

\[ Z_{1} \rightarrow Z_{2}(x) - \text{hom}
\]
\[ G \times G \]
\[ Z_{2} \rightarrow Z_{1}(x) - \text{hom}
\]
\[ G \times G \]

Correspondingly, we'll now have four covariant
connections and 1-form components:

\[ \theta_{\mu \nu}(x) = \left( \frac{1}{2} \delta_{\mu \nu} - \epsilon_{\mu \nu \lambda} \right) \Rightarrow \theta_{\mu \nu}(x) = \theta_{\mu \nu}(x) - V_{\mu \nu}(x) \]
\[ \theta_{\mu \nu}(x) = \theta_{\mu \nu}(x) + \left[ \delta_{\mu \nu} V_{\mu \nu}(x) \right] \]
\[ \theta_{\mu \nu}(x) = \theta_{\mu \nu}(x) - \left[ \delta_{\mu \nu} V_{\mu \nu}(x) \right] \]
\[ \theta_{\mu \nu}(x) = \theta_{\mu \nu}(x) - \left[ \delta_{\mu \nu} V_{\mu \nu}(x) \right] \]

From this, there are \( G \) Gilmab × Hilmab invariants:

\[ L_{\text{1,2}} = \int_{\text{1,2}} \text{tr} \left( \theta_{\mu \nu}(x) - \theta_{\mu \nu}(x) \right)^{2} \]
\[ L_{\text{3,4}} = \int_{\text{3,4}} \text{tr} \left( \theta_{\mu \nu}(x) - \theta_{\mu \nu}(x) \right)^{2} \]

\[ L_{\text{1,2,3,4}} = \int_{\text{1,2,3,4}} \text{tr} \left( \theta_{\mu \nu}(x) - \theta_{\mu \nu}(x) \right)^{2} \]

\[ \Rightarrow Z = a \theta_{\text{1,2}} + b \theta_{\text{3,4}} + c \theta_{\text{1,2}} + d \theta_{\text{3,4}} + e \theta_{\text{1,2,3,4}} \]

\[ Z \text{ is the most general } Z \]
Now parameterize $\hat{\Sigma}(x)$ by

$$\hat{\Sigma}(x) = \delta(x) \hat{\Sigma}(0) = e^{-i \hat{\Sigma} \cdot \hat{p} \cdot \hat{X}}$$

$$\hat{\Sigma}_a \cdot \hat{\Sigma}_b = \hat{\Sigma}_a^b \cdot \hat{\Sigma}_b^a = e^{i \hat{\Sigma} \cdot \hat{p} \cdot \hat{X}}$$

Sing theannel gauge to $\hat{\Sigma}(0) = 1$, the last 3 variables transform under the residual Gubser × Gross-Strassler

$$\hat{\Sigma}(x) = \hat{\Sigma}(0) + h(\hat{\rho}(x), \hat{\eta}(x)) \hat{\Sigma}(0)$$

$$\hat{\Sigma}(0) = h(\hat{\rho}(0), \hat{\eta}(0)) \hat{\Sigma}(0)$$

$$\hat{\Sigma}(x) = h(\hat{\rho}(x), \hat{\eta}(x)) \hat{\Sigma}(0)$$

These are the $A$-form backgrounds are the same as the Gubser × Gross-Strassler model.

→ So $G_g \times H_i \times G_i$ is gauge equiv. to $G_g \ast G_i$, which in turn is """" to $G_g \times H_i$ (seen by using $\hat{\Sigma}(0) = 1$), which can also be gauge fixed to $G / H$ model.

6. The more compensating ($\equiv$ 0 cm)

we introduce, the longer hidden local symmetries that can be present.

The gauge group of hidden local symmetry can be made as large as possible using methods like above. But the dynamics of the system determine whether further introduced local symmetries become physical or just gauge freedoms to eliminate the compensators.
Dynamical Group Breaking in QCD

Consider low-energy effective $Z_2 \times P$ QCD

$$SU(2)_L \times SU(2)_R / SU(2)_W \sim (\epsilon / \theta),$$

where the global symmetry is

spontaneously broken by the Digg. Subgroup

$SU(2)_W$.

(Note: this is for massless $Z_2 \times P$ QCD.

It can be generalized to

$$SU(N) \times SU(N) / SU(N)$$

for massless

$Z_2 \times P$ QCD.)

The model is built with two $SU(2)_W$ valued variables:

$$\tilde{3}_L(x), \tilde{3}_R(x) = U(x) e^{\frac{Z_2(x) / \theta}{\sqrt{2}}}$$

Parameterizing as

$$\tilde{3}_L(x) = e^{	heta \sigma_3 / \sqrt{2} \tau},$$

and transforming as

$$\tilde{3}_L(x) \rightarrow \tilde{3}_L(x) = h(x) \tilde{3}_L(x) h^{-1}(x) \frac{g_4}{\sqrt{2}}$$

The current derivative is then

$$D_{\mu} \tilde{3}_{L,R}(x) = \partial_{\mu} \tilde{3}_{L,R}(x) + \lambda(x) \delta_{\mu}(x)$$

The $U$ forms as

$$\tilde{3}_L(x) = \frac{1}{Z_2(x) \theta / \sqrt{2}}$$

$$\lambda_{\mu}(x) \rightarrow \lambda_{\mu}(x) = h(x) \lambda_{\mu}(x) h^{-1}(x) \frac{g_4}{\sqrt{2}}$$

The are two resulting $SU(2)_L \times SU(2)_R$

$$\tilde{3}_L(x), \tilde{3}_R(x), \lambda_{\mu}(x)$$

$$J_\nu = \int dx \left( \tilde{3}_L(x) \lambda_{\mu}(x) \right)^2 + \int \left(\lambda_{\mu}(x) - \mu_0(x)\right)^2$$

$$Z_\nu = \int dx \left( \tilde{3}_L(x) \lambda_{\mu}(x) \right)^2 + \int \left(\lambda_{\mu}(x) - \mu_0(x)\right)^2$$

$$\Rightarrow \frac{\int \left( \tilde{3}_L(x) \lambda_{\mu}(x) \right)^2 + \left(\lambda_{\mu}(x) - \mu_0(x)\right)^2}{Z_\nu}$$
Unfortunately, finding a solution to the $V_{mu}$ term and fixing the gauge with $\sigma(w, x)$ changes to $\bar{x}$ and $\text{Re}(x) = \text{Re}(x) + \text{Im}(x) = \text{Im}(x)$, so

$$L = \text{Re}(\omega(x, y))^2 = |\varphi|^2$$ as before.

In this framework, it's also possible to consider a gauge group $\mathbb{G} = [SU(3) \times SU(4)]_{global}$ with the elementary gauge fields

$$L = \text{Re}(\omega(x, y))^2 = |\varphi|^2$$

So now there are two types of gauge bosons: External ($V_{mu} + A_{mu}$) and Hidden ($\bar{V}_{mu}$), which couple independently from each other.

The covariant derivatives must change to reflect this ($\bar{V}_{mu}$ will be added to only the weak gauge fields $W_{mu}$)

$$D_{\mu} = (\bar{V}_{mu} + V_{mu}) \partial_{\mu} + i \frac{1}{2} \sigma_{ab} F_{\mu a} = \bar{D}_{\mu} + V_{mu}$$

Accordingly, the $L$ becomes

$$L = L_A + \omega(x) + \text{Lin} (L_{\mu}, \bar{R}_{\mu})$$

$$L_{\text{kin}} = \frac{1}{2} (\text{Re}(\omega(x, y))^2)$$

$$L_A = \frac{1}{4} g_4 \text{Re} (\frac{(\bar{V}_{\mu a} \cdot \lambda_a - \bar{V}_{\mu a} \cdot \lambda_a)}{2})$$

Here can be associated with $\gamma, \omega, \bar{Z}$ for electroweak SU(3) x SU(2) model.
Next, assume the kinetic term of $\psi_{\mu}$ is dynamically generated via underlying QCD dynamics and add it to the field $\phi_\mu$ and $\psi_\mu$ with the p-meson field $\phi$.

In the case $\gamma_\mu \cdot P_{\mu} = \gamma_\mu$ and $\gamma_\nu \cdot P_{\nu} = \gamma_\nu$ (photons),

\[
\mathcal{L}_V = \bar{\psi}_\mu D_\mu \psi - \frac{1}{2} \left( \gamma_\mu \cdot \phi - \frac{1}{2} M^2 \phi \right) \gamma_\mu \psi - \frac{1}{2} \gamma_\mu \cdot \chi \gamma_\mu \chi - \frac{1}{2} \gamma_\mu \cdot \phi \gamma_\mu \phi - \frac{1}{4} \left( \gamma_\mu \cdot \phi \right)^2 - \frac{1}{2} \gamma_\mu \cdot \chi \gamma_\mu \chi - \frac{1}{2} \gamma_\mu \cdot \phi \gamma_\mu \phi - \frac{1}{2} \gamma_\mu \cdot \chi \gamma_\mu \chi - \frac{1}{2} \gamma_\mu \cdot \phi \gamma_\mu \phi - \frac{1}{2} \gamma_\mu \cdot \chi \gamma_\mu \chi - \frac{1}{2} \gamma_\mu \cdot \phi \gamma_\mu \phi - \frac{1}{2} \gamma_\mu \cdot \chi \gamma_\mu \chi - \frac{1}{2} \gamma_\mu \cdot \phi \gamma_\mu \phi - \frac{1}{2} \gamma_\mu \cdot \chi \gamma_\mu \chi - \frac{1}{2} \gamma_\mu \cdot \phi \gamma_\mu \phi - \frac{1}{2} \gamma_\mu \cdot \chi \gamma_\mu \chi - \frac{1}{2} \gamma_\mu \cdot \phi \gamma_\mu \phi - \frac{1}{2} \gamma_\mu \cdot \chi \gamma_\mu \chi - \frac{1}{2} \gamma_\mu \cdot \phi \gamma_\mu \phi - 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$g_{\text{eff}} = g$ (p-coupling universality)

$\mathbf{35}) \quad m_{\pi}^2 = 2 g_{\text{eff}} \, f_{\pi}^2 \quad (\text{KSEF II})$

$\mathbf{36}) \quad g_{\text{eff}} = 0 \quad (\text{p-dominance of EM force for SM type forces})$

The Lagrangian can be written again as the

$$\mathcal{L} = \sum_{i} \text{tr} \left( \frac{\bar{\psi}_i \gamma_\mu \psi_i}{2} \right)^2$$

where: In this chiral limit, $G/H = U(3) \times U(3) / U(1)_\text{em}$

$\rho \to \text{not simple, \rho} = \text{gravitons, \rho} = \text{gravitons}$

In this sense, $G/H$, the coset space, has the form $(\mathbb{S}, \mathbb{S}^*)$. So the coset space is

$C \, \left\{ \begin{array}{l} \gamma_5, \gamma_5, \gamma_5 \end{array} \right\} \Rightarrow \gamma_5 \, \left\{ \begin{array}{l} \gamma_5, \gamma_5, \gamma_5 \end{array} \right\}$

This is the chiral SU(2) case as well.

Adding ext. gauge fields $A_\mu$ and $B_\mu$ to $U(1)$

as usual:

$$D_\mu U(1) = \frac{2}{3} \mathbf{V_{(1)}}_{\mu \nu}$$

leads to

$$\mathcal{L} = \frac{1}{f_\pi^2} \left( D_\mu U(1) D^\mu U(1) \right)$$

Einstein's field equation:

$$\nabla^\mu \nabla_\mu \phi = \lambda \phi$$

$$D_\mu \phi = \frac{2}{3} \mathbf{V_{(1)}}_{\mu \nu} \phi$$

$$\mathbf{V_{(1)}}_{\mu \nu} = \mathbf{V_{(1)}}_{\mu \nu}$$
Realistically, our extended Gauge Bosons are the EW bosons & Glashow-Salam-Weyl (\( \gamma \sim B^a_{\mu}, W \sim Y^a_{\mu}, Z \sim W^0 \)). These are incorporated by today

\[
Z_{\nu} = e \left[ B_{\mu
u} - \frac{1}{2} \xi_{\mu\nu} + \frac{1}{2} \xi_{\mu\nu} T_3 \cdot Z_{\mu\nu} \right] + \frac{e}{\sin 2\theta_W} T_3 \cdot Z_{\mu\nu} + \frac{e}{\sqrt{2} \sin 2\theta_W} H_{\mu\nu}^\alpha
\]

\[
P_{\mu\nu} = e \left[ F_{\mu\nu} - \frac{1}{2} \xi_{\mu\nu} - \frac{1}{2} \xi_{\mu\nu} T_3 \cdot Z_{\mu\nu} \right]
\]

The relevant loops in \( Z \) to get masses & couplings are noted

\[
Z_{\nu\mu} = 0 \quad \text{for } \nu = B
\]

\[
Z_{\nu\mu} = (\alpha g^2_{\nu B}) \frac{1}{2} \epsilon_{\mu\nu} (\nu_{\mu} \nu_{\nu} B_{\nu}^{\mu})
\]

\[
Z_{\nu\mu} = (\alpha g^2_{\nu B}) \frac{1}{2} \epsilon_{\mu\nu} (\nu_{\mu} \nu_{\nu} B_{\nu}^{\mu})
\]

\[
Z_{\nu\mu} = (\alpha g^2_{\nu B}) \frac{1}{2} \epsilon_{\mu\nu} (\nu_{\mu} \nu_{\nu} B_{\nu}^{\mu})
\]

\[
\text{That one could recover phenomena results with specific parameter choices served as evidence (at least support) for dynamical gauge bosons in low-energy hadronic physics.}
\]

An example given is prediction of \( \beta \) constants for parity-odd processes like \( \gamma \to e^+e^- \) and \( \gamma \to 3\pi \) via anomalous low-energy theorems for \( P = 2\pi \), \( \gamma = 3\pi \), to which the Weinberg-Zimmerman Non-Abelian anomaly contributes.
General outline of the WZ anomaly example:

$[U(\mathbb{C}) \times U(\mathbb{C})]_{\text{global}} \times [U(\mathbb{C})]_{\text{local}}$, global fully gauged.

Let a transformation of the above symmetry be

$s = S_L(c_L) \cdot S_V(v) \cdot S_R(c_R)$

s.t. $s_L \rightarrow e^{iV} s_L e^{-iV}$,

$S_V = \delta V + i[V, V]$,

$S_R = \delta c_R = \delta e_R + i[\delta e, R]$.

The WZ action is

$\Gamma_{WZ}[U, \omega, \sigma] = \frac{N_c}{2(4\pi)^2} \int_M tr(\omega^2) + \text{(commutator)}$

main terms containing $L$ & $R$

$\alpha = \frac{1}{4}(\partial \omega) U^{-1} \partial \omega = \frac{1}{4}(\partial U) U^{-1}$, $U = U^+ S_R$

Then the WZ action gives a solution of the WZ anomaly eqn., i.e.

$\Gamma_{WZ}[U, \sigma, \partial \omega, \partial \sigma] = -\frac{N_c}{2(4\pi)^2} \int_M tr[\delta(\omega^2) - \frac{1}{2} \delta V^2]$

$- (L \leftrightarrow R)$

to which the sol. can be written as

$\Gamma_{WZ}[U, \sigma, \partial \omega, \partial \sigma] = \Gamma_{WZ}[U, \sigma, \omega, \partial \omega]$ + $\int_M \frac{2}{N_c} C \omega$

$\alpha_L = \frac{1}{4} D_{\sigma_L} e^{\frac{i}{2} a_L v} \alpha_L e^{\frac{i}{2} \bar{a}_L \bar{v}}$

$F_L = \partial U - igU^2$, $F_L = g L \partial e_L - g \bar{L} \partial \bar{e}_L$

other terms will be linear combinations of

$\alpha, \bar{\alpha}$.
\[ \sum_{\mu} \bar{\psi} \gamma_{\mu} D_{\mu} \psi + \frac{i}{2} \psi \left( \mathcal{L}_{1} + \mathcal{L}_{2} \right) \psi 
abla_{\mu} \mathcal{A}_{\mu} \cdot \mathbf{B} \nabla_{\mu} \mathcal{A}_{\mu} \cdot \mathbf{B} \]

where \( Z_{\mu} \) and \( R_{\mu} \) are the photon fields and \( B_{\mu} \) and \( C_{\mu} \) are the electric and magnetic fields, respectively.

If we consider the \( \mathcal{L}_{2} \) of the \( \left( \mathcal{L}_{1} + \mathcal{L}_{2} \right) \) model, one could find Feynman rules for the behavior of the vector mesons obtained by the \( \mathcal{L}_{2} \):

a) Vector Propagator
\[ \mathcal{V}_{\mu}^{\nu} = \frac{g_{\mu\nu}}{k_{\nu}^{2} - m_{v}^{2}} \]

b) Vector - photon Vertex
\[ \mathcal{V}_{\mu}^{\nu} = g_{\mu\nu} \left( \frac{\gamma_{\nu}}{k_{\nu}^{2} - m_{v}^{2}} \right) \]

c) Vector - pseudoscalar - pseudoscalar Vertex
\[ \mathcal{V}_{\mu}^{\nu} = \epsilon_{\mu\nu\alpha\sigma} \left( \frac{p_{\nu} \cdot k_{\mu}}{k_{\nu}^{2} - m_{v}^{2}} \right) \left( 2 \gamma_{\alpha} \cdot \mathbf{k} + \frac{1}{2} \gamma_{\sigma} \cdot \mathbf{k} \right) \]
Consider the two processes:

\[ V_{\mu}^\nu = \frac{g (\not{q} - \frac{P_{\mu} P_{\nu}}{m^2})}{i m q^2} (\not{q} \gamma_{\nu}) \mathcal{Z} \text{Tr}(T^a T^b) \delta^{ab} \]

\[ p^2 = 0 \quad \mathcal{Z} \text{Tr}(T^a T^b) \delta^{ab} \]

This makes that \( V_{\mu}^\nu \) act as a vector (coupled to \( V \)-\( A \) vector) in effectively replaced by the photon \( B_{\mu}^\nu \) through a factor.

If the \( V_{\mu}^\nu \) appear in a vertex form \( \mathcal{Z} \text{Tr}(F_{\mu\nu}) \), the replacement is

\[ g \text{Tr}(F_{\mu\nu}) = g \sum_a \left( \text{Tr}(M^{ab}) \right) V^{ab}_{\mu\nu} \]

\[ \Rightarrow \sum_a \left( \text{Tr}(M^{ab}) \right) \text{Tr}(T^a T^b) \delta^{ab} \]

\[ = \text{Tr}(M^{ab}) \]

In general

\[ \frac{p^2}{m^2} \frac{1}{i} \frac{1}{2} (\not{q} \gamma_{\nu}) \mathcal{Z} \text{Tr}(T^a T^b) \delta^{ab} \]

\[ \Rightarrow \frac{i}{2} \sum_a \left( \text{Tr}(M^{ab}) \right) \text{Tr}(T^a T^b) \delta^{ab} \]

\[ \Rightarrow \sum_a \left( \text{Tr}(M^{ab}) \right) \text{Tr}(T^a T^b) \delta^{ab} \]

\[ = \sum_a \left( \text{Tr}(M^{ab}) \right) \text{Tr}(T^a T^b) \delta^{ab} \]

Together, this means we can replace

\[ \sigma_{\mu} = e B_{\mu}^\nu + e (2 i \gamma^\nu) \{ \Pi_{\mu}, J_{\mu\nu} \} \]

approximately when \( V_{\mu}^\nu \) for two pseudoscalars we times two vertices.
These are the first two terms in the calculation:

\[ 2 \gamma e^{\pm} T_x (\lambda \nu \bar{\nu} \gamma_5) \]

\[ -\gamma \gamma \frac{1}{2} T_x (\bar{\nu} \gamma_5 \nu) - \bar{\gamma} \gamma \frac{1}{2} T_x (\bar{\nu} \gamma_5 \nu) \]

Which cancel the last two terms in \( T_y \).

The remaining terms have come similarly but shown not to contribute to the processes due to cancellation b/w direct terms & various meson mediated terms.

Note: it can also be shown that these do not contribute to amplitudes of \( Z \nu \to B \bar{\nu} \) or other gauge bosons.