Name:______________________________ Number:___________________________

QUALIFYING EXAMINATION

SPRING 2006

General Instructions: You will be assigned a qualifier ID number at the start of the test. This will be the same for Parts I through IV. No reference materials (other than a calculator) are permitted. Do all the work in the answer booklet. Turn in the answer booklet with ONLY your ID number on it and the test with BOTH your name and ID number on it. Ninety minutes are allowed for each part, and there will be a 30-minute break between the parts.

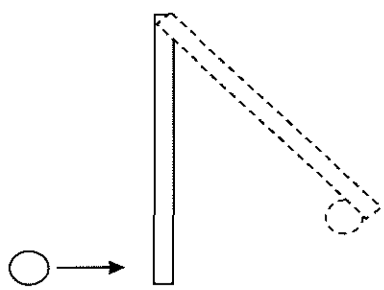
PART I: Classical Mechanics

Work any 5 of the 6 problems.

1. A block of mass m is given an initial speed v_0 up an inclined plane that makes an angle θ with respect to the horizon. The coefficient of kinetic friction between the block and the plane is μ.

   (a) How far up the incline does the block travel?
   (b) Assuming that the static coefficient of friction is small enough that the block returns down the incline. What is the speed of the block when it returns to its original position?
   (c) How long does it take for the block to go up the incline?
   (d) How long does it take for the block to come back down?

2. A ball of mass m_1 moving horizontally with speed v_0 strikes the lower end of a vertical rod of mass m_2 and length L. The rod is free to rotate about a fixed pivot at the upper end of the rod. The ball sticks to the rod during the collision and remains attached to it. Through what maximum angle does the rod rotate?



3. An object of mass m is dropped from a height h above the surface of a planet of mass M and radius R. Assume that the planet has no atmosphere so that friction can be neglected.

   (a) What is the speed of the mass just before it strikes the surface of the planet? Do not assume that h is small compared with R.
   (b) Show that this expression reduces to $v = \sqrt{2gh}$ for $h << R$.
   (c) Write down an integral expression for the time required for the object to fall to the surface for arbitrary size of h. (You don't have to evaluate the integral.)

4. A projectile is fired with initial speed v_0 at an angle θ_0 above the horizon. Assume that the air resistance varies linearly with velocity.

   (a) Derive expressions for the velocity and position of the projectile as a function of time in the horizontal (x) and vertical (y) directions.
   (b) Describe how you would use these equations to determine the time of flight and the range of the projectile over flat ground. (Don't try to obtain an explicit solution – numerical techniques or analytical approximations would be required.)

5. A particle with mass $M = 2$ kg has a potential given by $V = -2(J\text{-}m)/x + 1(J\text{-}m^2)/x^2$, where x is in meters (m) and V is in joules (J).

   (a) Find the equilibrium position of the mass.
   (b) Calculate the frequency for small oscillations about the equilibrium position.

6. A particle is subject to a central force potential $V(r)$.

   (a) Write down the Lagrangian for the particle in plane polar coordinates.
   (b) Determine the Langrangian equations of motion.
   (c) Show from these equations that the angular momentum is constant.

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PART II: Electricity and Magnetism

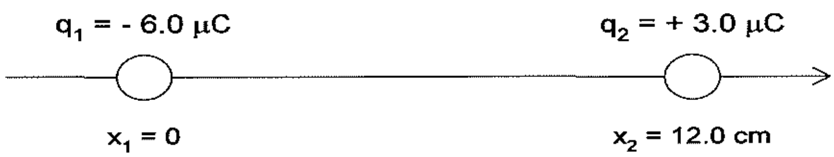
Work any 5 of the 6 problems.

1. Calculate the capitance of two concentric spherical metal shells with radii a and b, where $a<b$.

2. Two electrical charges, $q_1 = -6.0\ \mu C$ and $q_2 = +3.0\ \mu C$, are located on the x-axis at $x_1 = 0$ and $x_2 = 12.0$ cm, respectively (see figure below).

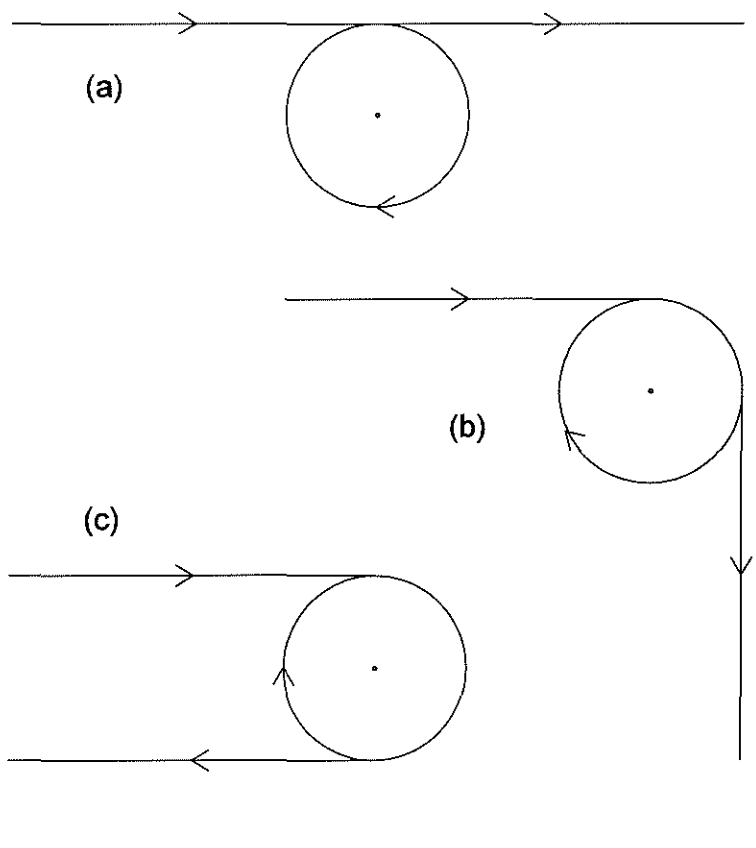
   (a) Find the points along the x-axis at which the electric potential vanishes identically.

   (b) Find the points along the x-axis at which the electric field vanishes identically.



3. A long coaxial cable carries a uniform volume charge density ρ on the inner cylinder of radius a, and a uniform surface charge density σ on the outer cylindrical shell of radius b. This surface charge density is negative and of just the right magnitude so that the cable as a whole is electrically neutral. Find the electric field **E** in each of the three regions: (i) inside the inner cylinder ($r < a$); (ii) between the cylinders ($a < r < b$); and (iii) outside the cable. Plot the magnitude of the electric field as a function of r..

4. An infinitely long conductor carrying a current I has a circular loop of radius R, as shown in the figures below. Calculate the magnetic flux density **B** at the center of the loop for the three configurations (a) – (c).

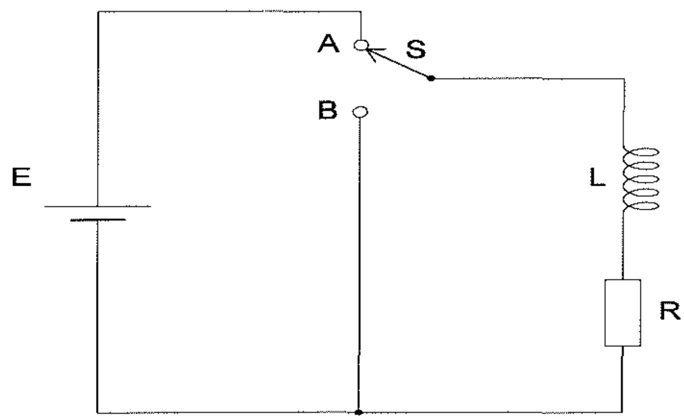


5. Suppose the circuit in the figure below has been connected for a long time when suddenly, at time $t = 0$, switch S is thrown, bypassing the battery.

(a) What is the current at any subsequent time t?

(b) What is the total energy delivered to the resistor?

(c) Show that this energy is equal to the energy originally stored in the inductor.



6. Write down the (real) electric and magnetic fields for a monochromatic plane wave of amplitude E_0, frequency ω, and vanishing phase angle that is traveling in the direction from the origin to the point (1, 1, 1), with polarization parallel to the x-y plane. Give the explicit Cartesian components of the propagation vector **k** and the unit polarization vector **n**.

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PART IV: Mixed Topics

Do problems from 4 of the 5 sections. Astrophysics can be chosen only in place of electronics. Do 6 of the 8 problems with at least 1 from each of the chosen sections.

A. Relativity

1. Neutrons from solar flares have been detected just above the Earth's atmosphere, 8 light-minutes away from their source. Free neutrons undergo β-decay with a half-life of about 615 seconds in its rest frame. Given: the neutron rest mass energy $m_n c^2$ = 939.5 MeV.

   (a) Find the minimum speed at which neutrons have to be moving for us to detect ½ of them emitted towards us from the nearest Sun-like star, α Centari A, at a distance of 4.3 light-years.

   (b) What would be the energy of such a neutron reaching our detectors?

2. An alien starship reaches our solar system on a direct linear course for Earth at a speed of 0.5c. As it passes the orbit of Neptune (distance 4.5 X 10^9 km from Earth), it sends us a modulated beam of particles that has been accelerated on-board the starship to 0.5c. How long in our reference frame does it take for us to receive this beam?

B. Thermodynamics

1. What is the coefficient of volume expansion for an ideal gas at constant pressure and at a temperature 373 K?

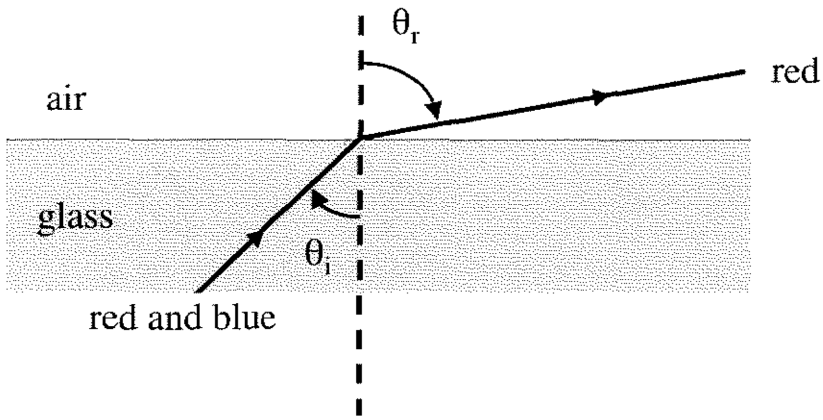
Hint: Make use of the ideal gas equation.

2. What is the entropy variation of 1000 g of water when its temperature is slowly raised from the freezing point to the boiling point? Assume that the specific heat of water (c = 4186 J/kg-K) is constant.

C. Optics

1. Red and blue light are incident on a glass-air interface, from the glass side, at an angle of incidence θ_i. The index of refraction for red light is n_{red}=1.50 and for blue light is n_{blue}=1.52. If θ_i is greater than some critical angle θ_c, the transmitted beam contains only red light.

   (a) What is the minimum angle of incidence θ_c for red light?



   (b)What is the minimum angle θ_r such that only red light emerges?

2. The headlights on automobiles are approximately 1.5m apart. If the diameter of the pupil of the eye is 3mm and an average wavelength of visible light is 500nm, what is the maximum distance at which the headlights will be resolved (appear as two separate lights) assuming that diffraction effects at the circular aperture of the eye are the limiting factors?

Hint: The solution to this problem involves an arbitrary constant, often taken to be 1.22. You need not worry about this constant. If you wish, you can set it to 1.0.

D. Astrophysics

1. A dust grain of radius $a_{grain} = 1 \times 10^{-5}$ cm resides in a molecular cloud of T = 350 K and is shielded from ultraviolet radiation. Estimate the number of excess electrons on the surface of the dust grain required to keep additional electrons from impacting and sticking to the surface. You may ignore protons and other ions there.

Hint: What is the electrostatic potential of the grain? What is the average kinetic energy of the electrons?

2. A pulsar consisting of a rotating neutron star with a mass $M = 2\ M_{sun} = 4 \times 10^{33}$ gm and radius R = 10 km has a rotational period P = 0.1 s. The period is observed to be changing with $|dP/dt| = 3.1 \times 10^{-6}$ s/year $= 1 \times 10^{-13}$ s/year. Assume that the moment of inertia is given by $I = (2/5)MR^2$.

   (a) What is the rotational energy E_0?

   (b) What is the rate, dE/dt, at which the rotational energy is decreasing?

   (c) What is the rotational period when the rotating neutron star has lost 0.99 E_0?

Various constants and quantities

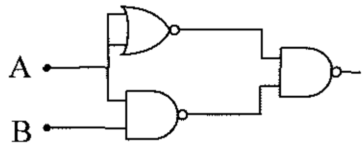
$k = 1.4 \times 10^{-16}$ erg/K, $h = 6.6 \times 10^{-27}$ erg/s, electron charge 4.8×10^{-10} esu,
$c = 3 \times 10^{10}$ cm/s, 1 eV $= 1.6 \times 10^{-12}$ erg, $m_p = 1.7 \times 10^{-24}$ gm, $m_e = 9.1 \times 10^{-28}$ gm,
$G = 6.7 \times 10^{-8}$ dyne cm^2/gm^2, $M_{sun} = 2 \times 10^{33}$ gm, Solar Luminosity $= 3.8 \times 10^{33}$ erg/s

E: Electronics

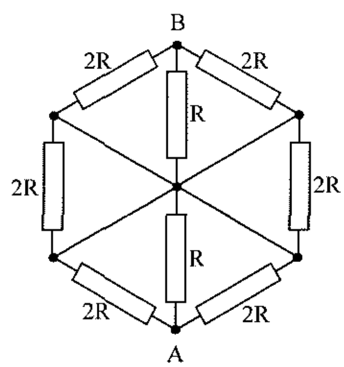
1. A. (a) Using only NOR gates construct a logic circuit that performs an OR function on the inputs A, B: the output should be true if A or B is true and false otherwise.

(b) Using only NAND gates construct a logic circuit that performs an AND Function on the inputs A, B: the output should be true if A and B are true and false otherwise.

B. The following logic circuit consists of one NOR gate and two NAND gates write down the truth table for the inputs A and B.



2. Calculate the total equivalent resistance between the points A and B in the following DC circuit:



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PART III: Quantum Mechanics

Work any 5 of the 6 problems

Formulas

$$\int_0^\infty x^n e^{-x} = n!$$

$$\int_0^\infty \frac{x^n}{(x^2+b^2)^m} dx = b^{n+1-2m} \int_0^{\frac{\pi}{2}} (\sin\theta)^n (\cos\theta)^{2m-2-n} d\theta$$

$$e^{ix} = \cos x + i \sin x$$

$$\cos(-x) = \cos(x)$$

$$\sin(-x) = -\sin(x)$$

$$\cos^2 x = \frac{1}{2}[1+\cos(2x)]$$

$$\sin^2 x = \frac{1}{2}[1-\cos(2x)]$$

$$\cos(mx)\cos(nx) = \frac{1}{2}[\cos(m+n)x + \cos(m-n)x]$$

$$\sin(mx)\sin(nx) = \frac{1}{2}[\cos(m-n)x - \cos(m+n)x]$$

$$\sin(mx)\cos(nx) = \frac{1}{2}[\sin(m+n)x + \sin(m-n)x]$$

$$\vec{S} = \frac{\hbar}{2}\vec{\sigma}$$

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$H \leftrightarrow -\frac{\hbar^2}{2m}\nabla^2 + V(x)$$

$$L^2 \leftrightarrow -\hbar^2 \left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right]$$

$$L_z \leftrightarrow \frac{\hbar}{i}\frac{d}{d\phi}$$

1. Consider a one-dimensional infinite square well ($V(x) = 0$ for $0 < x < a$, $V(x) = \infty$ for $x < 0$ or $x > a$). Derive formulas for the energy eigenvalues and eigenfunctions.

2. Particles of mass m and energy E are incident from the left on the potential barrier $V(x) = \alpha_0 \delta(x)$, where $\delta(x)$ is the Dirac delta function. In terms of E, α_0, m, and physical constants, find the probability that a particle is transmitted across the barrier. (Hint: the wave function is continuous but its first derivative is discontinuous at $x = 0$, but the discontinuity is calculable by integrating the time–independent Schrodinger equation over $(-\epsilon, \epsilon)$ and taking the limit $\epsilon \to 0$.)

3. Consider a particle of mass m confined to an infinite square well of width a. Take the potential at the bottom of the well to be $V = 0$. At t=0, its wave function is

$$\Psi = Ax(x-a)/a^2$$

for $0 < x < a$ and 0 everywhere else.

   (a) Determine A.

   (b) Calculate the expectation value of the energy for this state.

4. A particle of mass m is constrained to move in a circle of radius R lying in the x–y plane. Let the position of the particle be denoted by the angle ϕ, which is measured with respect to the positive x–axis. The classical Hamiltonian for this system is

$$H = \frac{L_z^2}{2mR^2}$$

At t=0

$$\Psi(\phi, 0) = \frac{1}{\sqrt{2\pi}}(\cos\phi - \sin\phi)$$

   (a) Calculate the probability that a measurement of L_z at $t = 0$ will yield the result $\hbar$.

   (b) Obtain $\Psi(x, t)$.

5. Consider a one–dimensional potential of the form

$$V = \alpha_0 x \quad x > 0$$

$$V = \infty \quad x < 0$$

where α_0 is a constant.

(a) Which trial variational wave function(s) from the set

$$\psi(x) = \{\frac{A}{x^2 + b^2}, Axe^{-bx}, Ae^{-bx}\}$$

(where A and b are constants) is (are) appropriate for this problem?

(b) Using an appropriate trial wave function and the variational method, estimate the ground state energy of a particle of mass m moving under the influence of this potential.

6. Consider the state described by the following column vector of coefficients of energy eigenstates:

$$\Psi(t) = \frac{1}{\sqrt{6}} \begin{pmatrix} e^{-iE_1 t/\hbar} \\ \sqrt{3}e^{-iE_2 t/\hbar} \\ \sqrt{2}e^{-iE_3 t/\hbar} \end{pmatrix}$$

where E_1, E_2, and E_3 are the allowed energies for the system. Take $E_1 = \lambda$, $E_2 = 2\lambda$, and $E_3 = 4\lambda$ where λ is a constant.

(a) If a large number (statistical ensemble) of systems were each prepared in this state and the energy of each is measured at time t, what would the mean and standard deviation of the result be? Express your answers in terms of λ.

(b) The operator $\hat{Q}$ associated with the observable Q is

$$\hat{Q} = \beta \begin{pmatrix} 0 & 3 & 0 \\ 3 & 0 & 4 \\ 0 & 4 & 0 \end{pmatrix}$$

where β is a constant. Find the possible outcomes of the measurement of Q. What is the probability that the measurement outcome will be 0 at $t = 0$?