## Qualifying Examination: January 10-11, 2011

## General Instructions

- No reference materials are allowed.
- Do all your work in the corresponding answer booklet.
- On the cover of each answer booklet put only your assigned number and the part number/subject.
- Turn in the questions for each part with the answer booklet.
- 120 minutes are allotted for each part, except for Thermal Physics (60 minutes).
- Calculator policy: Use of a graphing or scientific calculator is permitted provided it lacks ALL of the following capabilities: (a) programmable, (b) algebraic operations, and (c) storage of ascii data. Handheld computers, PDA's, and cell phones are specifically prohibited.

# Part I: Electricity and Magnetism

## Do any 5 of the 6 problems; if you try all 6, indicate clearly which 5 you want marked.

1. In a charge free region, two infinite, concentric cylinders of radii $R_1$ and $R_2$ centered on the z-axis are held at potentials:

$$V(R_1,\theta) = V_1[1 + \ln(R_1/R_2)] \text{ and } V(R_2,\theta) = V_2[1 + \cos\theta] .$$

What is the potential in the three regions:

$$0 \leq r \leq R_1 \quad R_1 \leq r \leq R_2 \quad \text{and} \quad R_2 = r < \infty \ ?$$

2. What work is done in moving a charge Q along a circular arc from the point (R, 0, 0) to the point ($\sqrt{3}$R/2, R/2, 0) against an electrical field of the form

$$\mathbf{E} = -(V_0/R^3)(2xy\, \mathbf{e}_x + x^2\, \mathbf{e}_y),$$

where $\mathbf{e}_x$ and $\mathbf{e}_y$ are unit vectors in the x and y directions?

3. An inverted hemispherical bowl of radius R carries a uniform surface charge density $\sigma$. Find the potential difference $V_{pole} - V_{center}$ between the "north pole" and the center of the circular base.

4. In an evacuated region defined by $0 < x < a$, $0 < y < a$, and $0 < z < \infty$, the electric field is given by

$$\mathbf{E}(x,y,z,t) = E_0 \sin(\pi y/a) e^{i(k_z z - \omega t)}\, \mathbf{e}_x ,$$

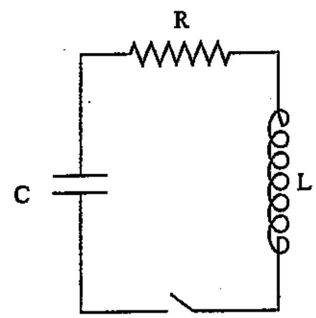
where $\mathbf{e}_x$ is the unit vector in the x direction. What must $\omega$ be as a function of the other parameters of the problem in order for **E** to satisfy the wave equation?

5. Consider a pair of parallel metallic plates each having an area A separated by a distance d. There is a constant potential difference V between the plates. A dielectric having a dielectric constant $K \equiv \varepsilon/\varepsilon_0 = 2$ fills ½ the volume between the plates.

(a) If the dielectric is parallel to the plates, find the capacitance.

(b) If the dielectric fills completely the left ½ of the volume, find the capacitance.

6. In the circuit shown below, the switch is closed at time $t = 0$ at which time the charge on the capacitor is $Q_0$. What is the current for later times t assuming that $R^2 = L/C$?



# Part II: Quantum Mechanics

## Do any 5 of the 6 problems; if you try all 6, indicate clearly which 5 you want marked.

Given: the Pauli spin matrices and the exponential integral I(n)

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$I(n) = \int_0^\infty x^n e^{-\alpha x} dx = n! / \alpha^{n+1}$$

1. In a parallel universe, it is observed that fundamental particles of charge Q are ejected from the surface of a metal illuminated with light. It is further observed that the ejected particles have a stopping potential $V_s$ (the reverse potential required to bring the most energetic ejected particles to rest) that varies linearly with the frequency of the light. Two of the data points (f, $V_s$) obtained are ($1.00 \times 10^{15}$ Hz, 5.00 V) and ($5.00 \times 10^{14}$ Hz, 1.00 V), respectively. Explain the dependence of the stopping potential on the light frequency in terms of the quantum model of light and use the above data to estimate H/Q, where H is Planck's constant in this universe.

2. A particle of mass m moves in one dimension under the influence of the potential

$$V(x) = \infty \qquad x < 0$$

$$V(x) = -\beta/x \quad x > 0,$$

where $\beta$ is a positive constant. One of the eigenfunctions for this potential is

$$\psi = A\, x\, e^{-\alpha x} \qquad x > 0,$$

where A and $\alpha$ are constants.

(a) Find the energy in this state in terms of $\hbar$, m, and $\beta$.

(b) Assume that a statistical ensemble of particles have been prepared, each in a state corresponding to the above eigenfunction. If the position of each particle were measured at some time t, what would be the average of the results?

3. A 10-eV electron propagates in the positive x-direction under the influence of a potential that is constant everywhere except for a downward step at $x = 0$. If the probability is 50% that the electron will be reflected from the step, what is the magnitude of the step?

4. The energy, magnitude of the total angular momentum, magnitude of the orbital angular momentum, and z-component of the total angular momentum are measured for an electron in a hydrogen atom. The magnitude of the total angular momentum is measured to be 2.96 $\hbar$. What are the possible outcomes for measurement of the energy, magnitude of the orbital angular momentum, and z-component of the total angular momentum? Note any correlations. Neglect the effects of the spin-orbit coupling on the electron energy.

5. Consider two non-interacting particles of mass m confined to a one-dimensional box spanning the interval $0 < x < L$.

   (a) If the energy of one of the particles were measured, what would be the possible outcomes? Give your answer in terms of m, L, and any physical and mathematical constants.

   (b) If the two particles were identical spin-0 bosons, write down the (time-independent) normalized wave function for the first excited state.

   (c) If the two particles were identical spin-1/2 fermions with total spin zero, write down the expression for the wave function describing the state such that a measurement of the total energy has a 50% probability of yielding the ground state energy, and a 10% probability of yielding the second excited state energy. (There is more than one correct way to answer this question.)

6. The time-independent Schrodinger equation for a fixed spin-1/2 particle is

$$\alpha S_z \psi = E\psi ,$$

where $\alpha$ is a constant.

   (a) What are the possible outcomes of the measurement of the energy? Give your answer in terms of $\alpha$ and any physical and mathematical constants.

   (b) Now a magnetic field along the x-direction is introduced, adding the term $\beta S_x$ to the Hamiltonian, where $\beta$ is a constant. What are the possible outcomes of the measurement of the energy in this case?

# Part IIIa: Classical Mechanics

## Do any 5 of the 6 problems; if you try all 6, indicate clearly which 5 you want marked.

1. A block of mass $m_1$ moves on a horizontal surface and is connected by a massless string over a cylindrical pulley of radius R and moment of inertia I with a second block of mass $m_2$ which is hanging vertically. The blocks are released from rest.

   (a) Use energy methods to find the common speed of the blocks after they have moved a distance L if the coefficient of kinetic friction between the surface and mass $m_1$ is $\mu_k$.

   (b) Explain under what conditions the masses will not move.

2. A particle of mass m moves along the x-axis under a force $F = -k/x^2$.

   (a) Find the potential energy, with the condition that it is zero at $x = \infty$.

   (b) Suppose that the particle is projected to the right from $x = 0$ with an initial speed $v_0$ to $x = b > 0$. Find the velocity as a function of position x.

   (c) What is the minimum initial speed $v_0$ for the particle to escape?

3. A bead of mass m slides on a smooth rigid circular wire of radius r. If the plane of the loop is vertical, and if the speed of the bead at the top of the loop is $v_0$, find:

   (a) the speed of the bead and (b) the reaction force (magnitude and direction) of the wire on the bead at the bottom of the wire. Assume the acceleration of gravity is g.

4. A particle of mass m is moving under a conservative central force F(r). Show that (a) the mechanical energy and (b) the angular momentum are conserved. That is, show that dE/dt and dL/dt are zero.

5. A block of mass m is attached to a massless spring having a spring constant k and moves on a horizontal surface. It oscillates along the x-axis about its equilibrium position at $x = 0$. There is a frictional force of constant magnitude f between the block and the surface. Suppose the mass is pulled to the right to $x = A$ and released at time $t = 0$.

(a) Find the position of the mass as it reaches the left turning point. Your answers should be expressed in terms of A, m, k and f. [One way to do this is to try a solution of the form $x(t) = C + D\cos(\omega t)$.]

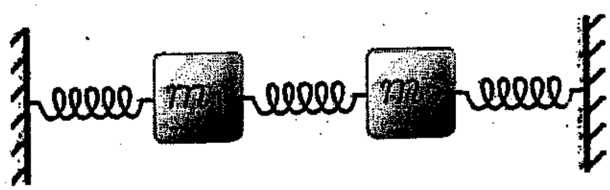
(b) Find the time it takes the mass to complete the first cycle.

(c) How much energy is lost to friction in this motion?

6. Consider two equal masses $m_1$ and $m_2$ that are connected by three identical springs having the same spring constant k as shown in the figure below. Let $x_1$ and $x_2$ be the displacements of the masses from their equilibrium positions.

(a) Find the Lagrangian in terms of coordinates $u_1 = x_1 + x_2$, and $u_2 = x_1 - x_2$.

(b) Find the corresponding Lagrange's equations of motion and the resulting frequencies of oscillation.



# Part IIIb: Thermal Physics

## Do any 2 of the 3 problems; if you try all 3, indicate clearly which 2 you want marked.

1. Two monatomic ideal gases A and B are placed in two chambers separated by a common wall. A has $2 \times 10^{23}$ particles with an initial temperature of 400 K and B has $10^{23}$ particles with initial temperature of 700 K.

   (a) If heat can pass through the common wall (and nowhere else), what is the final temperature of the gases after they reach thermal equilibrium?

   (b) If the common wall is replaced by a piston that is allowed to move and the initial volumes of A and B are 40 $cm^3$ and 50 $cm^3$, respectively, what are the final volumes of A and B?

2. Assume that the Earth's atmosphere is in a thermal steady state, radiating as much energy as it receive from the sun (averaged over a day). If the sun radiates as a blackbody with a temperature of 6,000 K, what is the average temperature of the Earth's atmosphere? Take the Earth-sun distance to be $\approx 1.4 \times 10^{11}$ m and the sun's radius to be $\approx 7 \times 10^{8}$ m.

3. An engine contains a monatomic ideal gas and operates on a three-stage cycle ABC consisting of:

   A) an isobaric expansion starting at a volume $V_1$ and temperature $T_1$ and ending at $T_2$;

   B) an isentropic expansion, which for an ideal gas is a reversible adiabatic expansion where no heat is exchanged, back to $T_1$.

   C) an isothermal contraction back to the starting volume.

   Define the engine efficiency as TotalWorkDone / Total Heat In, where the TotalHeatIn includes only the parts of the cycle during which heat flows into the gas, and excludes parts in which heat flows out. Derive the expression for the engine efficiency in terms of just $T_1$ and $T_2$.