Course and Section_______ Names ___________________________

Date___________ _________________________________

ROTATIONAL DYNAMICS EXPERIMENT

Introduction

In this experiment, you will measure the moment of inertia of the disk in two different ways, and compared the results. First by using the rotational apparatus, then by direct measurements of its mass and radius. The 'moment of inertia' is just the rotational equivalent of mass for the linear motion. Any mass rotating along any axis is associated to its moments differently, therefore, the moment of inertia is different along the different axes.

Equipment

Rotational dynamics apparatus, mass set, mass hanger, block mass, caliper, no-clamp pulley, smart cart, ruler.

Theory

The set up of the rotational apparatus consists of the disk mounted on top of a rotational vertical axis. Its angular acceleration α is caused by the tension in a string wrapped around a spindle and connected to a hanging mass.

The linear acceleration a of the mass m is related to the angular acceleration α of the rotational system by the equation:

$$a = \alpha r \qquad (1)$$

where r is the radius of the spindle.

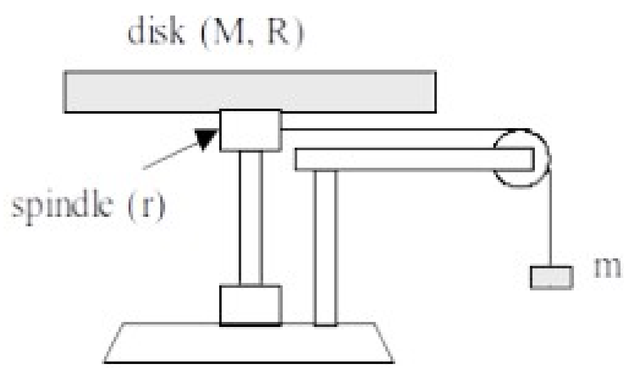

Fig 1

1. Draw below two free-body diagrams: one for the hanging mass m and the other for the rotating system (disk + rotational axis). Be precise indicating where the forces are applied on the objects.

Newton's 2nd law applied to the falling mass m yields

$$mg - T = ma\,, \quad or \quad T = m(g-a) \quad , \tag{2}$$

where T is the tension in the string. Similarly, for the rotating system, if we ignore friction in the spindle bearing

$$Tr = \frac{Ia}{r}\,, \quad or \quad T = \frac{Ia}{r^2} \quad , \tag{3}$$

where I is the moment of inertia of the rotating system, r is the spindle radius, and Tr is the torque applied by the string to the spindle. Combining Eq. (2) and (3) and ignoring the moment of inertia of the pulley yields

$$m(g-a) \;=\; \frac{I}{r^2}a \quad . \tag{4}$$

You will use the above equation to find I, by taking measurements of the angular acceleration.

Preliminary Questions

2. In the Fig 1, a disk is shown on the platform. If the disk is replaced by a hoop (hollow disk) of the same mass M and radius R, describe how the acceleration of the mass m would be effected

3. If instead, a different value of the mass m is considered, how the acceleration of the system would be effected?

Procedure

I_{CB} : Moment of Inertia of the Cart + Blocks

We use the cart to measure the angular acceleration, we start by finding the moment of inertia of the system given by the cart + block mass.

Step1. Mount the cart (not the disk) on top of the rotational axis, and place the blocks on top of it. Add a small mass on the hanger observes the rotation.

Step 2. Open the file:"**w-t.cap**" contained in the T:\Capstone folder. Use Capstone to take measurements of the angular velocity ω of the cart, and make a plot of ω vs t.

Step 3. Make a liner fit of the plot ω vs t, to find the angular acceleration α.

Step 4. Repeat for increasing values of the mass m on the hanger. NOTE: the cart cannot record angular velocities greater than 5 radian/sec. Use small values of m and consider only the initial motion.

4. Use the caliper and ruler to measure the radius of the spindle, r =__________
The linear acceleration a is obtained from equation (1).

Moment of Inertia of the Cart + Blocks

m (kg)	a (m/s^2) x-axis	$m(g\text{-}a)$ (N) y -axis

Analysis

To find the momentum of inertia I_{CB}, first notice that the equation

$$m(g-a) = \frac{I}{r^2}a$$

takes the form of a line $y = (slope) \cdot x$ in which

$$y \leftrightarrow m(g\text{-}a),\ \ (slope) \leftrightarrow \frac{I}{r^2} \text{ and } x \leftrightarrow a.$$

To find the numerical value of the slope:

Step 1. Enter your data into Excel.

Step 2. Plot *m(g-a)* versus *a*. That means *m(g-a)* on the vertical axis, *a* on the horizontal axis. Label these quantities on the graph.

Step 3. Do a linear fit, display the equation on the plot. **Print** the plot.

5. What is the numerical value of the slope?

6. Using the slope, calculate the moment of inertia of the system Cart + Blocks

$$I_{CB} = \_\_\_\_\_\_\_\_\_\_$$

I_{DCB} : <u>Moment of Inertia of the Disk + Cart + Blocks</u>

Step 1. Remove the cart from the rotational axis. Place the disk on the rotational axis and then the cart with the blocks on top of the disk. The four wheels of the cart should perfectly fit on top of the disk, such that the cart has the same angular velocity of the disk.

Step 2. Repeat the previous steps to calculate I_{DCB} and collect your data in the following table

Moment of Inertia of the Disk + Cart + Blocks

m (kg)	*a* (m/s²) *x*-axis	*m(g-a)* (N) *y* -axis

7. What is the numerical value of the slope?

8. Using the slope, calculate the moment of inertia of the system Disk + Cart + Blocks?

$$I_{DCB} = \_\_\_\_\_\_\_\_\_\_\_\_$$

I_D : <u>Moment of Inertia of the Disk</u>

Since the cart and the disk have equal rotational axis, the total moment of inertia I_{DCB} is related to I_{CB} and I_D simply as

$$I_{DCB} = I_D + I_{CB}$$

9. Use the equation above to find I_D =__________

The moment of inertia of the disk rotating around its center can also be calculated from direct measurement of its mass and radius.

10. Measure the mass of the disk (or ask the TA) M_{disk} = _______

11. Measure the radius of the disk, R_{disk} = ________

12. Calculate the momenta of inertia of the disk using the following equations

$$I'_D = \frac{1}{2} M_{disk} R_{disk}^2 = \_\_\_\_\_\_ \quad \text{(expected value)}$$

13. Calculate the percentage error of your value of I_D (question 9) and I'_D

Questions

14. What factors might account for the % differences between these values? List the possible source of errors for each method.

15. Review your answers to the preliminary questions, and explain any changes you wish to make to them after having done this experiment.