QUALIFYING EXAMINATION

FALL 1994

General instructions. No reference materials (other than a calculator) are permitted. Do all work in your answer booklet. Turn in the questions for each part with the answer booklet. You may finish Part I early, turn it in, and start work on Part II.

DO NOT PUT YOUR NAME ON THE ANSWER BOOKLET. INSTEAD WRITE THE NUMBER THAT HAS BEEN ASSIGNED TO YOU.

PART I -- CLASSICAL MECHANICS

Work any five of the following six problems.

1. When a light spring supports a block in a vertical position, the spring is found to stretch a distance d. The block is further pulled down a distance d and released. What is its acceleration at the top of its oscillatory motion? Explain your answer.

2. a) Find the center of mass of the volume bounded by the paraboloid of revolution $z = (x^2+y^2)/b$ and the plane $z=b$. (Assume uniform density)

   b) Find the moment of inertia about the z axis in terms of the total mass m.

3. A particle of mass m moves in two dimensions under the influence of the potential

$$V(x,y) = \tfrac{1}{2}kx^2 + 2ky^2 \quad .$$

   Solve for the resulting motion and plot the trajectory of the particle given the following initial conditions at t=0:

$$x=A \qquad y=0 \qquad \dot{x} = 0 \qquad \dot{y} = v_0 \quad .$$

4. Two objects with the same mass and speed collide inelastically and move away together. If 25% of the total kinetic energy is lost during the collision, find the angle between the initial velocities of the objects.

5. A uniform spherical shell has a few turns of light string wound around its diameter. If the end of the string is held steady and the ball is allowed to fall under gravity, what is the acceleration of the center of the ball? (the moment of inertia about the center is $2MR^2/3$.)

6. a) A gun is fired straight up. Assuming that the magnitude of the air drag on the bullet of mass m is mkv^2, verify that the upward velocity as a function of height x is given by

$$v = \sqrt{\alpha e^{-2kx} - g/k} \qquad \text{(upward motion)}$$

$$v = -\sqrt{g/k - \beta e^{2kx}} \qquad \text{(downward motion)}$$

where α, β, and k are constants and g is the acceleration of gravity.

b) Use the above formulae to show that, if the initial upward velocity of the bullet was v_0, its final velocity upon returning to the ground is

$$-\frac{v_0}{\sqrt{1 + kv_0^2/g}}$$

QUALIFYING EXAMINATION
FALL 1994

Part II. ELECTRICITY AND MAGNETISM

Answer any five of the following six questions. You may use either SI or cgs units.

1. A parallel-plate capacitor consists of two circular plates of radius R separated by a distance d. A current I(t) flows into the capacitor. Find the direction and magnitude of the magnetic field B(r) at a distance r from the axis of the circular plates. Assume $r \ll R$ and $d \ll R$.

2. A spherical capacitor consists of two concentric metal spherical shells of radii a and b. They are separated by a dielectric material with dielectric constant K. Find the capacitance.

3. a) Write down the Maxwell equations for a vacuum.

   b) From the Maxwell equations, derive a relation between the electric and magnetic field amplitudes E_0 and B_0 in a sinusoidal plane electromagnetic wave. If $E_0 = 1000$ v/m , calculate B_0.

   c) Derive an expression for the average energy flux in a plane wave in terms of E_0.

4. Calculate the mutual inductance between a long straight wire and a square loop that lies in the same plane and has one of its sides parallel to the long wire. The square loop has sides of length s, and the closest side is at a distance d from the long wire.

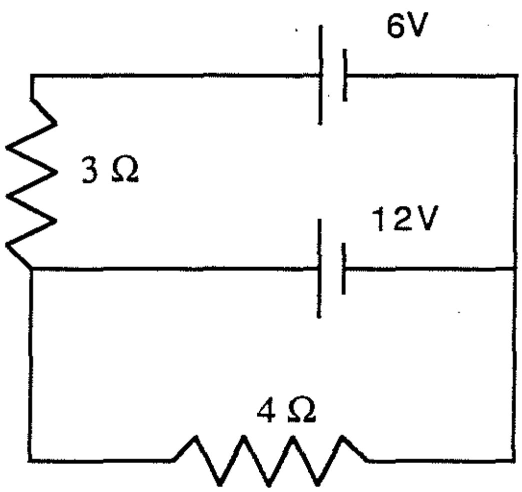
5. A proton has initial velocity v in the y direction, in a region with constant magnetic field B along the x direction.

a) Describe and sketch the trajectory of the proton, giving the dimensions (i.e., size) of the orbit.

b) What is the motion of the proton if $v = 0$?

c) If we turn on a uniform electric field E along the z direction, what is then the motion for zero initial velocity? Sketch the orbit and give the relevant dimensions.

6. In the circuit shown, calculate the current in each resistor.



PART III: QUANTUM MECHANICS
Do any five of the six problems.

1. Determine the energy levels and the normalized wave functions for a particle of mass m in a one dimensional potential well defined by

$V = 0$ for $0 < x < a$, $V = \infty$ otherwise.

2. A particle of mass m and energy E (>0) is in a one dimensional potential well defined by

$V = \infty$ for $x < 0$, $V = 0$ for $0 < x < a$, $V = V_0 > 0$ for $x > a$.

(a) Find the wave function for $0 < x < a$. Define $k^2 = 2mE/\hbar^2$.

(b) Find the wave function for $x > a$. Assume that $V_0 > E > 0$. It is not necessary to normalize the wave function, but it is necessary to find the relation between the wave functions in the two regions.

3. (a) Calculate the difference in energy between the two allowed electron spin orientations in a constant magnetic field **B**.

(b) What frequency of radiation can induce transitions (spin flips) between these two states when B = 0.5T?

4. Define in words or equations (for which you define the symbols used) the following:
(a) Pauli exclusion principle, (b) degeneracy, (c) stationary state, (d) orthonormality, (e) Zeeman effect, (f) Stark effect.

5. A particle (mass m, energy E) is incident from the left ($x<0$) on a one dimensional potential barrier defined by $V = 0$ if $x < 0$ and $V = V_0$ for $x > 0$. Show that if $E > V_0 > 0$, the transmission coefficient is given by

$$T = 4k_1k_2/(k_1 + k_2)^2.$$

k_1 and k_2 are the wave vectors for $x < 0$ and $x > 0$ respectively.

6. Discuss the normal Zeeman effect. Draw an energy level diagram showing the splitting of the P and D states of an atom in a magnetic field **B**. Show the allowed transitions on the diagram.

QUALIFYING EXAMINATION

FALL 1994

PART IV. MIXED TOPICS

Do problems from 4 of the 5 sections. Astrophysics may be chosen only in place of electronics. Do 6 of the 8 problems with at least one from each of the 4 chosen sections. Use a different answer book for each lettered section.

IV. A. Relativity

1. Measurements of the redshifts of the light emitted from two quasars Q_1 and Q_2 show that Q_1 is moving away from us (Earth) at a speed of $0.8c$ and that Q_2 is moving away from us at a speed of $0.4c$. Q_2 lies in the same direction in space as Q_1 but is closer to us than Q_1. What velocity for Q_2 would be measured by an observer on Q_1?

2. A proton synchrotron accelerates protons to a kinetic energy of 500 GeV. At this energy, calculate

(a) the Lorentz factor, γ,

(b) and the magnetic field at the proton orbit that has a radius of 750m. (The rest mass of the proton is 938.3 MeV.)

1. 10 kg of water at a temperature of 20°C is mixed with 2 kg of ice at −5°C at 1 atm of pressure until equilibrium is reached. Find the final temperature and the change in entropy of the system.
C_p (water) = 4.18×10^3 J/kg-K; C_p (ice) = 2.09×10^3 J/kg-K; Latent heat = 3.34×10^5 J/kg.

2. The free energy of a paramagnetic crystal in an external field H can be written as

$$F = U - MH - TS$$

where M is the magnetic moment and U is the internal energy. The work done in a process in which M is changed by dM is $dW = -HdM$. Neglect changes in volume.
(a) Show that

$$S = -\left(\frac{\partial F}{\partial T}\right)_H$$

$$M = -\left(\frac{\partial F}{\partial H}\right)_T$$

(b) Consider a system for which

$$F = -aT - \frac{bH^2}{2T},$$

where a and b are positive constants. Sketch M as a function of T at constant field. If the magnetic field H is increased adiabatically, will the temperature rise or fall?

IV. C. Optics

1. A beam of unpolarized light passes through a series of n perfectly efficient (linear) polarizers. If each has its plane of polarization rotated by the same amount counter-clockwise with respect to its predecessor, and the angle between the first and last is 90°, what is the emergent intensity beyond the nth polarizer?

2. At a plane interface between a medium of refractive index n and vacuum, derive the minimum angle of incidence for total internal reflection.

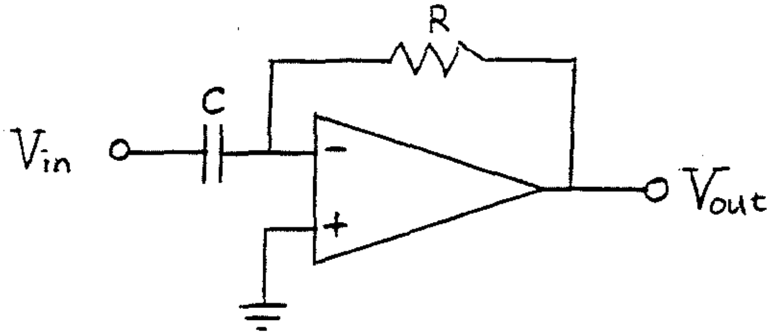
1. A comet with diameter 10 km and mass 5×10^{17} gm, held together only by its self-gravity, approaches Jupiter, which has a mass of 1.9×10^{30} gm and diameter 1.4×10^{5} km. At what distance from Jupiter will the comet fragment?

2. What is the scale height of an isothermal atmosphere surrounding a planet with mass M and radius R? Assume that the extent of the atmosphere is small compared to the size of the planet.

1a. What is the basic definition of an operational amplifier?

1b. Show that the output of the operational amplifier circuit below is given approximately by

$$V_{out} = -RC\frac{dV_{in}}{dt}$$



1c. Sketch the output waveform if the input were a symmetrical triangle wave.

2. What is the magnitude, I, and phase, φ, of the current at an arbitrary frequency ω in the following circuit? What is the maximum current, I_{max}, and at what frequency, ω_{max}, does it occur? Comment on what this condition means physically in the circuit.